NDBI001: **Query Languages I**

http://www.ksi.mff.cuni.cz/~svoboda/courses/231-NDBI001/

Lecture

Query Evaluation

Martin Svoboda

martin.svoboda@matfyz.cuni.cz

14. 11. 2023

Charles University, Faculty of Mathematics and Physics

Lecture Outline

Algorithms

- **Access methods**
- **External sort**
- **Nested loops join**
- **Sort-merge join**
- **Hash join**

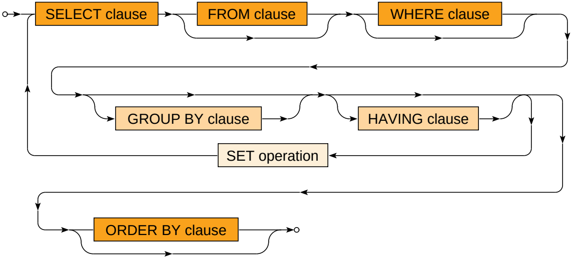
Evaluation

- **Query evaluation plans**
- **Optimization** techniques

Introduction

SQL queries

- `SELECT` **statements**



Introduction

Relational algebra

- Basic and inferred operations
  - Selection σ_φ, projection $\pi_{a_1,\ldots,a_n}$, renaming $\rho_{b_1/a_1,\ldots,b_n/a_n}$
  - Set operations: union $\cup$, intersection $\cap$, difference $\setminus$
  - Inner joins: cross join $\times$, natural join $\bowtie$, theta join $\bowtie_\varphi$
  - Left / right natural / theta semijoin $\ltimes$, $\rtimes$, $\ltimes_\varphi$, $\rtimes_\varphi$
  - Left / right natural / theta antijoin $\triangleright$, $\triangleleft$, $\triangleright_\varphi$, $\triangleleft_\varphi$
  - Division $\div$
- Extended operations
  - Left / right / full outer natural join $\leftouterjoin$, $\rightouterjoin$, $\fullouterjoin$
  - Left / right / full outer theta join $\leftouterjoin_\varphi$, $\rightouterjoin_\varphi$, $\fullouterjoin_\varphi$
  - Sorting, grouping and aggregation, distinct, …

Naïve Algorithms

Selection: $\sigma_{\varphi}(E)$

- Iteration over all tuples and removal of those filtered out

Projection: $\pi_{a_1,\ldots,a_n}(E)$

- Iteration over all tuples and removal of excluded attributes
  - But also removal of duplicates within the traditional model

Distinct

- Sorting of all tuples and removal of adjacent duplicates

Inner joins: $E_R \times E_S$, $E_R \bowtie E_S$, $E_R \bowtie_{\varphi} E_S$

- Iteration over all the possible combinations via nested loops

Sorting

- Quick sort, heap sort, bubble sort, insertion sort, …

Challenges

Blocks

- **Tuples** stored in data files are **not accessible directly**
  - Since we have **read / write operations** for **whole blocks** only
- That is true for all types of files…
  - And so not just **data files** for tables
  - But also files for **index structures** or **system catalog**

Latency

- Traditional **magnetic hard drives are extremely slow**
  - Efficient management of cached pages is hence essential

Memory

- Size of **available system memory** is always limited

$\Rightarrow$ **external algorithms** are needed

Objectives

Query evaluation plan

- Based on the **database** context and **available** memory…
  … suitable **evaluation** algorithms need to be selected…
  … so that the overall **evaluation** cost is **minimal**

Database context

- **Relational schema**: tables, columns, data types
- **Integrity constraints**: primary / unique / foreign keys, …
- **Data organization**: heap / sorted / hashed file
- **Index structures**: B^+ tree, bitmap index, hash index
- **Available statistics**: min / max values, histograms, …

Objectives

Available system memory

- **Number of pages** allocated for the **execution of a given query**
- There are two possible scenarios…
  - Having a particular **memory size**…
    - Propose its usage and estimate the evaluation cost
  - Having a particular **cost expectation**…
    - Determine the required memory and propose its usage

Evaluation algorithms

- **Access methods**
- **Sorting**: external sort approaches
- **Joining**: nested loops, merge join, and hash join approaches
- …

Objectives

Cost estimation

- Expressed in terms of **read / write disk operations**
  - Since hard drives are extremely slow, as already stated…
    - And so everything else can boldly be ignored
- We are interested in **estimates** only
  - Since it is unlikely we could provide accurate calculations
  - But still…
    - **The more accurate estimates, the better evaluation plans**
  - And there can really be huge differences in their efficiency…
    - Even up to **several orders of magnitude**!
- In other words…
  - **Query optimization is crucial for any database system**
  - As well as we also need to know what we are doing…

Available Statistics

Environment

- B: size of a block / page, usually $\approx 4\,kB$
- M: number of available **system memory** pages

Relation $\mathcal{R}$

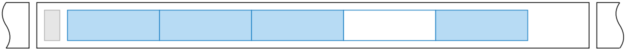
- n_R: **number of tuples**
- s_R: average / fixed tuple size
- $b_R \approx \lfloor B/s_R \rfloor$: **blocking factor**
  - Number of tuples that can be stored within one block
- $p_R \approx \lceil n_R/b_R \rceil$: **number of blocks**
- $V_{R.A}$: cardinality of the **active domain** of attribute A
  - Number of distinct values of A occurring in $\mathcal{R}$
- $min_{R.A}$ and $max_{R.A}$: minimal and maximal values for A

Access Methods

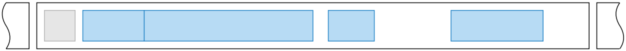
Data Files

Internal structure

- **Blocks** of data files for tables are **divided into** **slots**
  - Each slot is intended for storing exactly one tuple
    - By the way, they can easily be uniquely identified
    - Using a pair of **block and slot logical ordinal numbers**
- **Fixed-size slots**
  - Usage status of each slot just needs to be remembered



- **Variable-size slots**
  - When at least one variable-size attribute is involved
  - Slot **beginnings** and **lengths** need to be remembered



Access Methods

Access method

- Particular approach for **finding the intended tuples**
  - I.e., reading blocks with such tuples into the system memory
    - Directly from **data files** for tables
    - But also indirectly using **index structures**
- **Full scan** (**sequential read**) is possible under all circumstances
  - However, we can do better in certain cases based on…
    - Involved **selection conditions**
    - Particular **data file organization**
    - Available **index structures** (if any)
  - I.e., number of blocks to be read can significantly be reduced
    - And so the evaluation cost
    - Since only relevant blocks are considered instead all of them

Access Methods

Data file organization

- **Heap file**, **sorted file**, **hashed file**

Index structures

- **B^+ tree**, …

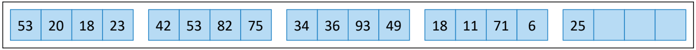
Selection conditions

- **Equality tests** with respect to **unique / non-unique attributes**
  - $A = v$, where v is a particular value (not another attribute)
- **Range queries** for **one-sided / two-sided intervals**
  - $v_1 \leq A$, $A \leq v_2$, and $(v_1 \leq A) \land (A \leq v_2)$
    - Analogously for other comparison operators ($\geq$, $<$, $>$)
    - As well as their mutual combinations in two-sided intervals
    - However, only fixed boundary values are assumed again
- …

Heap File

Heap file

- Tuples are put into individual slots entirely **arbitrarily**
  - I.e., we do not have any specific knowledge of their position



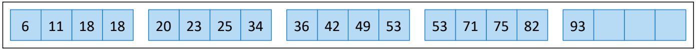
Selection costs

- **Full scan** is inevitable in **almost all situations**
  - $c = p_R$
- **Equality test** with respect to a **unique attribute**
  - $c = \lceil p_R/2 \rceil$
    - Since we can stop at the moment a given tuple is found
    - However, uniform distribution of data and queries is assumed
    - And values outside of the active domain may also be queried

Sorted File

Sorted file

- Tuples are **ordered** with respect to a particular attribute



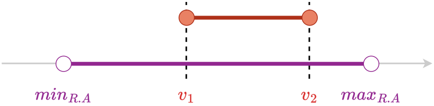
Selection costs

- Binary search (**half-interval search**) can be used in general
  - However, only when the same attribute is queried, of course
    - I.e., the same attribute as the one used for sorting
    - Otherwise, sequential read as in a heap file would be needed
- **Equality test**
  - $c = \lceil \log_2 p_R \rceil$ for a **unique** attribute
  - $c = \lceil \log_2 p_R \rceil + \lceil p_R / V_{R.A} \rceil$ for a **non-unique** attribute

Sorted File

Selection costs (cont'd)

- **Range query** for two-sided intervals $[v_1, v_2]$ and other

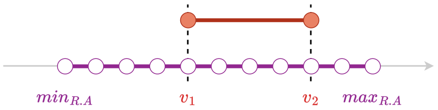


- For "**continuous**" domains…
  - Number of values between any two of them is not limited
    - At least potentially
    - In practical terms, there can simply be far too many of them
    - E.g.: `FLOAT`, `VARCHAR`, …
  - $c = \lceil \log_2 p_R \rceil + \lceil p_R \cdot (v_2 - v_1)/(max_{R.A} - min_{R.A}) \rceil$
    - Boundary types (inclusive / exclusive) are unimportant

Sorted File

Selection costs (cont'd)

- **Range query** for two-sided intervals $[v_1, v_2]$ and other



- For "**discrete**" domains…
  - Number of values between any two of them is finite
    - E.g.: `INTEGER`, `CHAR`, `DATE`, …
  - $c = \lceil \log_2 p_R \rceil + \lceil p_R \cdot (v_2 - v_1 + \varepsilon)/(max_{R.A} - min_{R.A} + 1) \rceil$
    - ε is 1 for closed intervals, -1 for open (unless $v_1 = v_2$), and 0 otherwise, i.e., half-open and zero-sized open

Sorted File

Selection costs (cont'd)

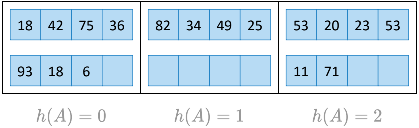
- **Range query** for one-sided intervals $(-\infty, v_2]$ and $(-\infty, v_2)$

  - $c = \lceil p_R \cdot (v_2 - min_{R.A})/(max_{R.A} - min_{R.A}) \rceil$
  - $c = \lceil p_R \cdot (v_2 - min_{R.A} + \varepsilon)/(max_{R.A} - min_{R.A} + 1) \rceil$
- **Range query** for one-sided intervals $[v_1, \infty)$ and (v_1, ∞)
  - Analogously…
  - $c = \lceil p_R \cdot (max_{R.A} - v_1)/(max_{R.A} - min_{R.A}) \rceil$
  - $c = \lceil p_R \cdot (max_{R.A} - v_1 + \varepsilon)/(max_{R.A} - min_{R.A} + 1) \rceil$

Hashed File

Hashed file

- Tuples are put into disjoint **buckets** (logical groups of blocks)
  - Based on a selected **hash function** over a particular attribute
    - E.g., $h(A) = A \bmod 3$



- **Hash function**
  - Its **domain** are values of a given attribute A
  - Its **range** provides H distinct values
    - There is exactly one bucket for each one of them
    - All tuples in a bucket always share the same hash value

Hashed File

File statistics

- H_R: **number of buckets**
- $C_R \approx \lceil p_R / H_R \rceil$: expected **bucket size**
  - Measured as a number of blocks in a bucket

Selection costs

- **Equality test** when the hashing attribute is queried
  - Only the corresponding bucket needs to be accessed
  - $c = C_R$ for a **non-unique** attribute
  - $c = \lceil C_R / 2 \rceil$ for a **unique** attribute
    - Similar assumptions as in the case of heap files
- Any other condition
  - $c = p_R$
    - I.e., **full scan** is needed

B^+ Tree Index

B^+ tree index structure = **self-balanced search tree**

- **Logarithmic height** is guaranteed (the same across all leaves)
- Moreover, very **high fan-out** is assumed
  - I.e., our trees will tend to be significantly **wider than taller**
    - $\Rightarrow$ **search times** will not only be **logarithmic**, but also really low

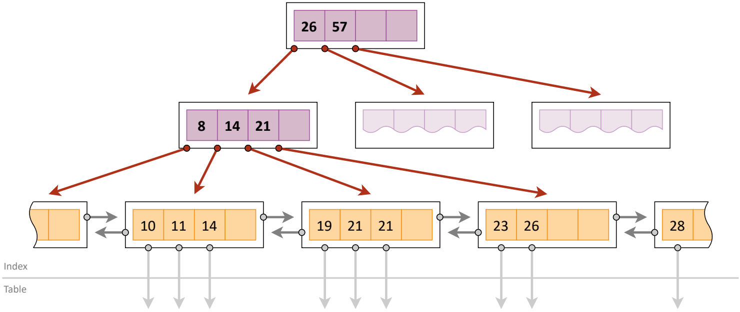
Logical structure

- **Internal node** (including a non-leaf root node)
  - Contains an ordered sequence of **dividing values** and **pointers to child nodes** representing the **sub-intervals** they determine
- **Leaf node**
  - Contains individual values and **pointers to tuples** in data file
  - Leaves are also interconnected by pointers in both directions

$\mathbf{B^+}$ Tree Index

$\mathbf{B^+}$ tree index structure (cont'd)

- Sample index for relation $\mathcal{R}$ and its attribute A



B^+ Tree Index

Physical structure

- **Each node** is physically represented by **one index file block**
  - And so they are treated the same way as data file blocks
    - I.e., loaded into the system memory one by one, etc.

