

Tomáš Horváth

# RECOMMENDER SYSTEMS

Tutorial at the conference

## Znalosti 2012

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Institute of Computer Science, Faculty of Science  
Pavol Jozef Šafárik University in Košice, Slovak Republic



Information Systems and Machine Learning Lab  
University of Hildesheim, Germany



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- Content-based techniques
- Collaborative-filtering
- Matrix factorization
- Issues worth to mention
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- Summary
  - ... and, if still alive,
- Questions & Answers



# Introduction



# What is a RS?



# Why do we need RS?

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# Why do we need RS?

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A **company** wants to

- sell more (diverse) items
- increase users' satisfaction and fidelity
- better understand users' needs



# Why do we need RS?

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A **company** wants to

- sell more (diverse) items
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- better understand users' needs

A **user** would like to

- find some (or all, in case of critical domains such as medicine) good items with a relatively small effort
- express herself by providing ratings or opinions
- help others by contribute with information to the community



# The Big Bang



- Contest begun on October 2, 2006
  - 100M ratings (1-5 stars) from 480K users on 18K movies
  - decrease RMSE of Cinematch (0.9525) at least with 10% ( $\leq 0.8572$ )
- Grand Prize \$1,000,000, Annual Progress Prizes \$50,000

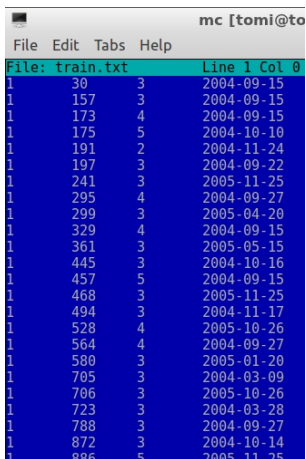
Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8567	10.06	2009-07-26 18:18:28
2	<a href="#">The Ensemble</a>	0.8567	10.06	2009-07-26 18:38:22
3	<a href="#">Grand Prize Team</a>	0.8582	9.90	2009-07-10 21:24:40
4	<a href="#">Opera Solutions and Vandelay United</a>	0.8588	9.84	2009-07-10 01:12:31
5	<a href="#">Vandelay Industries!</a>	0.8591	9.81	2009-07-10 00:32:20
6	<a href="#">PragmaticTheory</a>	0.8594	9.77	2009-06-24 12:06:56
7	<a href="#">BellKor in BigChaos</a>	0.8601	9.70	2009-05-13 08:14:09
8	<a href="#">Dace</a>	0.8612	9.59	2009-07-24 17:18:43
9	<a href="#">Feeds2</a>	0.8622	9.48	2009-07-12 13:11:51
10	<a href="#">BigChaos</a>	0.8623	9.47	2009-04-07 12:33:59
11	<a href="#">Opera Solutions</a>	0.8623	9.47	2009-07-24 00:34:07
12	<a href="#">BellKor</a>	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				





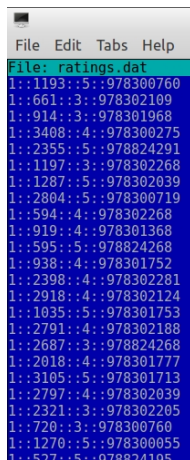
# Netflix and Movielens data (1/2)

Netflix



```
mc [tomi@to]
File Edit Tabs Help
File: train.txt Line 1 Col 0
1 30 3 2004-09-15
1 157 3 2004-09-15
1 173 4 2004-09-15
1 175 5 2004-10-10
1 191 2 2004-11-24
1 197 3 2004-09-22
1 241 3 2005-11-25
1 295 4 2004-09-27
1 299 3 2005-04-20
1 329 4 2004-09-15
1 361 3 2005-05-15
1 445 3 2004-10-16
1 457 5 2004-09-15
1 468 3 2005-11-25
1 494 3 2004-11-17
1 528 4 2005-10-26
1 564 4 2004-09-27
1 580 3 2005-01-20
1 705 3 2004-03-09
1 706 3 2005-10-26
1 723 3 2004-03-28
1 788 3 2004-09-27
1 872 3 2004-10-14
1 886 5 2005-11-25
```

