① $a^i w : \ i > 1 \ , \ w \in \{b,c\}^* , \ 2|w|_b = 3|w|_c$

$n \geq 1$? $u = aa\, b^{3n} c^{2n}$

	x	y	z	
a)	ε	a	$a\, b^{3n} c^{2n}$	? $k = 0$
b)	ε	$aa\, b^{\Delta}$ $0 \leq \Delta \leq n-2$	$b^{3n-\Delta} c^{2n}$	? $k = 0$
c)	a	$a\, b^{\Delta}$ $0 \leq \Delta \leq n-2$	$b^{3n-\Delta} c^{2n}$	? $k = 0$
d)	$aa\, b^{r}$ $0 \leq r \leq n-3$	b^{Δ} $1 \leq \Delta \leq n-2$	$b^{3n-(r+\Delta)} c^{2n}$ $r+\Delta \leq n-2$	? $k \neq 1$

} NENÍ RJ

② $a^m b^n c^{2n} \ : \ m + n \geq 2$

$n \geq 1$

? $u = a^m \ldots$

? $u = a^5 b^n c^{2n}$

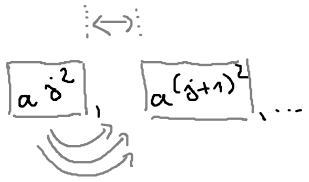
? $u = aa\, b^n c^{2n}$

? $u = b^n c^{2n}$

? $u = b^{n-1} c^{2(n+1)}$? $k \neq 1$ } $\Rightarrow$ NENÍ RJ

③ a^{i^2} : $i \geq 0$

$a^0 = \varepsilon$, $a^1 = a$, a^4, a^9, a^{16}, ...



$n \geq 1$ ∴ $w = a^{n^2}$

x	y	z
a^r	a^Δ	$a^{n^2-(r+\Delta)}$
$0 \leq r \leq n-1$	$1 \leq \Delta \leq n$	$r+\Delta \leq n$

∴ $k = 2$ $\quad x y^k z = x y^2 z = a^r \cdot (a^\Delta)^2 \cdot a^{n^2-(r+\Delta)}$ CHCEME ∉ L

POČET a:

$r + 2\Delta + n^2 - (r+\Delta) = n^2 + \Delta$

$(n)^2$ CHCEME $< n^2+\Delta <$ CHCEME $(n+1)^2$

VÍME $1 \leq \Delta \Rightarrow$ ✓

$n^2 + \Delta < (n+1)^2$ CHCEME

$n^2 + \Delta < n^2 + 2n + 1$

$\Delta <$ CHCEME $2n+1$

VÍME $\Delta \leq n \leq 2n < 2n+1$ ✓

MENÍ SE

(4) $v\,a^i\,w$: $i \geq 0$, $v, w \in \{a,b\}^*$, $|v| = 2|w|$ $[\,v \mid w\,]$

$n \geq 1$

? $u = b^{2n} \mid a \mid b^n$ (v, $i=1$, w)

x	y	z
b^r	b^s	$b^{2n-(r+s)}\, a\, b^n$
$0 \leq r \leq n-1$	$1 \leq s \leq n$	$r+s \leq n$

? k

$$x y^k z = b^r \cdot (b^s)^k \cdot b^{2n-(r+s)}\, a\, b^n$$

$$= b^{r+ks+2n-r-s}\, a\, b^n = b^{2n+s(k-1)}\, a\, b^n \overset{\text{CHCEME}}{\notin} L$$

$i=1$: $b^{2n+s(k-1)} \cdot \boxed{a} \cdot b^n$ (v, $i=1$, w)

$$|v| \overset{\text{CHCEME}}{\neq} 2|w|$$

$$2n + s(k-1) \neq 2n$$

$$s(k-1) \neq 0 \quad \Rightarrow \quad k \neq 1$$

$i=0$: $b^{2n+s(k-1)} \cdot a \cdot b^n$ $\overset{\text{CHCEME}}{\notin} L$ (v, w, 2)