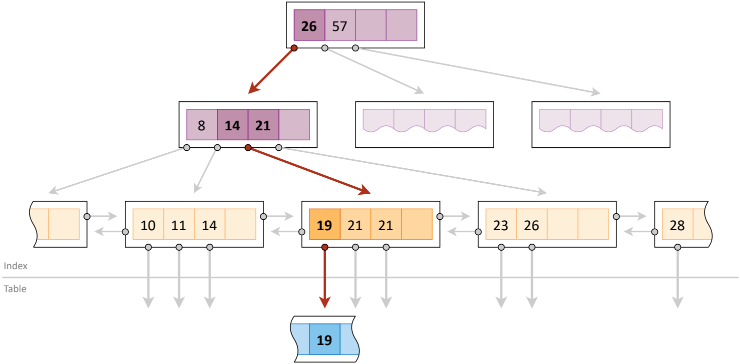
Index statistics

- $m_{R.A}$: maximal **number of children** (order of tree)
  - Usually up to small hundreds in practice
  - Actual number is guaranteed to be at least $\lceil m_{R.A}/2 \rceil$
    - Except for the root node
- $I_{R.A}$: **index height**
  - Usually just $\approx 2-3$ for typical real-world tables
- $p_{R.A}$: number of **leaf nodes**

B$^+$ Tree Index

Search algorithm

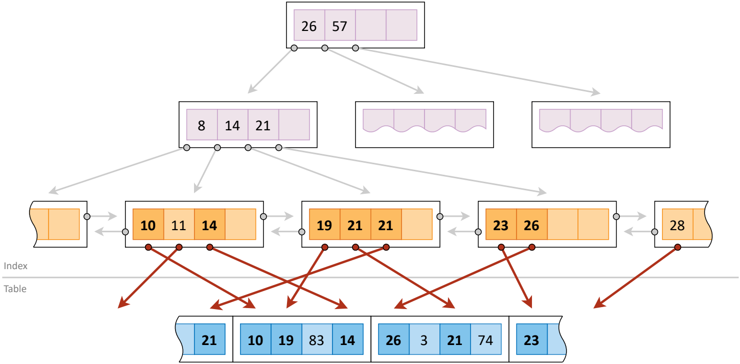
- Index is traversed from its root toward the corresponding leaf
  - Data tuple then needs to be fetched from the data file



Non-Clustered B$^+$ Tree Index

Non-clustered **index**

- Order of items within the leaves and data file is not the same
  - I.e., data file is organized as a **heap file** of **hashed file**



Non-Clustered B$^+$ Tree Index

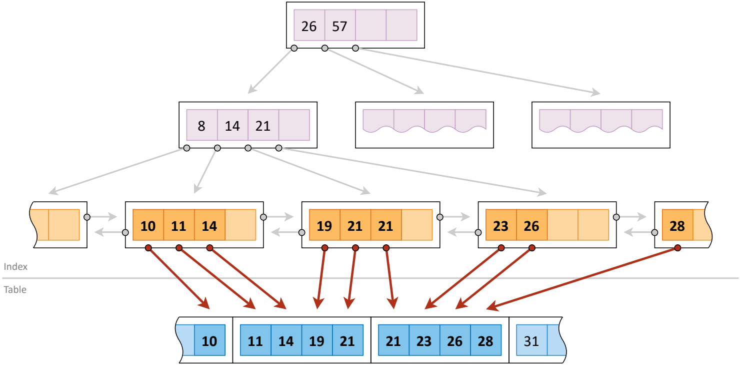
Selection costs

- **Equality test** for a **unique / non-unique** attribute
  - $c = I_{R.A} + 1$
  - $c = I_{R.A} + \lceil p_{R.A} / V_{R.A} \rceil + \min(p_R, \lceil n_R / V_{R.A} \rceil)$
- **Range query** for two-sided intervals $[v_1, v_2]$ and other
  - $c = I_{R.A} + \lceil p_{R.A} \cdot (v_2 - v_1)/(max_{R.A} - min_{R.A}) \rceil +$
    $\min(p_R, \lceil n_R \cdot (v_2 - v_1)/(max_{R.A} - min_{R.A}) \rceil)$
    - Analogously for discrete domains
- However, for small domains $V_{R.A}$ or large intervals…
  - Full scan of the data file is better
    - I.e., index is not utilized at all
- Conditions not involving the indexed attribute
  - Full scan again, of course

Clustered B$^+$ Tree Index

Clustered **index**

- On the contrary, order of items is (at least almost) the same
  - I.e., data file is a **sorted file** (with respect to the same attribute)



Clustered B$^+$ Tree Index

Selection costs

- **Equality tests**
  - $c = I_{R.A} + 1$ for a **unique** attribute
  - $c = I_{R.A} + \lceil p_R / V_{R.A} \rceil$ for a **non-unique** attribute
- **Range query** for two-sided intervals $[v_1, v_2]$ and other
  - $c = I_{R.A} + \lceil p_R \cdot (v_2 - v_1)/(max_{R.A} - min_{R.A}) \rceil$
    - Analogously for discrete domains
- **Range query** for one-sided intervals
  - Data file is read directly as an ordinary sorted file
- Conditions not involving the indexed attribute
  - Full scan again, of course

Examples

Sample scenario #1

- **Movie** (<u>id</u>, title, year, …)
  - Basic statistics
    - $n_M = 100\,000$ tuples, $b_M = 10$, $p_M = 10\,000$ blocks
    - $V_{M.id} = n_M = 100\,000$ values (since they are unique)
  - Heap file
  - Sorted file (using ids)
  - Hashed file
    - $h(M.id) = M.id \bmod 50$
    - $H_M = 50$ buckets, $C_M = 200$ blocks
  - B$^+$ tree index (using ids)
    - $m_{M.id} = 100$ followers
    - $I_{M.id} = 3$, $p_{M.id} = 1\,500$ blocks

Examples

Equality test: movie with a particular identifier

- Heap file
  - $c = \lceil p_M/2 \rceil = 5\,000$
- Sorted file
  - $c = \lceil \log_2 p_M \rceil = 14$
- Hashed file
  - $c = \lceil C_M/2 \rceil = 100$
- Non-clustered index (B$^+$ tree & heap file)
  - $c = I_{M.year} + 1 = 4$
- Clustered index (B$^+$ tree & sorted file)
  - $c = I_{M.year} + 1 = 4$

Examples

Sample scenario #2

- **Movie** (id, title, year, …)
  - Basic statistics
    - $n_M = 100\,000$ tuples, $b_M = 10$, $p_M = 10\,000$ blocks
    - $V_{M.year} = 50$ values
    - $min_{M.year} = 1943$, $max_{M.year} = 2022$ (i.e., 80 values)
  - Heap file
  - Sorted file (using years)
  - Hashed file
    - $h(M.year) = M.year \bmod 20$
    - $H_M = 20$ buckets, $C_M = 500$ blocks
  - B^+ tree index (using years)
    - $m_{M.year} = 100$ followers
    - $I_{M.year} = 3$, $p_{M.year} = 1\,500$ blocks

Examples

Equality test: movies filmed in a particular year

- Heap file
  - $c = p_M = 10\,000$
- Sorted file
  - $c = \lceil \log_2 p_M \rceil + \lceil p_M / V_{M.year} \rceil = 214$
- Hashed file
  - $c = C_M = 500$
- Non-clustered index (B$^+$ tree & heap file)
  - $c = I_{M.year} + \lceil p_{M.year} / V_{M.year} \rceil + \min(p_M, \lceil n_M / V_{M.year} \rceil)$ $= 2\,033$
- Clustered index (B$^+$ tree & sorted file)
  - $c = I_{M.year} + \lceil p_M / V_{M.year} \rceil = 203$

Examples

Range query: movies filmed during years $[y_1 = 2016, y_2 = 2020]$

- Heap file
  - $c = p_M = 10\,000$
- Sorted file
  - Let $r \leftarrow (y_2 - y_1 + 1)/(max_{M.year} - min_{M.year} + 1) = 5/80$
  - $c = \lceil \log_2 p_M \rceil + \lceil p_M \cdot r \rceil = 639$
- Hashed file
  - $c = p_M = 10\,000$
- Non-clustered index (B$^+$ tree & heap file)
  - $c = I_{M.year} + \lceil p_{M.year} \cdot r \rceil + \min(p_M, \lceil n_M \cdot r \rceil) = 6\,347$
- Clustered index (B$^+$ tree & sorted file)
  - $c = I_{M.year} + \lceil p_M \cdot r \rceil = 628$

External Sort

External Sort

N-way external merge sort

- **Sort phase** (**pass 1**)
  - Groups of input blocks are loaded into the system memory
  - Tuples in these blocks are then sorted
    - Any **in-memory** **<u>in-place</u>** **sorting algorithm** can be used
    - E.g.: *quick sort*, *heap sort*, *bubble sort*, *insertion sort*, …
  - Created initial **runs** are written into a temporary file
- **Merge phase** (**passes 2 and higher**)
  - Groups of runs are loaded into the memory and merged
  - Newly created (longer) runs are written back on a hard drive
  - Merging is finished when exactly one run is obtained
    - And so the entire input table is sorted

Sort Phase

Pass 1

- Input data file
  - **Relational table $\mathcal{R}$**
    - E.g., $n_R = 18$ tuples, $b_R = 4$ tuples/block, $p_R = 5$ blocks

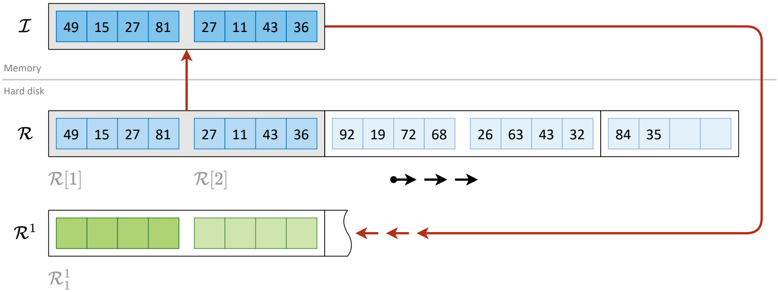
	$\mathcal{R}[1]$	$\mathcal{R}[2]$	$\mathcal{R}[3]$	$\mathcal{R}[4]$	$\mathcal{R}[5]$
$\mathcal{R}$	49, 15, 27, 81	27, 11, 43, 36	92, 19, 72, 68	26, 63, 43, 32	84, 35, ,

- System memory layout
  - **Input buffer $\mathcal{I}$**
    - E.g., size $M = 2$ pages

Sort Phase

Pass 1

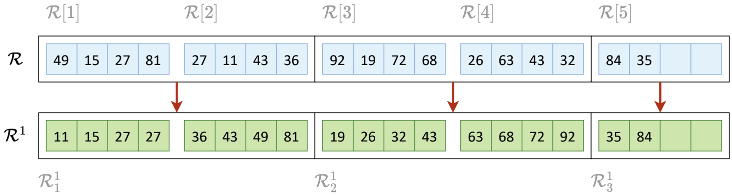
- **Groups of M blocks** are presorted and so **initial runs** created
  - Input blocks from $\mathcal{R}$ are first loaded to $\mathcal{I}$
    - Individual tuples in $\mathcal{I}$ are then sorted
    - Created runs are stored to a temporary file $\mathcal{R}^1$



Sort Phase

Pass 1

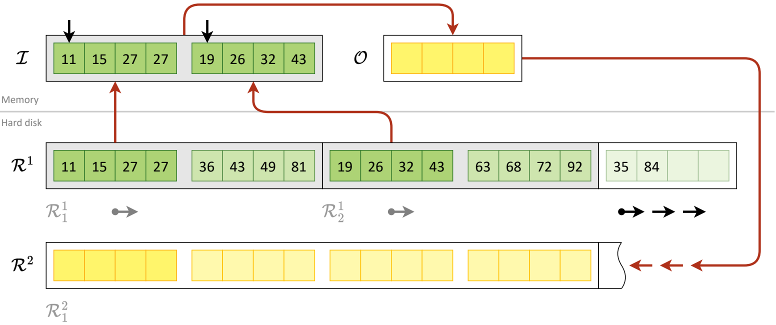
- **Resulting runs** in $\mathcal{R}^1$ within our sample scenario



Merge Phase

Pass 2

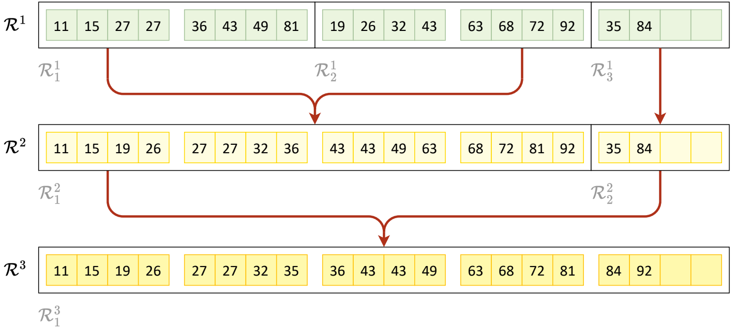
- **Groups of M runs** are iteratively **merged together**
  - Blocks from these input runs are gradually loaded into $\mathcal{I}$
    - Minimal items are then iteratively selected and moved to $\mathcal{O}$
    - Merged (longer) runs are written to a new temporary file $\mathcal{R}^2$



Merge Phase

Passes 2 and 3

- Merging continues until just a single run is acquired
  - And so the entire input table is sorted



Algorithm

Sort phase (pass 1)

```
p ← 1
foreach group of blocks B_1, ..., B_M (if any) from R do
    read these blocks to I
    sort all items in I
    write all blocks from I as a new run to R^p
```

Algorithm

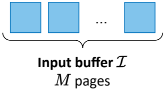
Merge phase (passes 2 and higher)

while $\mathcal{R}^p$ has more then just one run **do**
    $p \leftarrow p + 1$
    foreach group of runs $R_1, \ldots, R_M$ (if any) from $\mathcal{R}^{p-1}$ **do**
        start constructing a new run in $\mathcal{R}^p$
        read the first block from each run R_x to $\mathcal{I}[x]$
        while $\mathcal{I}$ contains at least one item **do**
            select the minimal item and move it to $\mathcal{O}$
            if the corresponding $\mathcal{I}[x]$ is empty **then**
                read the next block from R_x (if any) to $\mathcal{I}[x]$
            if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{R}^p$ and empty $\mathcal{O}$
        if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{R}^p$ and empty $\mathcal{O}$

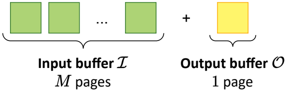
Summary

Memory layout

- Sort phase (**pass 1**): M
  - **Input buffer $\mathcal{I}$**: M pages



- Merge phase (**passes 2 and higher**): $M + 1$
  - **Input buffer $\mathcal{I}$**: $M \geq 2$ pages
  - **Output buffer $\mathcal{O}$**: 1 page



Summary

Time complexity

- **Single pass** (regardless of the phase)
  - $c_{\texttt{read}} = c_{\texttt{write}} = p_R$
- **Number of passes**
  - $t = \lceil \log_M(p_R) \rceil$
- **Overall cost**
  - $c_{\texttt{ES}} = t \cdot (c_{\texttt{read}} + c_{\texttt{write}}) = \lceil \log_M(p_R) \rceil \cdot 2p_R$

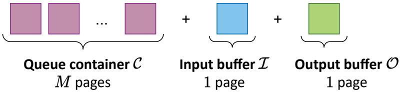
Limitation of the overall **number of passes**

- In general…
  - $M = \lceil \sqrt[t]{p_R} \rceil$
- Specifically for $t = 2$ (i.e., **exactly 2 passes**)
  - $M = \lceil \sqrt{p_R} \rceil$