Movielens (100K, 1M)



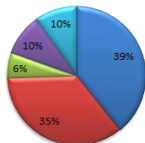
```
mc [tomi@to]
File Edit Tabs Help
File: ratings.dat
1::1193::5::978300760
1::661::3::978302109
1::914::3::978301968
1::3408::4::978300275
1::2355::5::978824291
1::1197::3::978302268
1::1287::5::978302039
1::2804::5::978300719
1::594::4::978302268
1::919::4::978301368
1::595::5::978824268
1::938::4::978301752
1::2398::4::978302281
1::2918::4::978302124
1::1035::5::978301753
1::2791::4::978302188
1::2687::3::978824268
1::2018::4::978301777
1::3105::5::978301713
1::2797::4::978302039
1::2321::3::978302205
1::720::3::978300760
1::1270::5::978300055
1::527::5::978824195
```



# Netflix and MovieLens data (2/2)

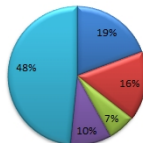
## RecSys 2010 - 2012

■ MovieLens data set ■ Netflix ■ Epinions.com ■ Amazon ■ other



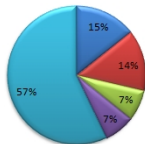
## WWW 2010 - 2012

■ MovieLens data set ■ Netflix ■ Epinions.com ■ Amazon ■ other



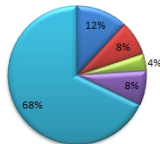
## UMAP 2010 - 2012

■ MovieLens data set ■ Netflix ■ Epinions.com ■ Amazon ■ other



## WSDM 2010 - 2012

■ MovieLens data set ■ Netflix ■ Epinions.com ■ Amazon ■ other



# Closely related fields

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## Information Retrieval

- unstructured data, various topics (IR) vs. repositories focused on a single topic (RS)
- relevant content for the query (IR) vs. relevant content for the user (RS)



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## Data mining & Machine Learning

- hardly measurable, subjective evaluation criteria (RS) besides some classic, objective evaluation measures (ML)



## **Information Retrieval**

- unstructured data, various topics (IR) vs. repositories focused on a single topic (RS)
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## **Data mining & Machine Learning**

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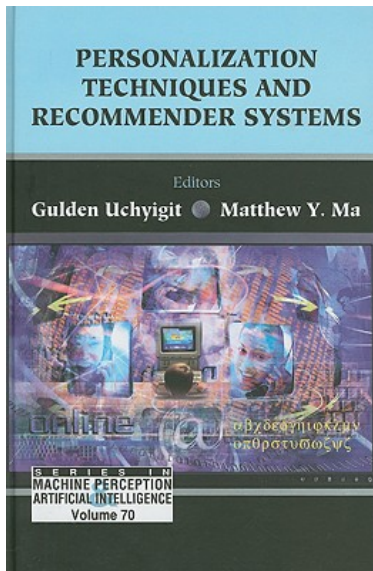
## **Human-Computer Interaction**

- RS should convince the user to try the recommended items
- clear, transparent and trustworthy system logic
- provide details about recommended items and opportunity to refine recommendations

