4.1

$$X = 01^*Y + 0X + 0$$
$$Y = 1X + 1$$

$$X = 01^*[1X+1] + 0X + 0$$
$$= 01^*1X + 01^*1 + 0X + 0$$
$$= (\underbrace{01^*1 + 0}) X + (01^*1 + 0)$$
$$= (01^*)\boxed{X} + (01^*)$$
$$X = (01^*)^* . (01^*)$$

$$Y = 1X + 1$$
$$= 1[(01^*)^*(01^*)] + 1$$
$$= 1[(01^*)^*(01^*) + \varepsilon]$$
$$\square^*\square + \varepsilon$$
$$Y = 1(01^*)^*$$

5.1

$$X = 0X + 1Y \rightsquigarrow \boxed{X = 0^* . 1Y}$$
$$Y = 1^*Z$$
$$Z = \cancel{0X + 1Y} + 0^*1^*Z + \varepsilon$$

$$= 0[0^*1Y] + 1Y + 0^*1^*Z + \varepsilon$$
$$= [00^*\boxed{1} + \boxed{1}]Y + 0^*1^*Z + \varepsilon$$
$$= 0^*\boxed{1}\,Y + 0^*1^*Z + \varepsilon$$

$$X = 0X + 1Y$$
$$Y = 1^*Z \rightsquigarrow \boxed{Y = 1^*Z}$$
$$Z = 0^*1Y + 0^*1^*Z + \varepsilon$$

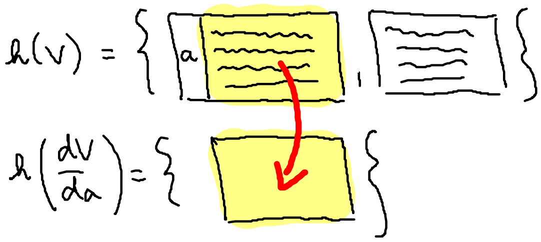
$$= 0^*1[1^*Z] + 0^*1^*Z + \varepsilon$$
$$= [0^*11^* + 0^*1^*]Z + \varepsilon$$
$$= 0^*1^*Z + \varepsilon$$

$$X = 0X + 1Y$$
$$Y = 1^*Z$$
$$Z = 0^*1^*Z + \varepsilon \rightsquigarrow Z = \ldots$$

$$Z = (0^*1^*)^* . \varepsilon = (0^*1^*)^* = (0+1)^*$$
$$Y = 1^*Z = 1^*(0+1)^* = (0+1)^*$$
$$X = 0^*1Y = 0^*1(0+1)^*$$

V

$\dfrac{dV}{da} = ?$



$x\ ,\ x \in \Sigma \qquad \dfrac{dx}{da} = \begin{cases} \dfrac{da}{da} = \varepsilon & x = a \\ \phi & x \neq a \end{cases}$

$\varepsilon \qquad \dfrac{d\varepsilon}{da} = \phi$

$\phi \qquad \dfrac{d\phi}{da} = \phi$

$r + s \qquad \dfrac{d(r+s)}{da} = \dfrac{dr}{da} + \dfrac{ds}{da}$

$r . s \qquad \dfrac{d(r.s)}{da} = \begin{cases} \dfrac{dr}{da} . s & \varepsilon \notin h(r) \\ \dfrac{dr}{da} . s + \dfrac{ds}{da} & \varepsilon \in h(r) \end{cases}$

$r^* \qquad \dfrac{dr^*}{da} = \dfrac{dr}{da} . r^*$

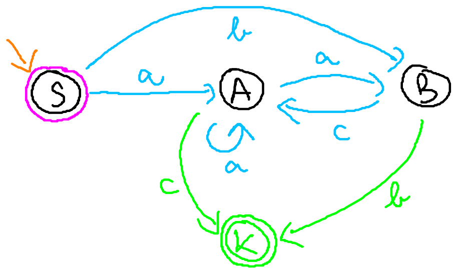
(6.1)
$$\frac{d\,\boxed{1}.\boxed{0^*1}}{d1} = \frac{d1}{d1}\cdot 0^*1 = \varepsilon\cdot 0^*1 = 0^*1$$

(6.2)
$$\frac{d\,\boxed{(01+10)}^*}{d0} = \frac{d\,(\boxed{01}+\boxed{10})}{d0}\cdot(01+10)^* = \left(\frac{d\boxed{0}\boxed{1}}{d0} + \frac{d10}{d0}\right)\cdot(01+10)^* =$$
$$= \left(\frac{d0}{d0}\cdot 1 + \frac{d1}{d0}\cdot 0\right)\cdot(01+10)^* = (\underbrace{\varepsilon.1}_{1} + \underbrace{\phi.0}_{\phi})(01+10)^* = 1(01+10)^*$$

(6.4)
$$\frac{d\ 0(01+1)^*}{d\ \boxed{0}\boxed{1}\boxed{0}\boxed{1}} = \frac{d}{d\boxed{1}}\left(\frac{d}{d\boxed{0}}\left(\frac{d}{d\boxed{1}}\left(\frac{d}{d\boxed{0}}\left(0.(01+1)^*\right)\right)\right)\right)$$
$$= \frac{d}{d1}\left(\frac{d}{d0}\left(\frac{d}{d1}\left((01+1)^*\right)\right)\right)$$
$$= \frac{d}{d1}\left(\frac{d}{d0}\left((01+1)^*\right)\right)$$
$$= \frac{d}{d1}\left(1(01+1)^*\right)$$
$$= (01+1)^*$$