Improved Approach

N-way external merge sort with priority queue

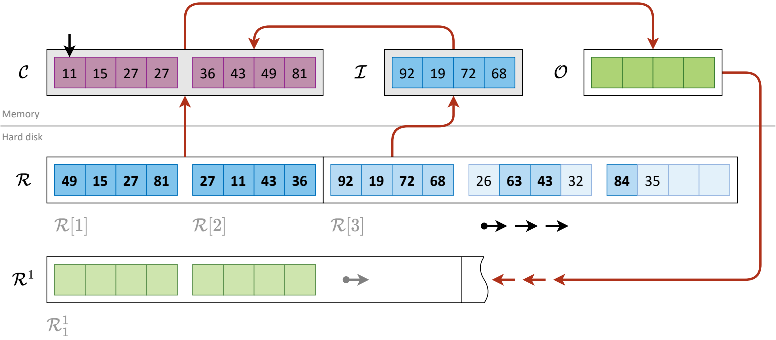
- **Sort phase** is modified
  - Instead of fixed-size initial runs…
  - … we generate them using a **priority queue**
    - In particular, **min-heap data structure** is used
  - The aim is to **make the initial runs longer**
- **Memory layout**: $M + 1 + 1$
  - **Queue container** $\mathcal{C}$: $M \geq 1$ pages
  - **Input buffer** $\mathcal{I}$: 1 page
  - **Output buffer** $\mathcal{O}$: 1 page



Sort Phase

Pass 1

- Once the queue is initialized, **runs are generated on the fly**
  - Minimal item greater than or equal to the last value is always extracted and replaced with another item from the input file



Sort Phase

Pass 1 (cont'd)

- Two runs are obtained in our scenario

$\mathcal{R}^1$	11	15	19	27	27	36	43	43	49	63	68	72	81	84	92		26	32	35	
	$\mathcal{R}^1_1$																$\mathcal{R}^1_2$			

Impact summary

- Created initial **runs will tend to be longer**
  - $2M$ blocks on average (instead of just M)
    - p_R in the best case
    - M in the worst case
- $\Rightarrow$ **number of the runs will tend to be lower**

Algorithm

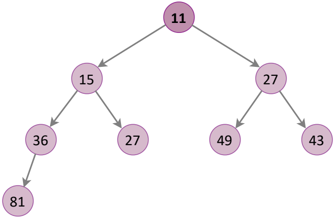
Improved **sort phase** (pass 1)

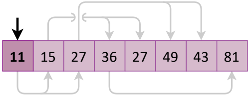
read blocks $\mathcal{R}[1], \ldots, \mathcal{R}[M]$ (if any) from $\mathcal{R}$ to $\mathcal{C}$
read block $\mathcal{R}[M+1]$ (if any) from $\mathcal{R}$ to $\mathcal{I}$
while $\mathcal{C}$ contains at least one item **do**
    start constructing a new run in $\mathcal{R}^1$, put $v \leftarrow -\infty$
    while $\mathcal{C}$ contains at least one item $i \geq v$ **do**
        let i be the minimal one, move i to $\mathcal{O}$, put $v \leftarrow i$
        move the next item from $\mathcal{I}$ (if any) to $\mathcal{C}$
        if $\mathcal{I}$ is empty **then**
            read the next block from $\mathcal{R}$ (if any) to $\mathcal{I}$
        if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{R}^1$ and empty $\mathcal{O}$
    if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{R}^1$ and empty $\mathcal{O}$

Priority Queue

Min-heap **data structure**

- Complete binary tree
  - Key associated with each node must be less than or equal to keys of all its child nodes
    - I.e., the **root node** contains the **minimal item** among them all
- **Array representation** is possible
  - Using a straightforward index arithmetic

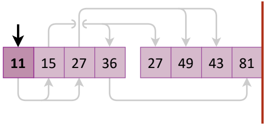




Queue Container

Queue container $\mathcal{C}$

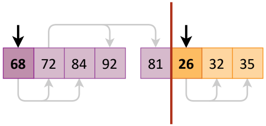
- **Two separate min-heap structures** are in fact used
  - Active **heap** with items greater than or equal to the last value
    - And so values that can still be (actually all really will be) used in the currently constructed run
  - Inactive **heap** with items not satisfying the condition
- Both are **represented as arrays**
  - Directly inside the container blocks
- **Container initialization** (line 1)
  - Active heap is built from the input items, inactive heap is empty



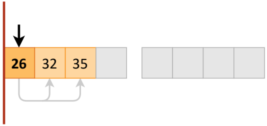
Queue Container

Queue container $\mathcal{C}$ (cont'd)

- Whenever an **item is added to the container** (line 7)
  - It is added to the active / inactive heap based on the condition



- Whenever the **active heap is fully depleted** (line 5)
  - I.e., the current run terminated, both the heaps are swapped



Nested Loops Join

Nested Loops

Binary nested loops

- **Universal approach** for all types of **inner joins**
  - Natural join, cross join, theta join
    - I.e., arbitrary joining condition can be involved
  - Support possible duplicates
  - Requires no index structures
- Not the best option in all situations, though
  - Suitable for tables with significantly different sizes

Basic idea

- **Outer loop**: iteration over the blocks of the **first** table
- **Inner loop**: iteration over the blocks of the **second** table

Nested Loops

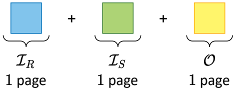
Sample input data

- Tables $\mathcal{R}$ and $\mathcal{S}$ to be joined using a **value equality** test

$\mathcal{R}$	**21** 84 **56** 19	41 **72** 69 **35**	**56** 84		
$\mathcal{S}$	31 **56** 75 43	88 **21** 43 14	92 52 25 81	**72** 37 64 **35**	14 64

Basic setup

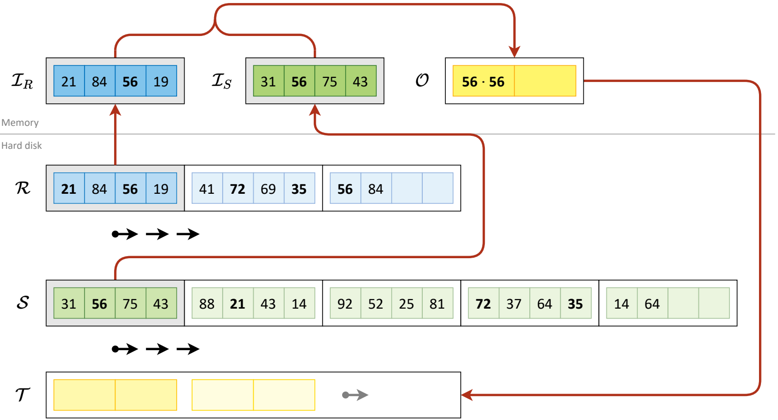
- Memory layout: $1 + 1 + 1$
  - **Input buffer** $\mathcal{I}_R$: 1 page
  - **Input buffer** $\mathcal{I}_S$: 1 page
  - **Output buffer** $\mathcal{O}$: 1 page



Nested Loops

Basic setup $(1 + 1 + 1)$

- Another pair of loops is used for joining tuples in the memory



Algorithm

Basic setup $(1 + 1 + 1)$

```
foreach block $R$ from $\mathcal{R}$ do
    read $R$ into $\mathcal{I}_R$
    foreach block $S$ from $\mathcal{S}$ do
        read $S$ into $\mathcal{I}_S$
        foreach item $r$ in $\mathcal{I}_R$ do
            foreach item $s$ in $\mathcal{I}_S$ do
                if $r$ and $s$ satisfy the join condition then
                    join $r$ and $s$ and put the result to $\mathcal{O}$
                    if $\mathcal{O}$ is full then write $\mathcal{O}$ to $\mathcal{T}$, empty $\mathcal{O}$
if $\mathcal{O}$ is not empty then write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$
```

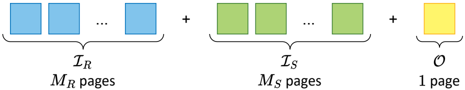
Observations

Time complexity

- Basic setup $(1 + 1 + 1)$
  - $c_{\texttt{NL}} = p_R + p_R \cdot p_S$
- $\Rightarrow$ **smaller table** should always be taken as the **outer** one

General setup

- Multiple pages are used for both the input buffers
- Memory layout: $M_R + M_S + 1$
  - **Input buffer $\mathcal{I}_R$**: M_R pages
  - **Input buffer $\mathcal{I}_S$**: M_S pages
  - **Output buffer $\mathcal{O}$**: 1 page



Algorithm

General setup ($M_R + M_S + 1$)

foreach group of blocks $R_1, \ldots, R_{M_R}$ (if any) from $\mathcal{R}$ **do**
    read these blocks into $\mathcal{I}_R$
    foreach group of blocks $S_1, \ldots, S_{M_S}$ (if any) from $\mathcal{S}$ **do**
        read these blocks into $\mathcal{I}_S$
        foreach item r in $\mathcal{I}_R$ **do**
            foreach item s in $\mathcal{I}_S$ **do**
                if r and s satisfy the join condition **then**
                    join r and s and put the result to $\mathcal{O}$
                    if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$, empty $\mathcal{O}$

if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$

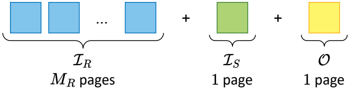
Observations

Time complexity

- General setup ($M_R + M_S + 1$)
  - $c_{\texttt{NL}} = p_R + \lceil p_R / M_R \rceil \cdot p_S$
- $\Rightarrow$ there is no reason of having $M_S \geq 2$

Standard setup

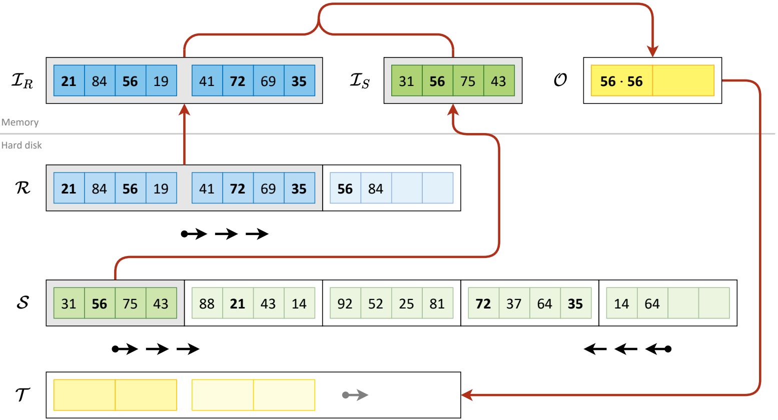
- Memory layout: $M_R + 1 + 1$
  - **Input buffer $\mathcal{I}_R$**: M_R pages
  - **Input buffer $\mathcal{I}_S$**: 1 page
  - **Output buffer $\mathcal{O}$**: 1 page



Standard Approach

Standard setup ($M_R + 1 + 1$) with **zig-zag** optimization

- Multiple pages are used just for the outer table



Observations

Zig-zag optimization

- Reading of the inner table $\mathcal{S}$
  - **Odd iterations** normally
  - **Even iterations** in reverse order

Time complexity

- Standard setup ($M_R + 1 + 1$)
  - $c_{\texttt{NL}} = p_R + \lceil p_R / M_R \rceil \cdot p_S$ (without zig-zag)
  - $c_{\texttt{NL}} = p_R + \lceil p_R / M_R \rceil \cdot (p_S - 1) + 1$ (with zig-zag)

Special Cases

Very small tables

- Smaller table fits entirely within the memory, i.e., $p_R \leq M_R$
  - $c_{\mathtt{NL}} = p_R + p_S$

Non-brute-force replacement for **inner loops**

- **B$^+$ tree index exists** in S on attribute A that is unique in S
  - $c_{\mathtt{NL}} = p_R + n_R \cdot (I_{S.A} + 1)$
    - If **R is organized as a heap**
  - $c_{\mathtt{NL}} = p_R + I_{S.A} + p_{S.A} + V_{R.A}$
    - If **R is sorted** with respect to A
- **S is a hashed file** over attribute A that is unique in S
  - $c_{\mathtt{NL}} = p_R + V_{R.A} \cdot C_S$
    - If **R is sorted** with respect to A
- …

Non-Binary Nested Loops

Non-binary nested loops

- Nested loops algorithm for **multiple tables at once**
  - In particular, let us have tables $\mathcal{R}_1, \ldots, \mathcal{R}_n$ for $n \geq 2$, $n \in \mathbb{N}$
    - Let their sizes be $p_1, \ldots, p_n$
- Solution
  - We just need to embed more loops into each other
- **Memory layout**: $M_1 + \cdots + M_n + 1$
  - **Input buffers** $\mathcal{I}_i$: M_i pages for each table $\mathcal{R}_i$
  - **Output buffer** $\mathcal{O}$: 1 page
- **Overall cost** with **zig-zag** optimization
  - $c_{\mathtt{NL}} = \Big(p_1\Big) + \Big(\lceil p_1/M_1 \rceil \cdot (p_2 - M_2) + M_2\Big) + \cdots + \Big(\lceil p_1/M_1 \rceil \ldots \lceil p_{n-1}/M_{n-1} \rceil \cdot (p_n - M_n) + M_n\Big)$

Memory Setup

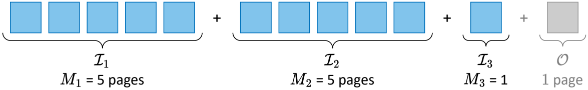
Memory layout: $M_1 + \cdots + M_n + 1$

- Optimization problem
  - Finding integer M_i **minimizing the overall cost** $c_{\texttt{NL}}$
- **Heuristics**
  - Let $M \geq n$ be all the available pages (for input buffers)
  - Let $p_1 \leq \cdots \leq p_n$ (without loss of generality)
  - Allocate **one page** for the **innermost table**, i.e., $M_n = 1$
  - Allocate the **remaining pages uniformly** to $\mathcal{R}_1, \ldots, \mathcal{R}_{n-1}$
    - I.e., let $m = \lfloor (M-1)/(n-1) \rfloor$
    - Then put $M_i = m$ for each $i \in \{1, \ldots, n-1\}$
    - It may happen that some pages will still be unallocated
    - There will be exactly $u = (M-1) - (n-1) \cdot m$ of them
    - Assign these **remaining pages** (if any) between **smaller tables**
    - I.e., $M_i \mathrel{+}= 1$ for each $i \in \{1, \ldots, u\}$

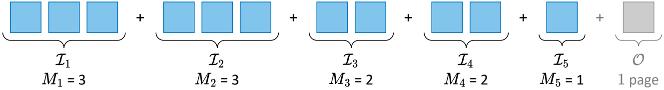
Memory Setup

Memory layout (cont'd)

- Example #1
  - $n = 3$ tables, $M = 11$ pages (for input buffers)
  - Allocation: $\langle 5, 5, 1 \rangle$



- Example #2
  - $n = 5$ tables, $M = 11$ pages
  - Allocation: $\langle 3, 3, 2, 2, 1 \rangle$



Sort-Merge Join

Sort-Merge Join

Sort-merge join algorithm (or just **merge join**)

- Inner joins based on **value equality tests** only
  - Basic version **without duplicates**
    - Could be extended to support them, though
- Suitable for tables with relatively similar sizes
  - Especially when they are already sorted
  - Or when the final result is expected to be sorted