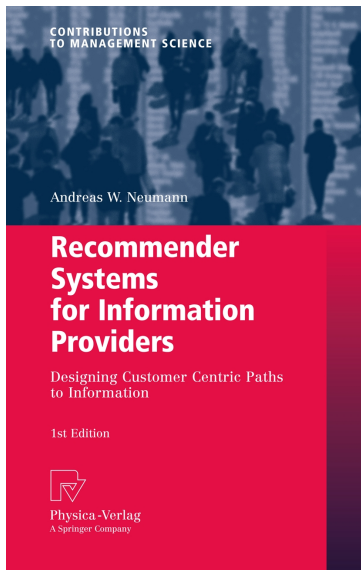


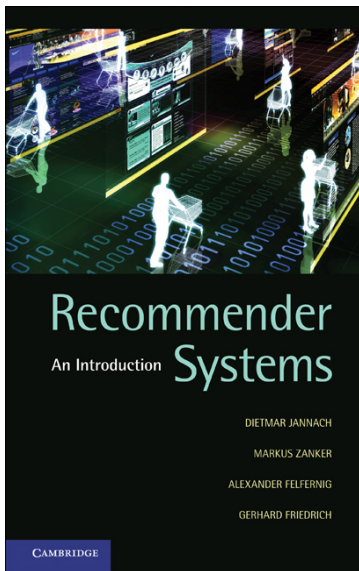
- ACM Recommender Systems (**RecSys**)
- User Modeling, Adaptation, and Personalization (**UMAP**)
- ACM Conference on Human Factors in Computing Systems (**CHI**)
- International World Wide Web Conference (**WWW**)
- ACM International Conference on Web Search and Data mining (**WSDM**)
- International Conference on Research and Development in Information Retrieval (**SIGIR**)
- ACM Conference on Information and Knowledge Management (**CIKM**)
- ...

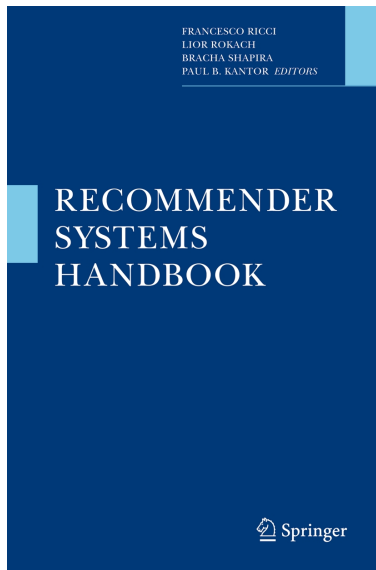












# Basic concepts



# Users, Items and their characteristics

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## Users

- set of users  $\mathcal{U}$
- user attributes  $\mathcal{A}^{user} \subset \mathbb{R}^k$ 
  - age, income, marital status, education, profession, nationality, ...
  - preferred sport, hobbies, favourite movies, ...
- user characteristics  $\chi^{user} : \mathcal{U} \rightarrow \mathcal{A}^{user}$ 
  - *sensitive* information, hard to obtain



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## Items

- set of items  $\mathcal{I}$
- item attributes  $\mathcal{A}^{item} \subset \mathbb{R}^l$ 
  - movies: title, genre, year, director, actors, budget, nominations, ...
- item characteristics  $\chi^{item} : \mathcal{I} \rightarrow \mathcal{A}^{item}$ 
  - quite *costly* to obtain



# User feedback

---

$$\phi : \mathcal{D} \rightarrow \mathcal{F}$$

- feedback values  $\mathcal{F} \subset \mathbb{R}$  observed on  $\mathcal{D} \subset \mathcal{U} \times \mathcal{I}$





# User feedback

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## Implicit feedback

- information obtained about users by watching their natural *interaction with the system*
  - view, listen, scroll, bookmark, save, purchase, link, copy&paste, ...
- no burden on the user



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## Explicit feedback

- *rating* items on a rating scale (Likert's scale)
- *scoring* items
- *ranking* a collection of items
- *pairwise ranking* of two presented items
- *provide* a list of preferred items



# The recommendation task

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Given

- $\mathcal{U}, \mathcal{I}$  and  $\phi$
- $\chi^{user}, \chi^{item}$
- some background knowledge  $\kappa$



