B0B36DBS: **Database Systems** | Class 11: **Functional Dependencies**

01: Closure of a Set of FDs

```
F+ = {
   // A1 triviality
   A→A, B→B, C→C,
   AB→A, AB→B, AB→AB, AC→A, AC→C, AC→AC, BC→B, BC→C, BC→BC,
   ABC→A, ABC→B, ABC→C, ABC→AB, ABC→AC, ABC→BC, ABC→ABC,
   // Assumptions
   A→B,
   // A3 composition
   A→AB,
   // A2 transitivity
   AC→B,
   // A3 composition
   AC→AB, AC→BC, AC→ABC
}
```

02: Cover of a Set of FDs

```
F = {
   A→C, // F1
   BC→D, // F2
   C→E, // F3
   E→A // F4
}
G = {
   A→CE, // G1
   C→A, // G2
   E→AE, // G3
   AB→D // G4
}
```

Successful derivation of dependency G1 (A→CE) using all the dependencies in F

```
R1: A→C (F1)
R2: C→E (F3)
R3: A→E (R1, R2, A2 transitivity)
R4: A→CE (R1, R3, A3 composition)
```

Successful derivation of dependency G2 (C→A) using all the dependencies in F

```
R1: C→E (F3)
R2: E→A (F4)
R3: C→A (R1, R2, A2 transitivity)
```

Successful derivation of dependency G3 (E→AE) using all the dependencies in F

```
R1: E→E (A1 triviality)
R2: E→A (F4)
R3: E→AE (R1, R2, A3 composition)
```

Successful derivation of dependency G4 (AB→D) using all the dependencies in F

```
R1: AB→A (A1 triviality)
R2: A→C (F1)
R3: AB→C (R1, R2, A2 transitivity)
R4: AB→B (A1 triviality)
R5: AB→BC (R3, R4, A3 composition)
R6: BC→D (F2)
R7: AB→D (R5, R6, A2 transitivity)
```

Analogously, we also need to verify that every single functional dependency in F can be successfully derived using the dependencies in G

Conclusion: yes, F is a cover of G, as well as G is a cover of F (this relation is symmetrical)

03: Redundant FDs

```
F = {
   AC→B, // F1
   E→B, // F2
   D→C, // F3
   AC→E, // F4
   E→AC // F5
}
```

Successful derivation of dependency F1 (AC→B) using all the remaining dependencies in the original F

```
R1: AC→E (F4)
R2: E→B (F2)
R3: AC→B (R1, R2, A2 transitivity)
```

Successful derivation of dependency F2 (E→B) using all the remaining dependencies in the original F

```
R1: E→AC (F5)
R2: AC→B (F1)
R3: E→B (R1, R2, A2 transitivity)
```

Conclusion: both the dependencies F1 and F2 are redundant when assessed individually, but after one of them is removed, the other will no longer be redundant as a result (F1 was needed for the derivation of F2 and vice versa)

04: Attribute Closures

```
A+ = {
   A, // A1 triviality
   C, E // F2
}

F+ = {
   F // A1 triviality
}

BC+ = {
   B, C, // A1 triviality
   A, // F4
   D, // F1
   E // F2
}
```

```
ABF+ = {
   A, B, F, // A1 triviality
   D, // F1
   C, E // F2
}
```

Observation: ABF is a super-key (since its attribute closure contains all the attributes), but not necessarily a key

---

05: Cover of a Set of FDs

Successful derivation of dependency F1 (A→BEF) using all the dependencies in G

```
A+ = {
   A, // A1 triviality
   B, // G1
   E, // G2
   C, // G6
   F, D // G5
} ⊇ {B, E, F}
```

Analogously for all the remaining functional dependencies in F using G and vice versa

Conclusion: yes, F is a cover of G, as well as G is a cover of F

06: Redundant FDs

F1 (A→C) is not redundant since A^+ using all the remaining FDs (all except F1) does not contain C

```
A+ using F2, F3, F4 and F5 = {
   A // A1 triviality
}
```

F2 (B→A) is not redundant since B^+ using all the remaining FDs (all except F2) does not contain A

```
B+ using F1, F3, F4 and F5 = {
   B, // A1 triviality
   C // F4
}
```

F3 (D→AB) is not redundant since D^+ using all the remaining FDs (all except F3) does not contain both A and B

```
D+ using F1, F2, F4 and F5 = {
   D, // A1 triviality
   C // F5
}
```

F4 (B→C) is redundant since B^+ using all the remaining FDs (all except F4) contains C, and so F4 can be removed

```
B+ using F1, F2, F3 and F5 = {
   B, // A1 triviality
   A, // F2
   C // F1
} ⊇ {C}
```

F5 (D→C) is also redundant since D^+ using all the remaining FDs (all except F5 and F4) contains C

```
D+ using F1, F2 and F3 = {
   D, // A1 triviality
   A, B, // F3
   C // F1
} ⊇ {C}
```

Conclusion: both F4 (B→C) and F5 (D→C) were redundant and could be removed

---

07: Redundant Attributes

Attribute A is not redundant in F1 (AB→D) since attribute closure of all the remaining attributes (i.e. just B) does not contain D, and so it cannot be removed

```
B+ = {
   B // A1 triviality
}
```

Attribute B is not redundant in F1 (AB→D), and so it cannot be removed as well

```
A+ = {
   A, // A1 triviality
   C, E // F2
}
```

Conclusion: there are no redundant attributes in F1 (AB→D)

Attribute B is redundant in F6 (BCEF→A), and so F6 can be replaced with F6' (CEF→A)

```
CEF+ = {
   C, E, F, // A1 triviality
   A, // F3
   B, // F5
   D // F1
} ⊇ {A}
```

Attribute C is redundant in F6' (CEF→A), and so F6' can be replaced with F6'' (EF→A)

```
EF+ = {
   E, F, // A1 triviality
   A, // F4
   C, // F2
   B, // F5
   D // F1
} ⊇ {A}
```

Attribute E is not redundant in F6'' (EF→A), and so it cannot be removed

```
F+ = {
   F, // A1 triviality
   B // F5
}
```

Attribute F is redundant in F6'' (EF→A), and so F6'' can be replaced with F6''' (E→A)

```
E+ = {
   E, // A1 triviality
   A, // F4
   C // F2
} ⊇ {A}
```

Conclusion: attributes B, C and F were redundant in F6 (BCEF→A), and so F6 could be replaced with F6''' (E→A)

08: Minimal Cover of a Set of FDs

Solution 1

```
BC→D, BC→E, DE→B, CE→A, CE→B
```

Solution 2

```
BC→A, BC→D, BC→E, DE→B, CE→B
```

09: Minimal Cover of a Set of FDs

```
AB→C, C→A, BC→D, D→E, D→G, BE→C, CG→B, CE→G
```

10: Minimal Cover of a Set of FDs

Solution: there are no redundant attributes and nor redundant dependencies

```
AB→H, EB→C, BC→A, C→F, F→G, A→E, A→C, E→D
```

11: First Key

```
ACE
```

12: All Keys

```
ACE, ABC
```

13: All Keys

```
ADF, ABF, ACF
```

14: Normal Forms

The provided relational schema is in 3NF

```
BC→D: BCNF
BC→E: BCNF
DE→B: 3NF
CE→B: BCNF
```