Phases

- **Sort phase**
  - Both tables are externally sorted, one by one (if not yet)
- **Join phase**
  - Items are joined while simulating the merge of the two tables

Basic Approach

Sample input data

- **Input tables $\mathcal{R}$ and $\mathcal{S}$**

$\mathcal{R}$	65	**19**	35	**92**	**49**	31														
$\mathcal{S}$	52	94	38	71	**92**	41	63	**19**	75	54	46	68	15	27	22	43	11	50	**49**	

Sort phase

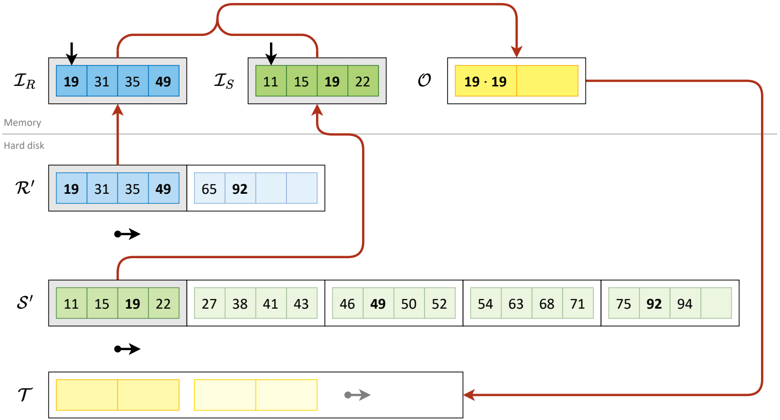
- Resulting sorted tables

$\mathcal{R}'$	**19**	31	35	**49**	65	**92**														
$\mathcal{S}'$	11	15	**19**	22	27	38	41	43	46	**49**	50	52	54	63	68	71	75	**92**	94	

Basic Approach

Join phase

- Blocks from the sorted tables are processed one by one



Algorithm

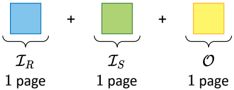
Join phase

read block $\mathcal{R}'[1]$ to $\mathcal{I}_R$ and block $\mathcal{S}'[1]$ to $\mathcal{I}_S$
while both $\mathcal{I}_R$ and $\mathcal{I}_S$ contain at least one item **do**
    let r be the minimal item in $\mathcal{I}_R$ and s minimal item in $\mathcal{I}_S$
    if r and s can be joined **then**
        join r and s and put the result to $\mathcal{O}$
        if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$
        remove both r from $\mathcal{I}_R$ and s from $\mathcal{I}_S$
    else remove the lower one of r from $\mathcal{I}_R$ or s from $\mathcal{I}_S$
    if $\mathcal{I}_R$ is empty **then** read the next block from $\mathcal{R}'$ (if any)
    if $\mathcal{I}_S$ is empty **then** read the next block from $\mathcal{S}'$ (if any)
if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$

Observations

Join phase

- Memory layout: $1 + 1 + 1$
  - **Input buffer** $\mathcal{I}_R$: 1 page
  - **Input buffer** $\mathcal{I}_S$: 1 page
  - **Output buffer** $\mathcal{O}$: 1 page



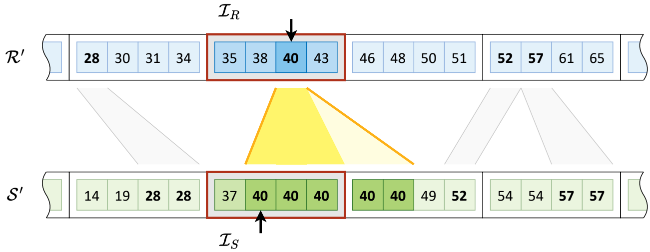
Time complexity

- Sort phase
- **Join phase**
  - $c_{\mathtt{MJ}} = p_R + p_S$

Extended Version

Duplicate items

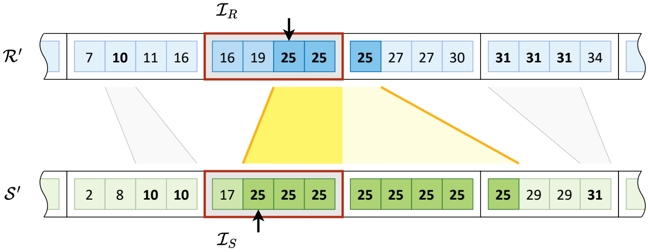
- Possible duplicates in **one table** only
  - Let it be $\mathcal{S}$ (without loss of generality)
  - Algorithm modification is straightforward…
    - Having successfully joined r and s, we just remove s from $\mathcal{I}_S$ and not r from $\mathcal{I}_R$ (line 7)



Extended Version

Duplicate items

- Possible duplicates in **both tables**
  - All matching pairs of r and s just need to be joined…
  - Unfortunately, **size of input buffers** might not be sufficient
    - Since we may span block boundaries, even repeatedly



Integrated Approach

2-pass integrated sort-merge join with priority queue

- **Sort phase** (**pass 1**)
  - Tables are processed one by one
    - They are not sorted entirely, though
  - **Only <u>initial runs</u> are constructed**
    - Using just the **sort phase** (pass 1) of the **external sort** algorithm
    - **Priority queue** is involved to make these runs longer
    - And so their overall number lower
- **Join phase** (**pass 2**)
  - The same idea as in the basic sort-merge approach
    - We only have more runs within each presorted table

Integrated Approach

Sort phase (pass 1)

- Resulting initial runs within tables $\mathcal{R}^1$ and $\mathcal{S}^1$

$\mathcal{R}^1$

19	31	35	**49**	65	**92**		

$\mathcal{R}^1_1$

$\mathcal{S}^1$

19	38	41	46	52	54	63	68	71	75	**92**	94

$\mathcal{S}^1_1$

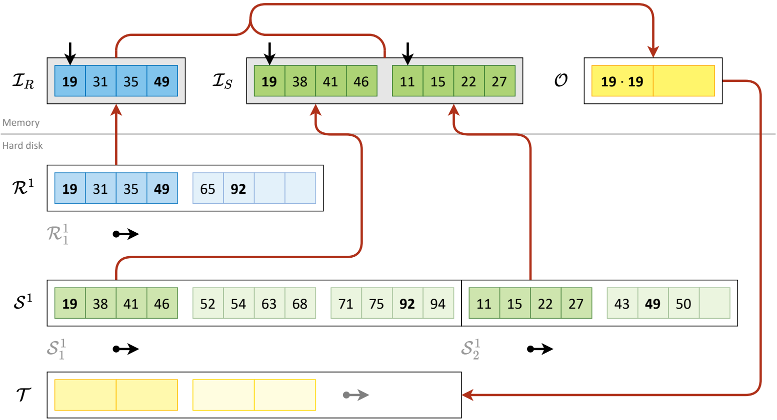
11	15	22	27	43	**49**	50	

$\mathcal{S}^1_2$

Integrated Approach

Join phase (pass 2)

- All runs from both the tables $\mathcal{R}^1$ and $\mathcal{S}^1$ are merged at once



Algorithm

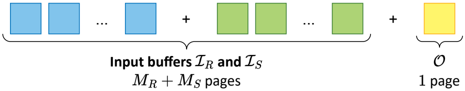
Join phase (pass 2)

read $\mathcal{R}^1_x[1]$ from each run in $\mathcal{R}^1$ to $\mathcal{I}_R[x]$, the same for $\mathcal{S}^1$
while both $\mathcal{I}_R$ and $\mathcal{I}_S$ contain at least one item **do**
    let r be the minimal item in $\mathcal{I}_R$ and s minimal item in $\mathcal{I}_S$
    if r and s can be joined **then**
        join r and s and put the result to $\mathcal{O}$
        if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$
        remove both r from $\mathcal{I}_R$ and s from $\mathcal{I}_S$
    else remove the lower one of r from $\mathcal{I}_R$ or s from $\mathcal{I}_S$
    if the given $\mathcal{I}_R[x]$ is empty **then** refill it from $\mathcal{R}^1_x$
    if the given $\mathcal{I}_S[x]$ is empty **then** refill it from $\mathcal{S}^1_x$
if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$

Observations

Join phase (pass 2)

- Memory layout: $M_R + M_S + 1$
  - **Input buffer $\mathcal{I}_R$**: M_R pages $=$ **number of runs** in $\mathcal{R}^1$
  - **Input buffer $\mathcal{I}_S$**: M_S pages $=$ **number of runs** in $\mathcal{S}^1$
  - **Output buffer $\mathcal{O}$**: 1 page



Time complexity

- Sort phase: $c_{\texttt{sort}} = 2p_R + 2p_S$
- Join phase: $c_{\texttt{join}} = p_R + p_S$
- **Overall cost**: $c_{\texttt{MJ}} = c_{\texttt{sort}} + c_{\texttt{join}} = 3(p_R + p_S)$

Observations

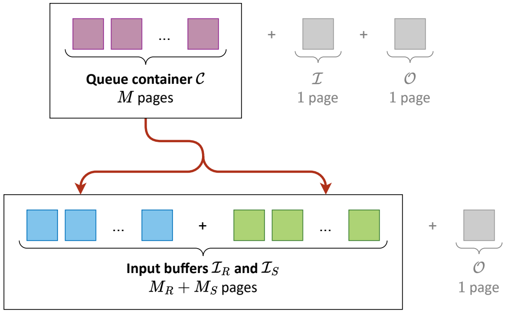
Optimized setup

- Motivation
  - Balanced memory usage across both phases
- **Sort phase** (pass 1)
  - Required memory: $M + 1 + 1$ pages
  - Let $M = \lceil \sqrt{p} \rceil$, where $p = \max(p_R, p_S)$
    - As if we wanted 2 passes for the external sort
  - If M **pages** are used for the **priority queue container**…
    - Expected **length of initial runs** should be $2M$
    - And so the expected **number of all runs** $p_S/2M + p_R/2M \leq p/2M + p/2M \approx 2p/2M = p/M \approx p/\sqrt{p} \approx \sqrt{p} \approx M$
- **Join phase** (pass 2)
  - Required memory: $M_R + M_S + 1$ pages
  - $\Rightarrow M_R + M_S \approx M$

Observations

Optimized setup (cont'd)

- In other words…
  - **The same number of M pages** should be sufficient for both…
    - **Queue container $\mathcal{C}$** during **pass 1**, and
    - **Input buffers $\mathcal{I}_R$** and $\mathcal{I}_S$ during **pass 2**



Hash Join

Hash Join

Hash join approaches

- Basic principle
  - Items of the first table are hashed into the system memory
  - Items of the second table are then attempted to be joined
- Limitations
  - Inner joins based on **value equality tests** only
    - Including possible duplicates
  - Not suitable for small **active domains**
- Particular approaches
  - **Classic** hash join, **Simple** hash join, **Partition** hash join, **Grace** hash join, and **Hybrid** hash join

Classic Hashing

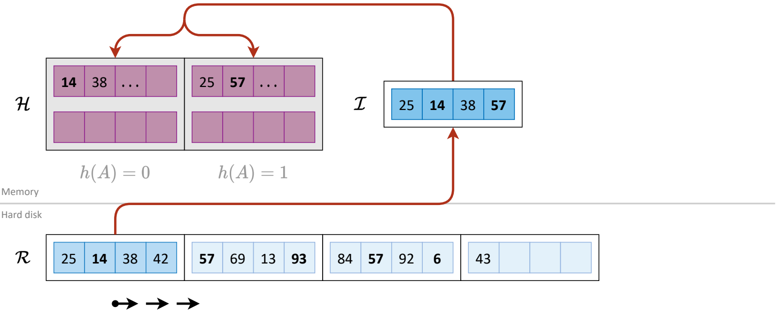
Classic hash join

- **Build phase**
  - **Smaller table** (let it be $\mathcal{R}$) is **hashed into the system memory**
    - I.e., it is sequentially loaded into the memory, block by block
    - All its tuples are then emplaced into the **hash container**
- **Hash function** h is assumed for this purpose
  - Its **domain** are values of the joining attribute A
  - Its **range** provides H distinct values
- **Hash container** internally contains H **buckets**
  - Its **overall size** will inevitably be somewhat larger than p_R
    - Let us say $M = \lceil F \cdot p_R \rceil$ pages for some small factor F
- **Probe phase**
  - Items from the **larger table** $\mathcal{S}$ are **attempted to be joined**

Build Phase

Build phase

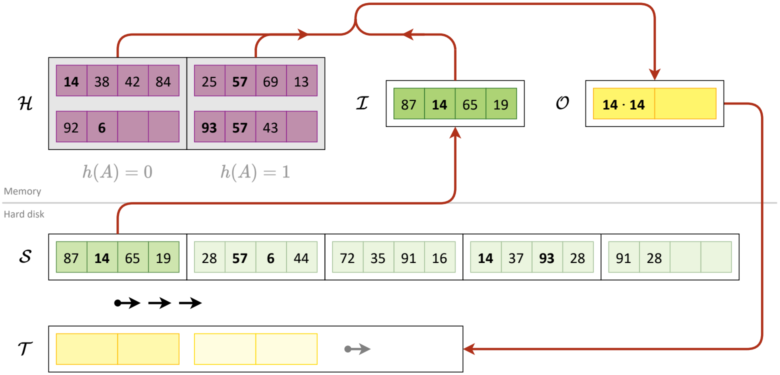
- Tuples from the smaller table are **hashed** into the memory
  - E.g., hash function $h(A) = A \bmod 2$ is assumed



Probe Phase

Probe phase

- Tuples from the larger table are attempted to be **joined**



Algorithm

Build phase

foreach block R from $\mathcal{R}$ **do**
    read R into $\mathcal{I}$
    foreach item r in $\mathcal{I}$ **do**
        calculate hash value $h \leftarrow h(r.A)$
        add r into bucket h in $\mathcal{H}$

Algorithm

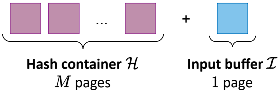
Probe phase

foreach block S from $\mathcal{S}$ **do**
    read S into $\mathcal{I}$
    foreach item s in $\mathcal{I}$ **do**
        calculate hash value $h \leftarrow h(s.A)$
        foreach item r in bucket h in $\mathcal{H}$ **do**
            if r and s can be joined **then**
                join r and s and put the result to $\mathcal{O}$
                if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$
if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$

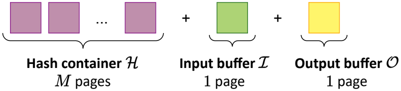
Observations

Memory layout

- Build phase: $M + 1$
  - **Hash container $\mathcal{H}$**: $M = \lceil F \cdot p_R \rceil$ pages
  - **Input buffer $\mathcal{I}$**: 1 page



- Probe phase: $M + 1 + 1$
  - **Hash container $\mathcal{H}$**: M pages (preserved from the build phase)
  - **Input buffer $\mathcal{I}$**: 1 page
  - **Output buffer $\mathcal{O}$**: 1 page