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To learn

- **model**  $\hat{\phi} : \mathcal{U} \times \mathcal{I} \rightarrow \mathbb{R}$  such that  $acc(\hat{\phi}, \phi, \mathcal{T})$  is maximal
  - a set of “unseen” (or future) user-item pairs  $\mathcal{T} \subseteq (\mathcal{U} \times \mathcal{I}) \setminus \mathcal{D}$
  - $acc$  is the accuracy of  $\hat{\phi}$  w.r.t.  $\phi$  measured on  $\mathcal{T}$



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It looks as a simple prediction task, **however**

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- usually,  $\mathcal{F} = \{1\}$  in case of implicit feedback



# Two distinguished tasks

---





## Two distinguished tasks

---

### Rating prediction from explicit feedback

- How would Steve rate the movie Titanic more likely?

	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Joe	1	4	5		3
Ann	5	1		5	2
Mary	4	1	2	5	
Steve	?	3	4		4

- $\hat{\phi}(u, i)$  – predicted rating of the user  $u$  for an item  $i$



## Two distinguished tasks

### Rating prediction from explicit feedback

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- $\hat{\phi}(u, i)$  – predicted rating of the user  $u$  for an item  $i$

### Item recommendation from implicit feedback

- Which movie(s) would does Steve see/buy more likely?

	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Joe	1	1	1		1
Ann	1	1		1	1
Mary	1	1	1	1	
Steve	?	1	1	?	1

- $\hat{\phi}(u, i)$  – predicted likelihood of a “positive” implicit feedback (ranking score) of the user  $u$  for an item  $i$



# Types of RS

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## Knowledge-based

- recommendations are based on knowledge about users' needs and preferences
  - $\chi^{item}, \kappa, \chi^{user}$



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- learn user's interests based on the features of items previously rated by the user, using supervised machine learning techniques
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## Collaborative-filtering

- recognize similarities between users according to their feedbacks and recommend objects preferred by the like-minded users
  - $\phi$  (also  $\chi^{item}$  and/or  $\chi^{user}$  can be utilized)



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## Hybrid



# Knowledge-based techniques





# Knowledge

---



## user requirements

- value ranges
  - *“the maximal accepted price should be lower than 8K EUR”*
- functionality
  - *“the car should be safe and suited for a family”*

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## user requirements

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## dependencies

- between user requirements and product properties
  - *“a family car should have big trunk size”*
- between different user requirements
  - *“if a safe family car is required the maximal accepted price must be higher than 2000 EUR”*



## possible user requirements $V_{user}$

- max-price (0, ..., 10K), usage (family, ...), safety (small, medium, big)

## possible item characteristics $V_{item}$

- price (0, ..., 100K), doors (3, 4, 5), terrain (yes, no), airbags (1, ..., 12)

## compatibility constraints $\kappa_C$

- allowed instantiations of user properties
  - safety = big  $\rightarrow$  max-price  $\geq$  2000

## filter conditions $\kappa_F$

- item-specific selection criteriae
  - safety = big  $\rightarrow$  airbags  $>$  4

## item characteristics $\chi^{item}$

- “item constraints”
  - (id=1  $\wedge$  price=4K  $\wedge$  doors=3  $\wedge$  terrain=no  $\wedge$  airbags=2)  $\vee$  ...  
...  $\vee$  (id=100  $\wedge$  price=6K  $\wedge$  doors=5  $\wedge$  terrain=no  $\wedge$  airbags=6)

# Recommendation

---

identifying products matching **user's requirements** *REQ*

- can be viewed as a kind of  $\chi^{user}$
- $REQ = \text{max-price}=7000 \wedge \text{usage}=\text{family} \wedge \text{safety}=\text{big}$



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## Constraint-based

- $RES = CSP(V_{user} \cup V_{item}, D, \kappa_C \cup \kappa_F \cup \chi^{item} \cup REQ)$ 
  - a set  $D$  of finite domains for  $V_{user}$  and  $V_{item}$
  - $RES = \{\text{max-price}=7000, \text{usage}=\text{family}, \text{safety}=\text{big}, \text{id}=100, \text{price}=6K, \text{doors}=5, \text{terrain}=\text{no}, \text{airbags}=6\}$