CELKOVÁ DÉLKA:

$$2n + s(k-1) + 1 + n$$

$$3n + s(k-1) + 1$$

$k = 3m+1$, $m \in \mathbb{N}$, $m \geq 1$

$$3n + s(3m+1-1) + 1$$

$$3n + 3sm + 1$$

$k = 4, 7, 10, \ldots$

$\Rightarrow k = 3m+1$, $m \in \mathbb{N}$

$\Downarrow$

NENÍ RJ

⑤ $a^i b^j$: $i \neq j$

$n \geq 1$ $\quad \exists\, u = a^n b^{n+n!}$

$|u| \geq n$

$u \in L$?

$x \mid y \mid z$

a^r $\quad a^s$ $\quad a^{n-(r+s)} b^{\square}$

$0 \leq r \leq n-1$ $\quad 1 \leq s \leq n$ $\quad r+s \leq n$

$\exists\, k = 1 + \frac{n!}{s}$

$k \in \mathbb{N}_0$

$k \geq 0$

$xy^k z = a^r \cdot (a^s)^{\square} \cdot a^{n-(r+s)} b^{\square} \overset{\text{CHCEME}}{\notin} L$

POČET a: $\quad$ POČET b:

$r + s\square + n - (r+s) \overset{\text{CHCEME}}{=} \square$

$n + s\left(1 + \frac{n!}{s} - 1\right) = n + n!$ $\quad \Rightarrow$ NENÍ RJ

A, B $\quad R \subseteq A \times B$

A $\quad R \subseteq A \times A$

EKVIVALENCE

PRAVÁ KONGRUENCE

Σ, Σ^*

$\sim$

$\forall u, v \in \Sigma^* : u \sim v \Rightarrow \forall w \in \Sigma^* : uw \sim vw$

MNV

② $a^i b^j : i \neq j$

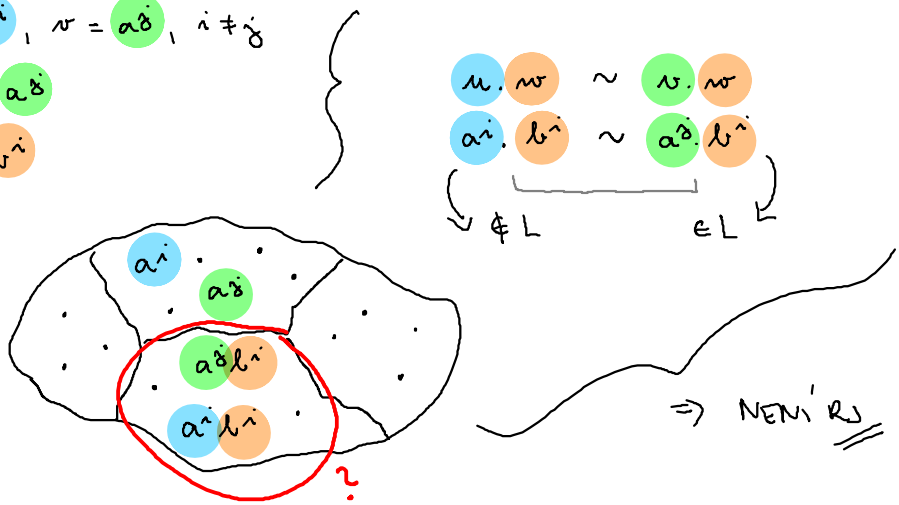
$\sim$, k

$a^1, a^2, \dots, a^k, a^{k+1}$

$u = a^i$, $v = a^j$, $i \neq j$

$a^i \sim a^j$

$w = b^i$



① $L = \{ a^i b^j : i \neq j \}$

$\overline{L} = \{a, b\}^* \setminus L \supseteq \{ a^i b^j : i = j \}$

↳ PL : $p \geq 1 \dots \exists u = a^p b^p \dots x y z \dots k \neq 1 \dots$

$\Rightarrow \overline{L}$ není RJ $\Rightarrow L$ není RJ