Observations

Time complexity

- Build and probe phases
  - $c_{\texttt{build}} = p_R$
  - $c_{\texttt{probe}} = p_S$
- **Overall cost**
  - $c_{\texttt{CH}} = c_{\texttt{build}} + c_{\texttt{probe}} = p_R + p_S$

Summary

- Interesting approach as for its efficiency
  - However, usable only when the smaller table can entirely be hashed into the system memory…

Simple Hashing

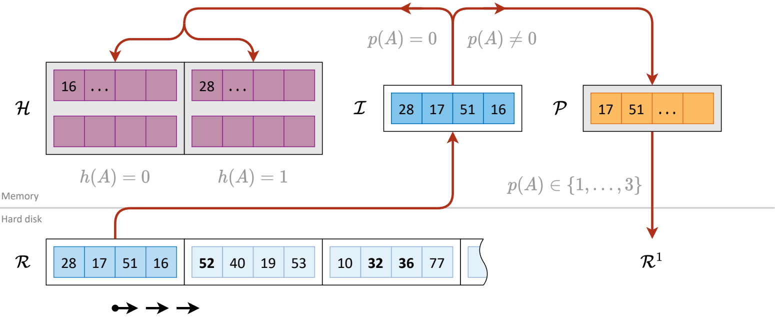
Simple hash join

- Basic idea
  - During each pass, **just a subset of all tuples is considered**
    - These are processed via analogous **build and probe routines**
    - The **remaining tuples are postponed** for the following passes
- **Partition function** p is assumed for this separation
  - Its **domain** are again values of the joining attribute A
  - Its **range** provides P distinct values
- Obvious requirement
  - **Both functions** p and h need to be **mutually <u>orthogonal</u>**
  - E.g.: $p(A) = A \bmod 4$ and $h(A) = A \bmod 2$ will not work
    - Because all items in a partition would either be even or odd

Build Phase

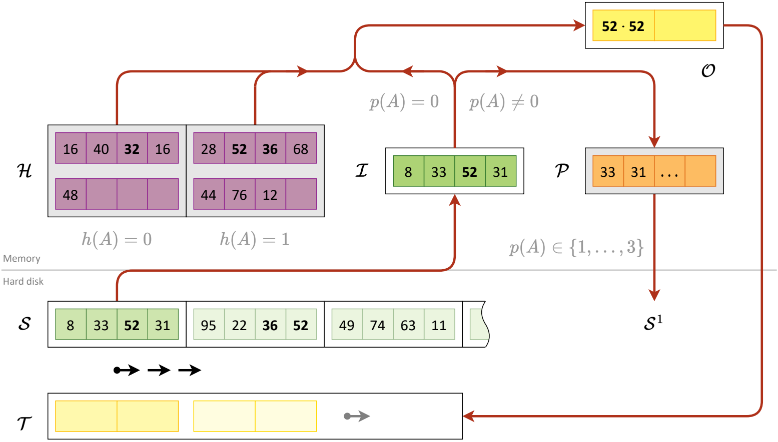
Build phase (partition 0)

- Items from the smaller table are either **hashed** or **postponed**
  - E.g., partition function $p(A) = A \bmod 4$ and hash function $h(A) = (A/4) \bmod 2$ are assumed



Probe Phase

Probe phase (partition 0)



Algorithm

Overall procedure

```
put $\mathcal{R}^0 \leftarrow \mathcal{R}$
put $\mathcal{S}^0 \leftarrow \mathcal{S}$
foreach partition $p \in \{0, \ldots, P-1\}$ do
    execute build phase for partition $p$ over $\mathcal{R}^p$ and create
      postponed $\mathcal{R}^{p+1}$
    execute probe phase for partition $p$ over $\mathcal{S}^p$ and create
      postponed $\mathcal{S}^{p+1}$
    empty hash container $\mathcal{H}$
```

Algorithm

Build phase (for partition K)

foreach block R from $\mathcal{R}^K$ **do**
    read R into $\mathcal{I}$
    foreach item r in $\mathcal{I}$ **do**
        calculate partition value $p \leftarrow p(r.A)$
        if $p = K$ **then**
            calculate hash value $h \leftarrow h(r.A)$
            add r into bucket h in $\mathcal{H}$
        else
            add r into partition buffer $\mathcal{P}$
            if $\mathcal{P}$ is full **then** write $\mathcal{P}$ to $\mathcal{R}^{K+1}$ and empty $\mathcal{P}$

if $\mathcal{P}$ is not empty **then** write $\mathcal{P}$ to $\mathcal{R}^{K+1}$ and empty $\mathcal{P}$

Algorithm

Probe phase (for partition K)

foreach block S from $\mathcal{S}^K$ **do**
    read S into $\mathcal{I}$
    foreach item s in $\mathcal{I}$ **do**
        calculate partition value $p \leftarrow p(s.A)$
        if $p = K$ **then**
            calculate hash value $h \leftarrow h(s.A)$
            foreach item r in bucket h in $\mathcal{H}$ **do**
                if r and s can be joined **then**
                    join r and s and put the result to $\mathcal{O}$
                    if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$, empty $\mathcal{O}$

▼▼▼

Algorithm

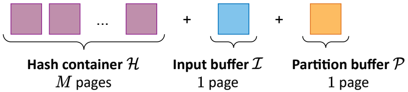
Probe phase (for partition K) (cont'd)

```
            else
                add s into partition buffer 𝒫
                if 𝒫 is full then  write 𝒫 to 𝒮^{K+1} and empty 𝒫
if 𝒪 is not empty then  write 𝒪 to 𝒯 and empty 𝒪
if 𝒫 is not empty then  write 𝒫 to 𝒮^{K+1} and empty 𝒫
```

Observations

Memory layout

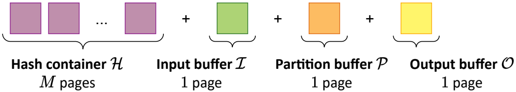
- Build phase: $M + 1 + 1$
  - **Hash container $\mathcal{H}$**: $M = \lceil F \cdot (p_R / P) \rceil$ pages
  - **Input buffer $\mathcal{I}$**: 1 page
  - **Partition buffer $\mathcal{P}$**: 1 page



Observations

Memory layout

- Probe phase: $M + 1 + 1 + 1$
  - **Hash container $\mathcal{H}$**: M pages (preserved from the build phase)
  - **Input buffer $\mathcal{I}$**: 1 page
  - **Partition buffer $\mathcal{P}$**: 1 page
  - **Output buffer $\mathcal{O}$**: 1 page



Observations

Time complexity

- Build and probe phases
  - $c_{\texttt{build}} \approx \left(p_R + \frac{P-1}{P}p_R\right) + \left(\frac{P-1}{P}p_R + \frac{P-2}{P}p_R\right) + \cdots + \left(\frac{1}{P}p_R\right)$
    $= p_R + 2\frac{1}{P}\Big[(P-1) + (P-2) + \cdots + (1)\Big]p_R$
    $= p_R + 2\frac{1}{P}\left[\frac{(P-1)+(1)}{2} \cdot (P-1)\right]p_R = p_R + (P-1)p_R$
    $= P \cdot p_R$
  - Analogously $c_{\texttt{probe}} = P \cdot p_S$
- **Overall cost**
  - $c_{\texttt{SH}} = c_{\texttt{build}} + c_{\texttt{probe}} = P \cdot (p_R + p_S)$

Summary

- We are now able to deal even with larger tables
  - However, overall cost is still not efficient enough…

Partition Hashing

Partition hash join

- Basic principle
  - **Both tables are first partitioned**
    - Using partition function p again
  - **Pairs of the corresponding partitions** are then **joined** together
    - Using the classic hash join approach
    - Or actually even nested loops if desired

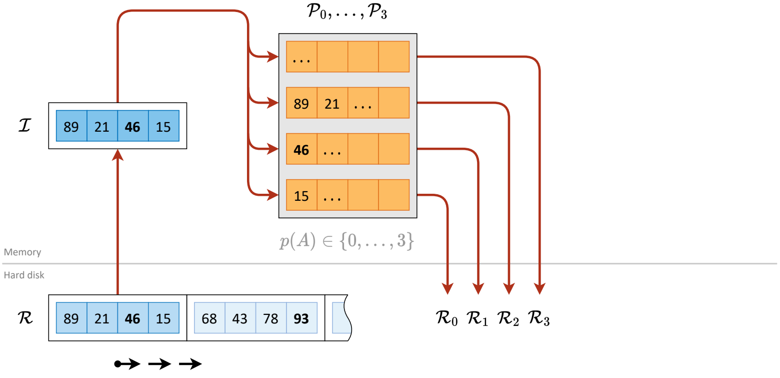
Overall procedure

split $\mathcal{R}$ and create partitions $\mathcal{R}_0, \ldots, \mathcal{R}_{P-1}$
split $\mathcal{S}$ and create partitions $\mathcal{S}_0, \ldots, \mathcal{S}_{P-1}$
foreach partition $p \in \{0, \ldots, P-1\}$ **do**
    join partitions $\mathcal{R}_p$ and $\mathcal{S}_p$

Partition Phase

Partition phase (for table $\mathcal{R}$)

- Tuples of a given table are split to **disjoint partitions**



Join Phase

Partition phase

- **Resulting partitions** for our sample scenario

$\mathcal{R}_0$	68 **52** 12 96	44		$\mathcal{S}_0$	28 72 84 **52**	
$\mathcal{R}_1$	89 21 **93** 69	73 **45** 33 21	37 53	$\mathcal{S}_1$	**93** 25 57 **45**	
$\mathcal{R}_2$	**46** 78 22 54			$\mathcal{S}_2$	**46** 30 **46** 18	
$\mathcal{R}_3$	15 43 79 35	71 55 43		$\mathcal{S}_3$	51 83 95 39	

Join phase

- Pairs of the <u>corresponding</u> partitions are then joined together
  - $\mathcal{R}_0$ and $\mathcal{S}_0$, $\mathcal{R}_1$ and $\mathcal{S}_1$, …

Algorithm

Partition phase

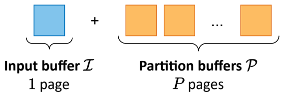
- Table $\mathcal{R}$ is assumed, partitioning of $\mathcal{S}$ is analogous

foreach block R from $\mathcal{R}$ **do**
    read R into $\mathcal{I}$
    foreach item r in $\mathcal{I}$ **do**
        calculate partition value $p \leftarrow p(r.A)$
        add r into partition buffer $\mathcal{P}_p$
        if $\mathcal{P}_p$ is full **then** write $\mathcal{P}_p$ to $\mathcal{R}_p$ and empty $\mathcal{P}_p$
foreach partition $p \in \{0, \ldots, P-1\}$ **do**
    if $\mathcal{P}_p$ is not empty **then** write $\mathcal{P}_p$ to $\mathcal{R}_p$ and empty $\mathcal{P}_p$

Observations

Memory layout

- Partition phase: $1 + P$
  - **Input buffer** $\mathcal{I}$: 1 page
  - **Partition buffers** $\mathcal{P}$: P pages



Time complexity

- **Partitioning phase**
  - $c_{\texttt{split}} \approx 2 \cdot p_R + 2 \cdot p_S$
- **Overall cost** (with classic hash join involved)
  - $c_{\texttt{PH}} = c_{\texttt{split}} + P \cdot c_{\texttt{CH}} \approx c_{\texttt{split}} + P\left[\frac{p_R}{P} + \frac{p_S}{P}\right] \approx 3 \cdot (p_R + p_S)$

Grace Hashing

Grace hash join

- Just ordinary **partition hash join**
  - … with **balanced memory** requirements across all the phases

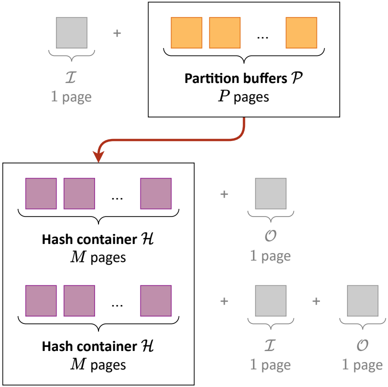
Memory setup

- Let $m \approx \sqrt{F \cdot p_R}$
  - I.e., square root of the size of an in-memory container that would roughly be needed for hashing of the smaller table $\mathcal{R}$
- **Partition function** p is chosen to ensure that $P = m$
  - $\Rightarrow$ m partitions will be created (for $\mathcal{R}$ as well as $\mathcal{S}$)
  - $\Rightarrow$ **expected size of each partition** of $\mathcal{R}$ should be…
    - $s = p_R/P = p_R/m = p_R/\sqrt{F \cdot p_R} \approx \sqrt{p_R/F}$ pages
  - $\Rightarrow$ **space needed for hashing** each of these partitions…
    - $F \cdot s = F \cdot \sqrt{p_R/F} \approx \sqrt{F \cdot p_R} \approx m$ pages

Grace Hashing

Memory setup (cont'd)

- I.e., size P of **partition buffers** $\mathcal{P}$ (partition phase) and size M of **hash container** $\mathcal{H}$ (build and probe phases) are equal to m



Hybrid Hashing

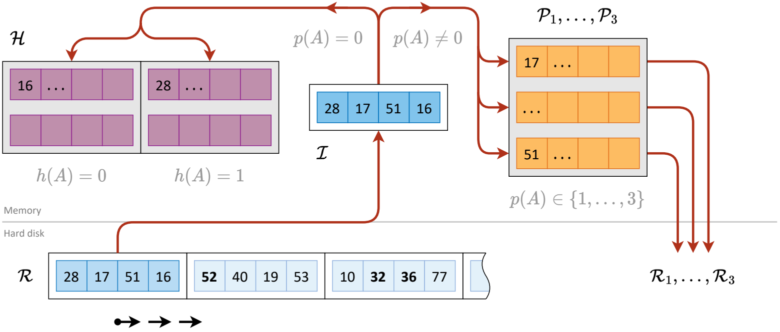
Hybrid hash join

- Basically an improvement of the **simple hash join** approach
  - Instead of using just one buffer for all items to be postponed…
  - … we directly split them to **separate partitions**
    - I.e., as in the **partition hash join** approach
- In other words…
  - **Partitions** 0 are joined directly during the first pass
    - Using the altered build and probe phases
  - All the **remaining partitions** are pairwise joined subsequently
    - Using the **classic hash join** approach