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## Conjunctive queries

- $\sigma_{[\text{airbags} \geq 4 \wedge \text{price} \leq 8000]}(\chi^{item})$





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## Conjunctive queries

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## Case-based

- $\text{similarity}(i, REQ) = \sum_{r \in REQ} w_r \cdot \text{sim}(i, r) / \sum_{r \in REQ} w_r$ 
  - weight  $w_r$  for requirements  $r$
  - similarity  $\text{sim}(i, r)$  of items  $i \in \chi^{item}$  to requirements  $r \in REQ$ 
    - different types of  $\text{sim}(i, r)$
    - user might maximize (e.g. safety) or minimize (e.g. price)



# Interaction – default requirement values

---



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---

**static** defaults for each user property

- `default(usage)=family`



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**static** defaults for each user property

- `default(usage)=family`

**dependent** defaults on combinations of user requirements

- `default(usage=family, max-price=6000)`



## Interaction – default requirement values

---

**static** defaults for each user property

- $\text{default}(\text{usage})=\text{family}$

**dependent** defaults on combinations of user requirements

- $\text{default}(\text{usage}=\text{family}, \text{max-price}=6000)$

**derived** defaults from user requirements log

- the known requirement of the current user is  $\text{REQ}=\{\text{price}=6000\}$
- nearest-neighbor
  - 1-NN:  $\text{REQ}=\{\text{price}=6000, \text{doors}=5, \text{terrain}=\text{no}, \text{airbags}=6\}$
  - 3-NN:  $\text{REQ}=\{\text{price}=6000, \text{doors}=4, \text{terrain}=\text{no}, \text{airbags}=4\}$

user id	price	doors	terrain	airbags
1	6000	5	no	6
2	2000	3	yes	2
3	5500	4	yes	4
4	6500	4	no	4



# Interaction – unsatisfiable requirements

---

*Which of the requirements should be changed?*

---

<sup>1</sup>D. Jannach (2006). Finding Preferred Query Relaxations in Content-based Recommenders. IEEE Int. Conf. on Intelligent Systems, pp.355-360.



*Which of the requirements should be changed?*

- the **MinRelax**<sup>1</sup> algorithm

PQRS = compute the partial query results for all atoms

$a_i$  of  $Q$  for the product catalog  $P$

MinRS =  $\emptyset$

**forall**  $p_i \in P$  **do**

PSX = Compute the product-specific relaxation

$PSX(Q, p_i)$  by using PQRS

% Check relaxations that were already found

SUB =  $\{r \in MinRS \mid r \text{ is subquery of } PSX\}$

**if** SUB  $\neq \emptyset$

    % Current relaxation is superset of existing

**continue** with next  $p_i$

**endif**

SUPER =  $\{r \in MinRS \mid PSX \text{ is subquery of } r\}$

**if** SUPER  $\neq \emptyset$

    % Remove supersets

$MinRS = MinRS \setminus SUPER$

**endif**

% Store the new relaxation

$MinRS = MinRS \cup PSX$

**endfor**

**return** MinRS

---

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**endfor**

**return** MinRS

REQ =  $\{r_1:\text{price} \leq 6000, r_2:\text{doors} = 5,$   
 $r_3:\text{terrain} = \text{no}, r_4:\text{airbags} \geq 6\}$

- $\sigma_{[r_1 \wedge r_2 \wedge r_3 \wedge r_4]}(\chi^{item}) = \emptyset$

- partial query results PQRS

req	$i_1$	$i_2$	$i_3$	$i_4$
1	1	0	1	0
2	0	1	0	1
3	0	0	1	0
4	1	1	0	1

- product-specific relaxation

- $PSX(\text{REQ}, i_1) = \{r_2, r_3\}$

<sup>1</sup>D. Jannach (2006). Finding Preferred Query Relaxations in Content-based Recommenders. IEEE Int. Conf. on Intelligent Systems, pp.355-360.