(1) RG → KA

$\rightarrow S \rightarrow aA \mid bB \mid \varepsilon$
$A \rightarrow aA \mid aB \mid c$
$B \rightarrow cA \mid b$



(2) KA → RG

		0	1	2
←→	S	{S A}	{B}	{}
	A	{B}	{C}	{A}
	B		{S}	{C}
←	C	{C}	{}	

$\rightarrow S' \rightarrow 0S \mid 0A \mid 1B \mid 0 \mid \varepsilon$
$\cancel{S} \rightarrow 0S \mid 0A \mid 1B \mid 0 \mid \cancel{\varepsilon}$
$A \rightarrow 0B \mid 1C \mid 2A \mid 1$
$B \rightarrow 1S \mid 2C \mid 1 \mid 2$
$C \rightarrow 0C \mid 0$

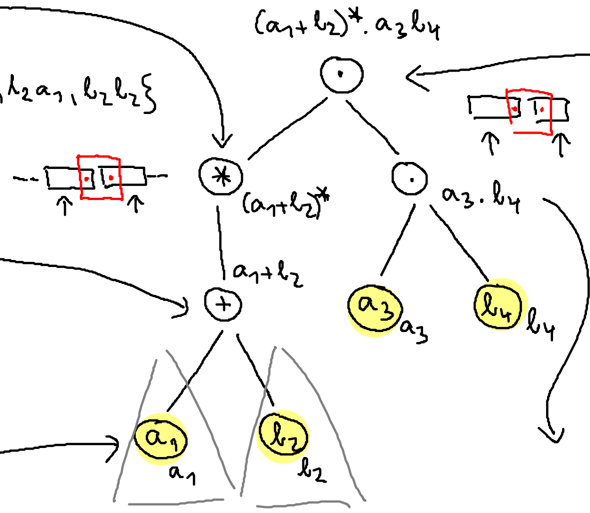
3B RV → KA

$$(a_1+b_2)^*\cdot(\underbrace{\phi^*+\varepsilon}_{\varepsilon})\cdot a_3 b_4 = (a_1+b_2)^*\cdot a_3 b_4$$

Z, K, P, E

$Z=\{a_1, b_2\}$
$K=\{a_1, b_2\}$
$P=\{a_1a_1, a_1b_2, b_2a_1, b_2b_2\}$
E = TRUE

$Z=\{a_1, b_2, a_3\}$
$K=\{b_4\}$
$P=\{a_1a_1, a_1b_2, b_2a_1, b_2b_2, a_3b_4, a_1a_3, b_2a_3\}$
E = FALSE



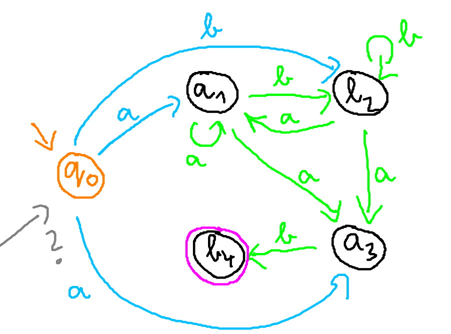
$Z=\{a_1, b_2\}$
$K=\{a_1, b_2\}$
$P=\{\}$
E = FALSE

$Z=\{a_1\}$
$K=\{a_1\}$
$P=\{\}$
E = FALSE

$Z=\{a_3\}$
$K=\{b_4\}$
$P=\{a_3b_4\}$
E = FALSE

	Z	K	P	E
a	$\{a\}$	$\{a\}$	$\{\}$	FALSE
ε	$\{\}$	$\{\}$	$\{\}$	TRUE
ϕ	$\{\}$	$\{\}$	$\{\}$	FALSE
$r+s$	$Z_r \cup Z_s$	$K_r \cup K_s$	$P_r \cup P_s$	$E_r \vee E_s$
$r.s$	↗ Z_r E_r==FALSE ↘ $Z_r \cup Z_s$	↗ K_s E_s==FALSE ↘ $K_s \cup K_r$	$P_r \cup P_s \cup \{kz \mid k\in K_r, z\in Z_s\}$	$E_r \wedge E_s$
r^*	Z_r	K_r	$P_r \cup \{kz \mid k\in K_r, z\in Z_r\}$	TRUE

$Z=\{a_1, b_2, a_3\}$
$K=\{b_4\}$
$P=\{a_1a_1, a_1b_2, b_2a_1, b_2b_2, a_3b_4, a_1a_3, b_2a_3\}$
E = FALSE



→ $S \to a\,a_1 \mid b\,b_2 \mid a\,a_3$
$a_1 \to a\,a_1 \mid b\,b_2 \mid a\,a_3$
$b_2 \to a\,a_1 \mid b\,b_2 \mid a\,a_3$
$a_3 \to b\,b_4 \mid b$
$b_4 \to$