Build Phase

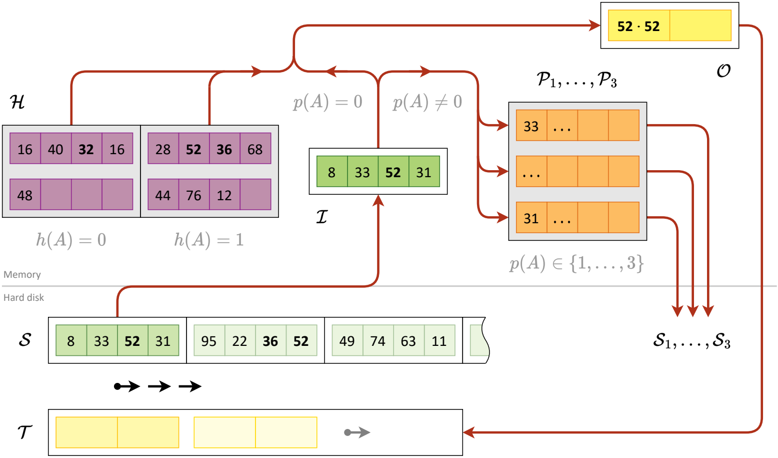
Build phase

- Items from the smaller table are either **hashed** or **postponed**
  - However, when they are to be postponed, they are branched to individual separated partitions



Probe Phase

Probe phase



Algorithm

Overall procedure

execute **build phase** over $\mathcal{R}$, hash items from partition 0
  and create postponed partitions $\mathcal{R}_1, \ldots, \mathcal{R}_{P-1}$
execute **probe phase** over $\mathcal{S}$, join items from partition 0
  and create postponed partitions $\mathcal{S}_1, \ldots, \mathcal{S}_{P-1}$
foreach partition $p \in \{1, \ldots, P-1\}$ **do**
  join partitions $\mathcal{R}_p$ and $\mathcal{S}_p$

Algorithm

Build phase

foreach block R from $\mathcal{R}$ **do**
    read R into $\mathcal{I}$
    foreach item r in $\mathcal{I}$ **do**
        calculate partition value $p \leftarrow p(r.A)$
        if $p = 0$ **then**
            calculate hash value $h \leftarrow h(r.A)$
            add r into bucket h in $\mathcal{H}$
        else
            add r into partition buffer $\mathcal{P}_p$
            if $\mathcal{P}_p$ is full **then** write $\mathcal{P}_p$ to $\mathcal{R}_p$ and empty $\mathcal{P}_p$

▼▼▼

Algorithm

Build phase (cont'd)

▲▲▲

foreach partition $p \in \{1, \ldots, P-1\}$ **do**

  if $\mathcal{P}_p$ is not empty **then** write $\mathcal{P}_p$ to $\mathcal{R}_p$ and empty $\mathcal{P}_p$

Algorithm

Probe phase

foreach block S from $\mathcal{S}$ **do**
    read S into $\mathcal{I}$
    foreach item s in $\mathcal{I}$ **do**
        calculate partition value $p \leftarrow p(s.A)$
        if $p = 0$ **then**
            calculate hash value $h \leftarrow h(s.A)$
            foreach item r in bucket h in $\mathcal{H}$ **do**
                if r and s can be joined **then**
                    join r and s and put the result to $\mathcal{O}$
                    if $\mathcal{O}$ is full **then** write $\mathcal{O}$ to $\mathcal{T}$, empty $\mathcal{O}$

▼▼▼

Algorithm

Probe phase (cont'd)

▲▲▲

else

add s into partition buffer $\mathcal{P}_p$

if $\mathcal{P}_p$ is full **then** write $\mathcal{P}_p$ to $\mathcal{S}_p$ and empty $\mathcal{P}_p$

if $\mathcal{O}$ is not empty **then** write $\mathcal{O}$ to $\mathcal{T}$ and empty $\mathcal{O}$

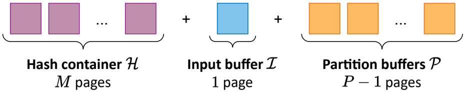
foreach partition $p \in \{1, \ldots, P-1\}$ **do**

if $\mathcal{P}_p$ is not empty **then** write $\mathcal{P}_p$ to $\mathcal{S}_p$ and empty $\mathcal{P}_p$

Observations

Memory layout

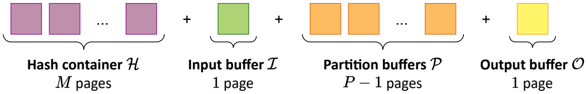
- Build phase: $M + 1 + (P - 1)$
  - **Hash container $\mathcal{H}$**: $M = \lceil F \cdot (p_R / P) \rceil$ pages
  - **Input buffer $\mathcal{I}$**: 1 page
  - **Partition buffers $\mathcal{P}$**: $P - 1$ pages



Observations

Memory layout

- Probe phase: $M + 1 + (P - 1) + 1$
  - **Hash container** $\mathcal{H}$: M pages (preserved from the build phase)
  - **Input buffer** $\mathcal{I}$: 1 page
  - **Partition buffers** $\mathcal{P}$: $P - 1$ pages
  - **Output buffer** $\mathcal{O}$: 1 page



Observations

Time complexity

- Build and probe phases for partition 0
  - $c_{\texttt{build}} \approx p_R + p_R \cdot \frac{P-1}{P} = p_R \cdot (1 + \frac{P-1}{P}) = p_R \cdot (2 - \frac{1}{P})$
  - Analogously $c_{\texttt{probe}} \approx p_S \cdot (2 - \frac{1}{P})$
- **Overall cost** (with classic hash join involved)
  - $$\begin{aligned} c_{\texttt{HH}} &= c_{\texttt{build}} + c_{\texttt{probe}} + (P-1) \cdot c_{\texttt{CH}} \\ &\approx p_R \cdot (2 - \tfrac{1}{P}) + p_S \cdot (2 - \tfrac{1}{P}) + (P-1)\left[\tfrac{p_R}{P} + \tfrac{p_S}{P}\right] \\ &\approx (3 - \tfrac{2}{P}) \cdot (p_R + p_S) \end{aligned}$$

Query Evaluation

Sample Query

Database schema

- **Movie** (<u>id</u>, title, year, …)
- **Actor** (<u>movie, actor</u>, character, …)
  - FK: Actor[movie] $\subseteq$ Movie[id]

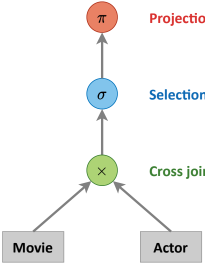
Sample query

- **Actors and characters they played in movies filmed in 2000**
  - ```
    SELECT title, actor, character
    FROM Movie JOIN Actor
    WHERE (year = 2000) AND (id = movie)
    ```
  - $\texttt{(Movie} \times \texttt{Actor)((year = 2000)} \wedge \texttt{(id = movie))}$
    $\texttt{[title, actor, character]}$
  - $\pi_{\text{title,actor,character}}\left(\varphi_{(\text{year}=2000)\wedge(\text{id}=\text{movie})}\left(\text{Movie} \times \text{Actor}\right)\right)$
    ```

Sample Query

Sample query (cont'd)

- **Actors and characters they played in movies filmed in 2000**
  - $\pi_{\text{title,actor,character}}\left(\varphi_{(\text{year}=2000)\wedge(\text{id}=\text{movie})}\left(\text{Movie} \times \text{Actor}\right)\right)$



Projection [title, actor, character]

Selection (year = 2000) ∧ (id = movie)

Cross join

Query Evaluation

Basic idea

- SQL query $\rightarrow$ RA query $\rightarrow$ evaluation plan $\rightarrow$ query result

Evaluation process

- (1) **Scanning** [scanner]
  - **Lexical analysis** is performed over the input SQL expression
    - Lexemes are recognized and then tokens generated
- (2) **Parsing** [parser]
  - **Syntactic analysis** is performed
    - Derivation tree is constructed according to the SQL grammar
- (3) **Translation**
  - **Query tree** with **relational algebra operations** is constructed

Query Evaluation

Evaluation process (cont'd)

- (4) **Validation** [validator]
  - **Semantic validity** is checked
    - Compliance of relation schemas with intended operations
- (5) **Optimization** [optimizer]
  - **Alternative evaluation plans** are devised and compared
    - In order to find the most efficient plan
    - Based on their evaluation **cost estimates**
- (6) **Code generation** [generator]
  - Execution code is generated for the chosen plan
- (7) **Execution** [processor]
  - Intended query is finally evaluated
    - And the yielded result provided to the user

Query Evaluation

Query tree

- Internal tree structure
  - **Leaf nodes** = input **tables**
  - **Inner nodes** = individual RA **operations** (σ, π, $\times$, $\bowtie$, …)
- Root node represents the entire query
  - Nodes are evaluated from leaves toward the root

Query evaluation plan

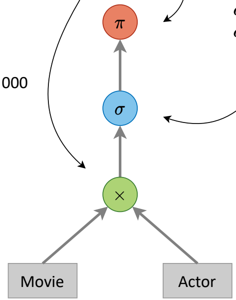
- **Query tree**
- For each inner node…
  - Calculated **statistics** (number of tuples, blocking factor, …)
  - Selected **algorithm** (limited by **context** and available **memory**)
  - Estimated **cost**
- **Overall cost**

Sample Plan #1

Projection [title, actor, character]
$n_3 = n_2 =$ 20 000
$b_3 \leftarrow$ 50
$p_3 = n_3/b_3 =$ 400
$c_3^{\mathtt{r}} = p_2 =$ 2 500
$c_3^{\mathtt{w}} = p_3 =$ 400

Cross join
$n_1 = n_M \cdot n_A =$ 100 000 000 000
$b_1 = (b_M \cdot b_A)/(b_M + b_A) =$ 8
$p_1 = n_1/b_1 =$ 12 500 000 000
Nested loops
$M_1 = 25 + 1 + 1 =$ 27
$c_1^{\mathtt{r}} = p_M + (p_M/25) \cdot p_A =$ 10 010 000
$c_1^{\mathtt{w}} = p_1 =$ 12 500 000 000

Selection (year = 2000) ∧ (id = movie)
$n_2 = n_1 \cdot (1/V_{M.year}) \cdot (1/n_M) =$ 20 000
$b_2 = b_1 =$ 8
$p_2 = n_2/b_2 =$ 2 500
$c_2^{\mathtt{r}} = p_1 =$ 12 500 000 000
$c_2^{\mathtt{w}} = p_2 =$ 2 500



Sorted file (year)
$n_M =$ 100 000
$b_M =$ 10
$p_M =$ 10 000
$V_{M.year} =$ 50
B$^+$ tree index (year)
$m_{M.year} =$ 100
$I_{M.year} =$ 3

Heap file
$n_A =$ 1 000 000
$b_A =$ 40
$p_A =$ 25 000

Evaluation Plan Cost

Overall **evaluation cost**

- Let us first assume that all **intermediate results are always written to temporary files** and so each involved operation…
  - Reads its inputs from / writes its output to a hard drive
- **Overall cost** then equals to the **sum of all the partial costs**

Cost of **Plan #1**

- $M = 25 + 1 + 1$ memory pages
- $c = [c_1^{\mathtt{r}} + c_1^{\mathtt{w}}] + [c_2^{\mathtt{r}} + c_2^{\mathtt{w}}] + [c_3^{\mathtt{r}}]$
- $c = [p_M + (p_M/25) \cdot p_A + p_1] + [p_1 + p_2] + [p_2]$
- $c = [10\,010\,000 + 12\,500\,000\,000] + [12\,500\,000\,000 + 2\,500] + [2\,500]$
- $c = 25\,010\,015\,000$

Sample Query

Intuitive optimization

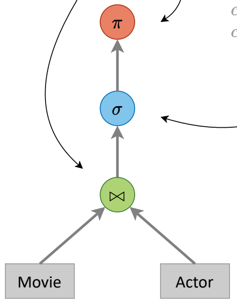
- **Actors and characters they played in movies filmed in 2000**
  - SQL expression