# Interaction – repairs for unsatisfiable requirements

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*How should the unsatisfiable requirements be changed?*



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- $\text{MinRS} = \{\{r_2, r_4\}, \{r_2, r_3\}\}$ 
  - $\pi_{[\text{doors}, \text{airbags}]}\sigma_{[r_1, r_3]}(\chi^{\text{item}}) = \{(\text{doors} = 3, \text{airbags} = 4), (\text{doors} = 4, \text{airbags} = 2)\}$
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## Contributions

- pre-defined set of dimensions

	value	quality	economy	safety
price	$\langle 0, 3000 \rangle$	2	3	3
	$\langle 3000, 7000 \rangle$	3	2	4
	$\geq 7000$	5	1	5
terrain	yes	3	2	3
	no	2	4	2
airbags	0	1	5	1
	2	2	4	2
	...		...	
doors	3	3	5	2
	...		...	



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	...		...	

- $contribution(item, dimension)$ 
  - $i = (price=4000 \wedge terrain=no \wedge airbags=2 \wedge doors=3)$
  - $contribution(i, quality) = 3+2+2+3 = 10, \dots$



## Interaction – ranking the retrieved items (2/2)

---

### **Interest** of the user in pre-defined dimensions

- user-defined
  - $\text{interest}(\text{quality}) = 0.3$
  - $\text{interest}(\text{economy}) = 0.6$
  - $\text{interest}(\text{safety}) = 0.1$



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- derived from requirements
  - $\text{REQ} = \{\text{price}=4000 \wedge \text{airbags}=2\}$ 
    - $\text{contribution}(\text{req}, \text{quality}) = 3+2 = 5$
    - $\text{contribution}(\text{req}, \text{economy}) = 2+4 = 6$
    - $\text{contribution}(\text{req}, \text{safety}) = 4+2 = 6$
  - $\text{interest}(\text{quality}) = 5/(5+6+6) = 5/17 = 0.3$
  - $\text{interest}(\text{economy}) = \text{interest}(\text{safety}) = 6/17 = 0.35$
- other approaches





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$$\text{utility}(i) = \sum_{d \in \text{dimensions}} \text{interest}(d) \cdot \text{contribution}(i, d)$$



# Interaction – Critiquing

---



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---

a **browsing-based** approach used in case-based systems

- requirements refined w.r.t. the recommended item
  - *“Show me cheaper cars” ... “cars with more airbags” ...*



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Dynamic critiquing

- suggests **critique patterns** according to the candidate items
  - association rules ( $>_{price} \rightarrow <_{doors}$ )
  - compound critique patterns ( $>_{price} \wedge <_{doors}$ )

	price	doors	terrain	airbags
entry item	3600	5	no	4
candidate item 1	4500	3	no	4
candidate item 2	5600	4	yes	6
...			...	
critique pattern 1	>	<	≠	=
critique pattern 2	>	<	=	>
...			...	



## Content-based techniques



# Content

---

Item features/characteristics ( $\chi^{item}$ )

- explicitly **defined**
  - attributes (price, airbags, doors, ...)



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  - keywords  $w$ , boolean vector space model ...





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$$TF - IDF(w, d) = TF(w, d) \cdot IDF(w, \mathcal{D})$$

- term frequency

$$TF(w, d) = \frac{freq(w, d)}{\max\{freq(w', d) | w' \neq w\}}$$

- inverse document frequency

$$IDF(w, \mathcal{D}) = \log \frac{|\mathcal{D}|}{|\{d \in \mathcal{D} | w \in d\}|}$$



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- $\chi^{item} = (TF - IDF(w_1, item), \dots, TF - IDF(w_k, item))$



# Similarity-based recommendation

---

*How to check if a user would like an item?*



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---

*How to check if a user would like an item?*

- If she **liked similar** items in the past...
  - feedback and similarity measures needed

cosine vector similarity

$$\text{sim}_{cv}(\chi^i, \chi^j) = \frac{\chi^i \cdot \chi^j}{\|\chi^i\| \cdot \|\chi^j\|} = \frac{\sum_{k=1}^n \chi_k^i \chi_k^j}{\sqrt{\sum_{k=1}^n \chi_k^i{}^2} \sqrt{\sum_{k=1}^n \chi_k^j{}^2}}$$



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- k most similar items user has got feedback on
  - recommend an item according to majority vote/average/etc.



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k-nearest-neighbors

- k most similar items user has got feedback on
  - recommend an item according to majority vote/average/etc.
- reflects on short-term preferences
  - considering only recent feedbacks
- simple to implement, small number of feedbacks is enough



## Rocchio's method

- find a **prototype** of “user’s ideal item”
- user-defined queries refined **iteratively**
  - good results already after the first iteration
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- $\mathcal{D}^-$  – documents with negative user feedback
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- $Q_i$  – actual query (**vector**) in the iteration  $i$
- $\alpha, \beta, \gamma$  – parameters





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$$Q_{i+1} = \alpha Q_i + \beta \left( \frac{1}{|\mathcal{D}^+|} \sum_{d^+ \in \mathcal{D}^+} d^+ \right) + \gamma \left( \frac{1}{|\mathcal{D}^-|} \sum_{d^- \in \mathcal{D}^-} d^- \right)$$



# Machine learning

**learn** a mapping  $\hat{\phi} : \mathcal{A}^{item} \rightarrow \mathbb{R}$  from

- item features/characteristics  $\chi^{item}$
- user's feedback  $\phi$

with appropriate **classification/regression** techniques

- nearest-neighbor
- probabilistic methods
- decision trees, SVM
- ...

item	$\mathcal{A}^i$	$\phi(u, item)$
$i_1$	$\chi^{item}(i_1)$	$\phi(u, i_1)$
$i_2$	$\chi^{item}(i_2)$	$\phi(u, i_2)$
$\vdots$	$\vdots$	$\vdots$
$i_n$	$\chi^{item}(i_n)$	$\phi(u, i_n)$



A little commercial ;)



# A fuzzy recommender system

---

First prototype developed during the NAZOU<sup>1</sup> project (2006 – 2008)

- 2009 – 2012, developed without funding (BSc, MSc theses)
- 2012 – now, development within the CEZIS project

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Main characteristics of the UPRE recommender module<sup>2</sup>

- **fuzzy preference models**
  - on attributes (local)
  - aggregated (global)
  - top-k item retrieval
- explicit user feedback
- conversational

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A **hybrid** of content-based and knowledge-based techniques...

- collaborative-filtering is planned

---

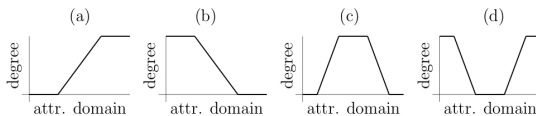
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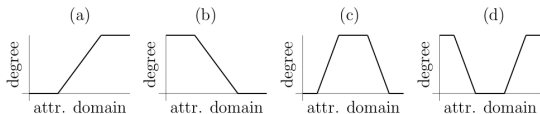


# Preferences on attributes

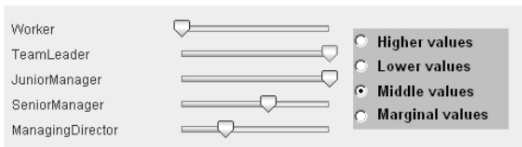
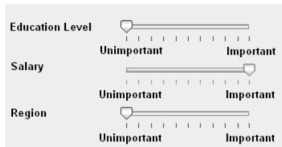
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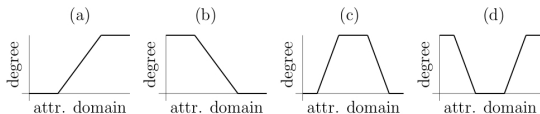