```
SELECT title, actor, character
FROM Movie JOIN Actor ON (id = movie)
WHERE (year = 2000)
```

  - RA expression

$$\pi_{\text{title,actor,character}}\left(\varphi_{(\text{year}=2000)}\left(\text{Movie} \bowtie_{(\text{id}=\text{movie})} \text{Actor}\right)\right)$$

Sample Plan #2



Projection [title, actor, character]
$n_3 = n_2 =$ 20 000
$b_3 \leftarrow$ 50
$p_3 = n_3 / b_3 =$ 400
$c_3^{\mathtt{r}} = p_2 =$ 2 500
$c_3^{\mathtt{w}} = p_3 =$ 400

Theta join [id = movie]
$n_1 = n_A =$ 1 000 000
$b_1 = (b_M \cdot b_A)/(b_M + b_A) =$ 8
$p_1 = n_1 / b_1 =$ 125 000
Nested loops
$M_1 = 25 + 1 + 1 = 27$
$c_1^{\mathtt{r}} = p_M + (p_M/25) \cdot p_A =$ 10 010 000
$c_1^{\mathtt{w}} = p_1 =$ 125 000

Selection (year = 2000)
$n_2 = n_1 \cdot (1/V_{M.year}) =$ 20 000
$b_2 = b_1 =$ 8
$p_2 = n_2 / b_2 =$ 2 500
$c_2^{\mathtt{r}} = p_1 =$ 125 000
$c_2^{\mathtt{w}} = p_2 =$ 2 500

Sorted file (year)
$n_M =$ 100 000
$b_M =$ 10
$p_M =$ 10 000
$V_{M.year} =$ 50
B$^+$ tree index (year)
$m_{M.year} =$ 100
$I_{M.year} =$ 3

Heap file
$n_A =$ 1 000 000
$b_A =$ 40
$p_A =$ 25 000

Sample Plan #2

Cost of **Plan #2**

- Again $M = 25 + 1 + 1$ memory pages
- $c = [c_1^{\mathtt{r}} + c_1^{\mathtt{w}}] + [c_2^{\mathtt{r}} + c_2^{\mathtt{w}}] + [c_3^{\mathtt{r}}]$
- $c = [p_M + (p_M/25) \cdot p_A + p_1] + [p_1 + p_2] + [p_2]$
- $c = [10\,010\,000 + 125\,000] + [125\,000 + 2\,500] + [2\,500]$
- $c = 10\,265\,000$
    - That is approximately $2\,400$ times better than the first plan

Pipelining

Pipelining mechanism

- **Intermediate results are passed** between the operations **directly without** the usage of **temporary files** on a disk
  - And so just within the system memory
    - It may even be possible to do it **in-place** without extra pages
- Unfortunately, such an approach is not always possible…

Cost of **Plan #2 with pipelining**

- Still $M = 25 + 1 + 1$ memory pages
- $c = [c_1^{\mathrm{r}} + \cancel{c_1^{\mathrm{w}}}] + [\cancel{c_2^{\mathrm{r}}} + \cancel{c_2^{\mathrm{w}}}] + [\cancel{c_3^{\mathrm{r}}}]$
  - Joined tuples are filtered and projected immediately in-place
- $c = 10\,010\,000$

Query Optimization

Objective = finding the most **optimal query evaluation plan**

- It is not possible to consider all plans, though
  - Simply because there are far too many of them
  - And so **pruning and heuristics** need to be incorporated

Optimization strategies

- **Algebraic**
  - Proposal of **alternative plans** using query tree transformations
- **Statistical**
  - Estimation of **costs and result sizes** based on available statistics
- **Syntactic**
  - Manual modification of query expressions by users themselves
    - In order to involve plans that would otherwise be unreachable
    - Breaches the principle of **declarative querying**, though

Statistical Optimization

Statistical Optimization

Objective

- Capability of **calculating necessary result characteristics**…
  - Of both the final result as well as all intermediate ones
    - I.e., all individual nodes within a given evaluation plan tree
- … so that the **overall cost** can be estimated
  - And thus alternative plans mutually compared

Basic statistics

- **Data file** for table $\mathcal{R}$
  - n_R number of tuples, s_R tuple size, b_R blocking factor
  - p_R number of pages
  - Hashed file: H_R number of buckets, C_R bucket size
- **Index file** for attribute A from table $\mathcal{R}$
  - B$^+$ tree: $I_{R.A}$ tree height, $p_{R.A}$ number of leaf nodes

Statistical Optimization

Additional statistics

- Provide deeper insight into the **active domain**
  - May even be implicitly **derivable** from index structures
  - Unfortunately, they may also be **missing or unavailable**
    - Especially as for intermediate results
- $V_{R.A}$ number of distinct values
- $min_{R.A}$ and $max_{R.A}$ minimal and maximal values
- **Histograms**
  - Provide even more accurate understanding of the domain
    - And so better estimates
  - Especially useful for **non-uniform distributions**

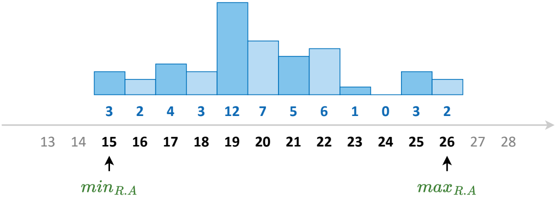
Histograms

Histogram = **approximate representation of data distribution**

- Active domain is split into **sub-intervals** called **buckets**
  - Usually consecutive and non-overlapping
- **Frequency of values** is determined for each one of them
  - I.e., count of values that fall into that bucket

Sample data

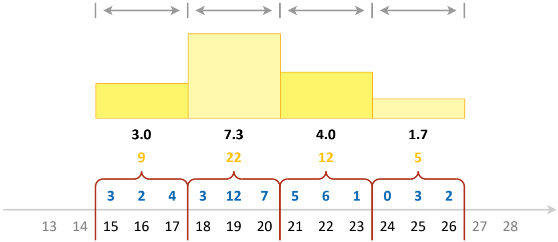
- Integer values from interval $[15, 26]$ and their frequencies



Histograms

Equi-width **histogram**

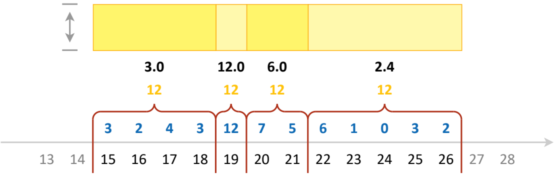
- Buckets have **equal widths** (count of distinct values)
- **Discrete** domains: average **frequencies** are stored
  - So that frequency $f_{E.A}(v)$ can be retrieved for any value v
- **Continuous** domains: **probabilities** are stored instead
  - So that probability $t_{E.A}(b)$ can be retrieved for any bucket b



Histograms

Equi-depth **histogram**

- **Buckets** are designed so that they have **equal depths**
  - I.e., absolute frequencies are the same
    - Or at least almost the same
    - Since real-world data will likely not be nice enough
- We also need to explicitly store **bucket placement** information
  - Since it is not derivable automatically



Size Estimates: Selection

Selection: $T = \sigma_\varphi(E)$

Tuple size

- $s_T = s_E$
  - Tuples are just filtered out and so their size remains untouched

Blocking factor

- $b_T = b_E$

Number of tuples

- Basic idea: $n_T = \lceil n_E \cdot r_\varphi \rceil$
- $r_\varphi \in [0, 1]$ is an estimated **reduction factor**
  - Describes how much the original tuples will be reduced
    - Depends on a particular condition φ
    - As well as particular available statistics…

Size Estimates: Selection

Reduction factors

- **Equality test** with respect to a **unique attribute**
  - $r_\varphi = 1/n_E$ (and so $n_T = 1$)
- **Equality test** with respect to a **non-unique attribute**
  - $r_\varphi = 1/V_{E.A}$
  - $r_\varphi = f_{E.A}(v)/n_E$ if histogram for discrete domains is available
    - As a consequence, $n_T = f_{E.A}(v)$
  - $r_\varphi = t_{E.A}(bucket(v))$ analogously for continuous domains
  - $r_\varphi = 1/10$ when no information is available at all
- **Estimates using constants** in general
  - May work well, not bad, as well as totally wrong…
    - But when nothing better is available, it must simply suffice
    - Of course, particular constant is just a matter of discussion

Size Estimates: Selection

Reduction factors (cont'd)

- **Range query** for two-sided intervals $I = [v_1, v_2]$ and other
  - $r_\varphi = (v_2 - v_1 + \varepsilon)/(max_{E.A} - min_{E.A} + 1)$
  - $r_\varphi = (\sum_{v \in I} f_{E.A}(v))/n_E$
  - $r_\varphi = (v_2 - v_1)/(max_{E.A} - min_{E.A})$
  - $r_\varphi = \sum_{b \in buckets(I)} t_{E.A}(b)$
  - $r_\varphi = 1/4$
- **Range query** for one-sided intervals $(-\infty, v_2]$ and $(-\infty, v_2)$
  - Works analogously…
  - $r_\varphi = 1/2$
    - Unfortunately, there are certain **undesired consequences**…
    - E.g., reduction factors of $A \leq 1$ and $A \leq 1000$ are the same
- **Range query** for one-sided intervals $[v_1, \infty)$ and (v_1, ∞)
  - Works analogously again…

Size Estimates: Selection

Reduction factors (cont'd)

- **Conjunction**: $\varphi_1 \wedge \varphi_2$
  - $r_\varphi = r_{\varphi_1} \cdot r_{\varphi_2}$
  - **Independence** of both conditions is assumed
- **Disjunction**: $\varphi_1 \vee \varphi_2$
  - $r_\varphi = r_{\varphi_1} + r_{\varphi_2} - r_{\varphi_1} \cdot r_{\varphi_2}$
- **Negation**: $\neg \varphi_1$
  - $r_\varphi = 1 - r_{\varphi_1}$
- …

Improved estimates might also be useful for **access methods**

- Since it is also about selection
  - However, technical possibilities of data files must be respected

Size Estimates: Projection

Projection: $T = \pi_{a_1,\ldots,a_n}(E)$

Tuple size

- s_T is simply calculated using sizes of all preserved attributes

Blocking factor

- $b_T = \lfloor B / s_T \rfloor$

Number of tuples

- Default SQL projection without the `DISTINCT` modifier
  - I.e., removal of potential duplicates is not performed
  - $n_T = n_E$
- With duplicates removal enabled
  - $n_T = n_E$ if at least one key of E is preserved
  - …

Size Estimates: Joins

Inner joins: $T = E_R \times E_S$ or $E_R \bowtie E_S$ or $E_R \bowtie_\varphi E_S$

Tuple size

- $s_T \approx s_R + s_S$
  - Less for natural join since shared attributes are not repeated

Blocking factor

- $b_T \approx \left\lfloor \frac{B}{s_T} \right\rfloor \approx \left\lfloor \frac{B}{s_R + s_S} \right\rfloor \approx \left\lfloor \frac{B}{B/b_R + B/b_S} \right\rfloor \approx \left\lfloor \frac{b_R \cdot b_S}{b_R + b_S} \right\rfloor$
  - Can be calculated exactly from the actual resulting tuple size
  - As well as **estimated just using the original blocking factors**

Number of tuples

- $n_T = \lceil n_R \cdot n_S \cdot r_\varphi \rceil$ with $r_\varphi \in [0, 1]$ for **joining condition** φ
  - Similar approach with **reduction factors** as in selections

Size Estimates: Joins

Reduction factors

- **Cross join**
  - $r_\varphi = 1$ (hence $n_T = n_R \cdot n_S$)
- **Foreign key** lookup
  - Let us assume that φ traverses a foreign key from $\mathcal{R}$ to $\mathcal{S}$
    - Then for each tuple $r \in \mathcal{R}$ there must exist exactly one $s \in \mathcal{S}$
  - And so $r_\varphi = 1/n_S$ (hence $n_T = n_R$)
- **Equality test** over an attribute A in $\mathcal{S}$
  - $r_\varphi = 1/V_{S.A}$
  - $r_\varphi = 1/n_S$ specifically for a **unique attribute** (again $n_T = n_R$)
- …

Algebraic Optimization

Equivalence Rules: Selection

Commutativity of selection

- $\sigma_{\varphi_2}(\sigma_{\varphi_1}(E)) \equiv \sigma_{\varphi_1}(\sigma_{\varphi_2}(E))$
- **Mutual order of selections** can be changed
  - Condition with higher **selectivity** can be applied first
    - I.e., condition which yields a fewer number of tuples

Cascade of selections

- $\sigma_{\varphi_2}(\sigma_{\varphi_1}(E)) \equiv \sigma_{\varphi_1 \land \varphi_2}(E)$
- Direction $\rightarrow$
  - **Selections can be merged together** into just one
    - Via a conjunction over the original conditions
- Direction $\leftarrow$
  - **Conjunctive selection can be split** into separate selections

Equivalence Rules: Projection

Cascade of projections

- $\pi_{A_2}(\pi_{A_1}(E)) \equiv \pi_{A_2}(E)$
- $\rightarrow$: only the **outermost projection** actually matters
  - And so the inner one can entirely be omitted as meaningless

Commutativity of selection and projection

- $\pi_A(\sigma_\varphi(E)) \equiv \sigma_\varphi(\pi_A(E))$
- **Selection and projection can be mutually swapped**
  - $\leftarrow$: without any limitation
  - $\rightarrow$: only when all attributes in φ are still available
    - When this assumption is not satisfied…
- $\pi_A(\sigma_\varphi(E)) \equiv \pi_A(\sigma_\varphi(\pi_{A \cup S}(E)))$
  - Attributes S from E are those that are needed for the selection

Equivalence Rules: Joins

Commutativity of joins

- Cross join: $E_1 \times E_2 \equiv E_2 \times E_1$
- Natural join: $E_1 \bowtie E_2 \equiv E_2 \bowtie E_1$
- Theta join: $E_1 \bowtie_{\varphi} E_2 \equiv E_2 \bowtie_{\varphi} E_1$
- **Operands of inner joins** can be mutually swapped
  - Such a thing is not possible for outer joins

Associativity of joins

- Inner joins are also associative (again, not outer)
- $(E_1 \times E_2) \times E_3 \equiv E_1 \times (E_2 \times E_3)$
- $(E_1 \bowtie E_2) \bowtie E_3 \equiv E_1 \bowtie (E_2 \bowtie E_3)$
- $(E_1 \bowtie_{\varphi_{12}} E_2) \bowtie_{\varphi_{13} \wedge \varphi_{23}} E_3 \equiv E_1 \bowtie_{\varphi_{12} \wedge \varphi_{13}} (E_2 \bowtie_{\varphi_{23}} E_3)$
  - Assuming that each φ_{ij} only involves attributes from E_i and E_j

Equivalence Rules: Joins

Integration of selection into joins

- **Any inner join can be rewritten using theta join**…
- … and then combined with selection
  - Intended for **conditions of <u>joining</u> nature**
    - I.e., conditions that involve attributes from both the operands
- $\sigma_{\varphi_S}(E_1 \times E_2) \equiv E_1 \bowtie_{\varphi_S} E_2$
- $\sigma_{\varphi_S}(E_1 \bowtie_{\varphi_J} E_2) \equiv E_1 \bowtie_{\varphi_J \wedge \varphi_S} E_2$
- $\sigma_{\varphi_S}(E_1 \bowtie E_2) \equiv E_1 \bowtie_{\varphi_N \wedge \varphi_S} E_2$
  - φ_N involves pairwise equality tests for all the shared attributes
    - I.e., attributes occurring in both the operands

Equivalence Rules: Joins

Distribution of selection over joins

- Let us have an inner join wrapped by a selection…
  - … and this selection contains a **condition of filtering nature**
    - I.e., condition with attributes from just one join operand
- It can then be **executed before the join** over just that operand
  - And so the join evaluation cost can be decreased
- $\sigma_{\varphi_S}(E_1 \times E_2) \equiv \sigma_{\varphi_S}(E_1) \times E_2$
  - Assuming that, in particular, φ_S involves attributes from E_1 only
- $\sigma_{\varphi_S}(E_1 \bowtie E_2) \equiv \sigma_{\varphi_S}(E_1) \bowtie E_2$
- $\sigma_{\varphi_S}(E_1 \bowtie_{\varphi_J} E_2) \equiv \sigma_{\varphi_S}(E_1) \bowtie_{\varphi_J} E_2$

Equivalence Rules: Joins

Distribution of projection over joins

- Let us assume that attributes A_1 are from E_1 and A_2 from E_2
- $\pi_{A_1 \cup A_2}(E_1 \times E_2) \equiv \pi_{A_1}(E_1) \times \pi_{A_2}(E_2)$
- $\pi_{A_1 \cup A_2}(E_1 \bowtie E_2) \equiv \pi_{A_1}(E_1) \bowtie \pi_{A_2}(E_2)$
  - $\rightarrow$: only works when all joining attributes are still available
- $\pi_{A_1 \cup A_2}(E_1 \bowtie E_2) \equiv \pi_{A_1 \cup A_2}(\pi_{A_1 \cup N}(E_1) \bowtie \pi_{A_2 \cup N}(E_2))$
  - Attributes N are those that are needed for the natural join
  - Despite looking strange, the impact may be significant
    - Since unnecessary attributes are removed earlier
- $\pi_{A_1 \cup A_2}(E_1 \bowtie_\varphi E_2) \equiv \pi_{A_1}(E_1) \bowtie_\varphi \pi_{A_2}(E_2)$
  - $\rightarrow$: analogous assumption again
- $\pi_{A_1 \cup A_2}(E_1 \bowtie_\varphi E_2) \equiv \pi_{A_1 \cup A_2}(\pi_{A_1 \cup J_1}(E_1) \bowtie_\varphi \pi_{A_2 \cup J_2}(E_2))$
  - Attributes J_i from E_i are those needed for the theta join

Equivalence Rules: Set Operations

Commutativity of set operations

- $E_1 \cup E_2 \equiv E_2 \cup E_1$
- $E_1 \cap E_2 \equiv E_2 \cap E_1$
- Set difference is not commutative

Associativity of set operations

- $(E_1 \cup E_2) \cup E_3 \equiv E_1 \cup (E_2 \cup E_3)$
- $(E_1 \cap E_2) \cap E_3 \equiv E_1 \cap (E_2 \cap E_3)$
- Set difference is also not associative

Equivalence Rules: Set Operations

Distribution of selection over set operations

- $\sigma_\varphi(E_1 \cup E_2) \equiv \sigma_\varphi(E_1) \cup \sigma_\varphi(E_2)$
- $\sigma_\varphi(E_1 \cap E_2) \equiv \sigma_\varphi(E_1) \cap \sigma_\varphi(E_2)$
- $\sigma_\varphi(E_1 \setminus E_2) \equiv \sigma_\varphi(E_1) \setminus \sigma_\varphi(E_2)$

Distribution of projection over set operations

- $\pi_A(E_1 \cup E_2) \equiv \pi_A(E_1) \cup \pi_A(E_2)$
- Such a thing is not possible for intersection and difference

Recommendations

Basic heuristics

- **Push** filtering selections as close as possible to leaves
  - To throw away **not needed tuples** as soon as possible
- **Push** projections toward leaves the same way
  - So that **size of intermediate results** is decreased
- **Integrate** joining selections into joins
  - I.e, **rewrite other types of joins to theta joins**
- **Simplify cascades** of projections or selections
- **Transform** sub-queries **to joins** whenever possible
  - Since optimization only works for simple `SELECT` blocks
- **Exploit** commutativity and associativity of operations
  - Especially **joins** but also set operations

Examples

Sample transformations

- $\pi_{\text{title,actor,character}}\left(\boxed{\varphi_{(\text{year}=2000)\wedge(\text{id}=\text{movie})}}\left(\text{Movie} \times \text{Actor}\right)\right)$ // #1
- $\pi_{\text{title,actor,character}}\left(\boxed{\varphi_{(\text{id}=\text{movie})}}\left(\boxed{\varphi_{(\text{year}=2000)}}\left(\text{Movie} \times \text{Actor}\right)\right)\right)$
- $\pi_{\text{title,actor,character}}\left(\varphi_{(\text{year}=2000)}\left(\boxed{\varphi_{(\text{id}=\text{movie})}}\left(\text{Movie} \boxed{\times} \text{Actor}\right)\right)\right)$
- $\pi_{\text{title,actor,character}}\left(\boxed{\varphi_{(\text{year}=2000)}}\left(\text{Movie} \bowtie_{(\text{id}=\text{movie})} \text{Actor}\right)\right)$ // #2
- $\boxed{\pi_{\text{title,actor,character}}}\left(\varphi_{(\text{year}=2000)}(\text{Movie}) \bowtie_{(\text{id}=\text{movie})} \text{Actor}\right)$
- $\pi_{\text{title,actor,character}}\left(\pi_{\text{id,title}}\left(\varphi_{(\text{year}=2000)}(\text{Movie})\right) \bowtie_{(\text{id}=\text{movie})}\right.$
  $\left.\pi_{\text{movie,actor,character}}(\text{Actor})\right)$ // #3

Algebraic Optimization

Objective

- Capability of **finding alternative query evaluation plans**
  - Based on the so far introduced **equivalence rules**
    - As well as other not covered rules and heuristics
- Ultimate challenge
  - **Space of all possible plans** may be enormous
  - And so **significant pruning** must be involved

Basic strategy for **SPJ queries** = **select-project-join** queries

- They allow to be approached at two separate levels…
  - Single-relation plans / multi-relation plans
- But still an **NP-complete problem**

Alternative Plans

Single-relation plans

- Finding the **best access method** for each **individual table**
  - Including optional filtering **selections** and **projections**

Multi-relation plans

- Finding the **best join plan** for a given set of tables
  - Only **binary joins** are usually assumed
  - And so we just need to take into account **all possible orderings**
    - Since inner joins are commutative and associative

Observation

- **Optimal plan** may not consist of **optimal sub-plans**
  - And so it may happen that the truly best plan will not be found

Algorithm

Basic top-down approach

- Finding the best plan for a set of relations S
  - Using a **dynamic programming** method

---

if the best plan for S is already calculated **then**
    $\mathcal{P} \leftarrow$ fetch the best plan for S
    return $\mathcal{P}$
else
    if S contains just a single relation $\mathcal{R}$ **then**
        $\mathcal{P} \leftarrow$ find the best access method for $\mathcal{R}$
        store $\mathcal{P}$ as the best plan for S
        return $\mathcal{P}$
▼▼▼

Algorithm

Basic top-down approach (cont'd)

▲▲▲

else

  foreach $S_L \subseteq S$ such that $S_L \neq \emptyset \wedge S_L \neq S$ **do**

    $\mathcal{P}_L \leftarrow$ recursively find the best plan for S_L

    $\mathcal{P}_R \leftarrow$ recursively find the best plan for $S \setminus S_L$

    $\mathcal{P} \leftarrow$ find the best join plan over $\mathcal{P}_L$ and $\mathcal{P}_R$

    if $\mathcal{P}$ is so far the best plan for S (if any) **then**

      store $\mathcal{P}$ as the best plan for S

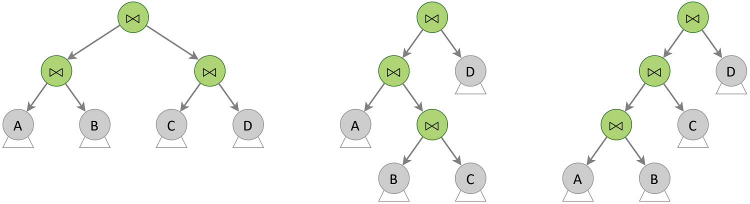
  $\mathcal{P} \leftarrow$ fetch the best plan for S

  return $\mathcal{P}$

Left-Deep Linear Trees

Only left-deep linear trees are usually taken into account…

- **Linear tree**
  - Each non-leaf node must have at least one child with relation
- **Left-deep linear tree**
  - Moreover, that child must be the right-hand one
    - Since that also increases the chance of attainable **pipelining**



$((A \bowtie B) \bowtie (C \bowtie D))$ $\quad$ $((A \bowtie (B \bowtie C)) \bowtie D)$ $\quad$ $(((A \bowtie B) \bowtie C) \bowtie D)$

Algorithm

Restricted top-down approach

- For left-deep linear trees only
  - This means there will be just $O(n \cdot 2^n)$ instead of $O(3^n)$ plans

---

if the best plan for S is already calculated **then**
    $\mathcal{P} \leftarrow$ fetch the best plan for S
    return $\mathcal{P}$
else
    if S contains just a single relation $\mathcal{R}$ **then**
        $\mathcal{P} \leftarrow$ find the best access method for $\mathcal{R}$
        store $\mathcal{P}$ as the best plan for S
        return $\mathcal{P}$

▼▼▼

Algorithm

Restricted top-down approach (cont'd)

▲▲▲

else

  foreach single relation $\mathcal{R} \in S$ **do**

    $\mathcal{P}_L \leftarrow$ recursively find the best plan for $S \setminus \{\mathcal{R}\}$

    $\mathcal{P}_R \leftarrow$ recursively find the best plan for $\{\mathcal{R}\}$

    $\mathcal{P} \leftarrow$ find the best join plan over $\mathcal{P}_L$ and $\mathcal{P}_R$

    if $\mathcal{P}$ is so far the best plan for S (if any) **then**

      store $\mathcal{P}$ as the best plan for S

  $\mathcal{P} \leftarrow$ fetch the best plan for S

  return $\mathcal{P}$

Algorithm

Restricted bottom-up approach

- We proceed by induction on the number of relations
  - All **single-relation plans** are found first
  - Then gradually all **multi-relation plans**
    - The best plan for n relations is found by considering all possible means of joining any of its $n-1$ relations with the 1 remaining

---

foreach single relation $\mathcal{R} \in S$ **do**
    $\mathcal{P} \leftarrow$ find the best access method for $\mathcal{R}$
    store $\mathcal{P}$ as the best plan for $\{\mathcal{R}\}$

▼▼▼

Algorithm

Restricted bottom-up approach (cont'd)

▲▲▲

foreach pass $p \in \{2, \ldots, |S|\}$ **do**
    foreach $T \subseteq S$ such that $|T| = p$ **do**
        foreach single relation $\mathcal{R} \in T$ **do**
            $\mathcal{P}_L \leftarrow$ fetch the best plan for $T \setminus \{\mathcal{R}\}$
            $\mathcal{P}_R \leftarrow$ fetch the best plan for $\{\mathcal{R}\}$
            $\mathcal{P} \leftarrow$ find the best join plan over $\mathcal{P}_L$ and $\mathcal{P}_R$
            if $\mathcal{P}$ is so far the best plan for T (if any) **then**
                store $\mathcal{P}$ as the best plan for T

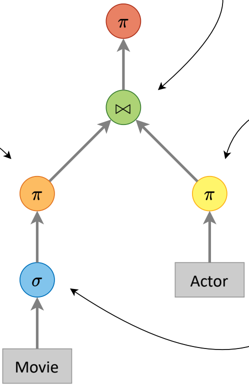
$\mathcal{P} \leftarrow$ fetch the best plan for S
return $\mathcal{P}$

Query Evaluation

Sample Plan #3

Projection [title, actor, character]
$n_5 = n_4 =$ 20 000
$b_5 \leftarrow$ 50
$p_5 = n_5 / b_5 =$ 400
$c_5^{\mathtt{w}} = p_5 =$ 400

Theta join [id = movie]
$n_4 = n_3 \cdot (1/V_{M.year}) =$ 20 000
$b_4 = (b_2 \cdot b_3)/(b_2 + b_3) \approx$ 35
Nested loops
$M_4 = p_2 + 1 + 1 =$ 27



Projection [id, title]
$n_2 = n_1 =$ 2 000
$b_2 \leftarrow$ 80
$p_2 = n_2 / b_2 =$ 25

Projection [movie, actor, character]
$n_3 = n_A =$ 1 000 000
$b_3 \leftarrow$ 65
$c_3^{\mathtt{r}} = p_A =$ 25 000

Heap file
$n_A =$ 1 000 000
$b_A =$ 40
$p_A =$ 25 000

Sorted file (year)
$n_M =$ 100 000
$b_M =$ 10
$p_M =$ 10 000
$V_{M.year} =$ 50
B$^+$ tree index (year)
$m_{M.year} =$ 100
$I_{M.year} =$ 3

Selection (year = 2000)
$n_1 = n_M \cdot (1/V_{M.year}) =$ 2 000
$b_1 = b_M =$ 10
$p_1 = n_1 / b_1 =$ 200
$c_1^{\mathtt{r}} = I_{M.year} + p_M \cdot (1/V_{M.year}) =$ 203

Sample Plan #3

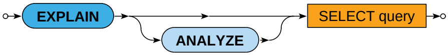
Cost of **Plan #3 with pipelining**

- $M = 25 + 1 + 1$ memory pages for buffers $\mathcal{I}_1$, $\mathcal{I}_2$ and $\mathcal{O}$
  - I.e., still the same amount of system memory pages used
- $c = [c_1^{\mathtt{r}} + \cancel{c_1^{\mathtt{w}}}] + [\cancel{c_2^{\mathtt{r}}} + \cancel{c_2^{\mathtt{w}}}] + [c_3^{\mathtt{r}} + \cancel{c_3^{\mathtt{w}}}] + [\cancel{c_4^{\mathtt{r}}} + \cancel{c_4^{\mathtt{w}}}] + [\cancel{c_5^{\mathtt{r}}}]$
  - $\mathcal{I}_2$ is used for index traversal and then reading of movies
  - All filtered and projected movies are put into $\mathcal{I}_1$
  - Actors are read into $\mathcal{I}_2$, their projection is postponed
  - Joined tuples are put into $\mathcal{O}$ and projected
- $c = [I_{M.year} + p_M \cdot (1/V_{M.year})] + [p_A]$
- $c = [203] + [25\,000]$
- $c = 25\,203$
  - That is approximately 400 times better than the second plan
    - And so almost 1 million times better than the first plan

Explain Statements

`EXPLAIN` **statement**

- Allows to **retrieve the evaluation plan** for a given query
  - When `ANALYZE` modifier is provided…
    - Query is also executed and the actual run times are returned



Example

- ```
  EXPLAIN
    SELECT title, actor, character
    FROM Movie JOIN Actor
    WHERE (year = 2000) AND (id = movie)
  ```
  ```

Observations

False assumptions and simplifications

- **Size of tuples**
  - Real-world **tuples usually have variable size**
    - Because data types such as `VARCHAR` are often used
  - That complicates internal block structure and cost estimates
- **Unused slots**
  - **Not all slots** within data file blocks **may really be used**
    - I.e., there can be gaps because of, e.g., deleted tuples
  - And so the actual file size may be greater than assumed
- **Inner fragmentation**
  - It may not be possible to utilize **inner block space** entirely
    - I.e., there can be **unused space** after the last slot
    - Or even around the slots in case of variable-size tuples

Observations

False assumptions and simplifications (cont'd)

- **Overflow areas** in **sorted files**
  - New tuples are usually not inserted to their correct positions
  - Instead, special dedicated area is used for that purpose
    - So that **time-complicated insertion** (up to linear) is avoided
  - Only time to time the whole file is reorganized (resorted)
- **Overflow areas** in **hashed files**
  - Allocated size of buckets may not be sufficient
- **Outer fragmentation**
  - **Layout of file blocks on a hard drive** may not be continuous
    - That may significantly increase time costs
    - Because of repeated seeks and rotational delays

Observations

False assumptions and simplifications (cont'd)

- **Impact of caching manager**
  - **Blocks** we require **may already be loaded into the memory**
    - And so the actual cost may be lower
- **Extent of available statistics**
  - **Not all statistics we worked with may be available**
    - Or derivable in case of inner nodes
  - And so less accurate estimates can then be made
- **Lazy maintenance of statistics**
  - **Statistics** we do have **may already be obsolete**
    - Simply because some of them are updated only occasionally

Observations

False assumptions and simplifications (cont'd)

- **Non-uniform distribution**
  - Assumption of **uniform distribution is often not realistic**
    - And it is not just about the **data**
    - But also **queries**
- **Independence of conditions**
  - When **reduction factors for conditions** are estimated…
    - Their independence is assumed
    - But this may not be realistic again
- **Cost estimation** in general
  - Our formulae provide only estimates, not precise calculations
    - Moreover, there was a lot of simplification
    - And the statistics we relied on may really be unavailable
  - And so despite the effort, they may not always work well

Conclusion

Evaluation **algorithms**

- Access methods
- Sorting
  - **External merge sort** with / without **priority queue**
- Joining
  - Binary / non-binary **nested loops join** with / without zig-zag
  - Basic / integrated **sort-merge join**
  - Classic / simple / partition / grace / hybrid **hash join**

Query **evaluation** and **optimization**

- Evaluation **plans**
  - Cost estimates, **pipelining**
- Statistical / algebraic optimization