defined by the user

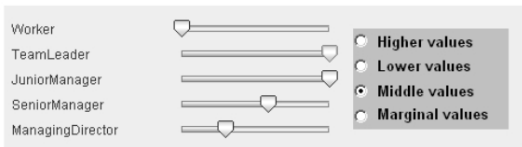
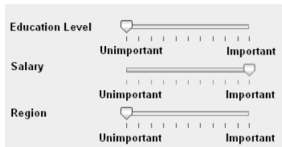




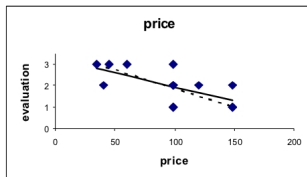
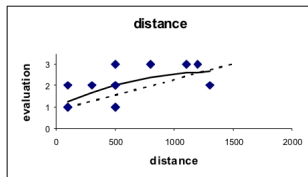
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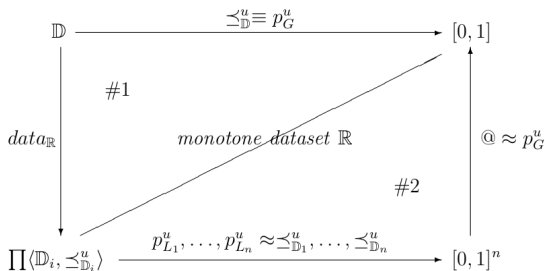


computed from explicit feedback



# Aggregated preferences

computed<sup>1</sup> with **monotone prediction** techniques

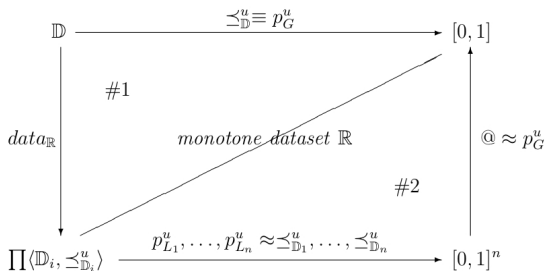


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# Aggregated preferences

computed<sup>1</sup> with **monotone prediction** techniques



preference rules integrated<sup>2</sup> with **top-k search**

- fast computation of pareto-optimal values
- implicit ranking of items in the resulting list

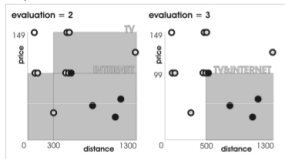
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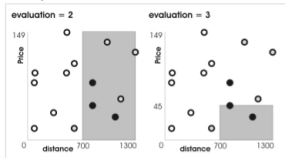


# Iterative recommendation

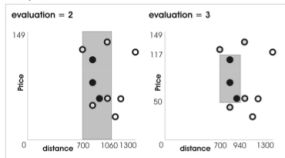
first phase



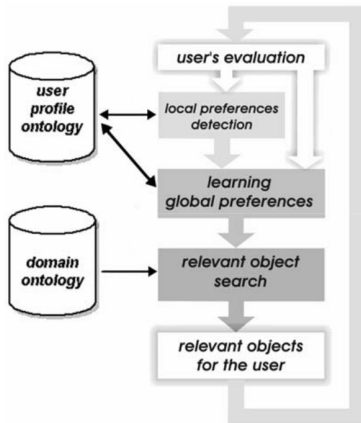
second phase



third phase



○ hotels evaluated by 1 ● hotels evaluated by 2 ● hotels evaluated by 3



# Collaborative filtering



# Neighborhood-based CF

---

Recommendation  $\hat{\phi}(u, i)$  for user  $u$  on item  $i$  using  $\phi$



# Neighborhood-based CF

---

Recommendation  $\hat{\phi}(u, i)$  for user  $u$  on item  $i$  using  $\phi$

- user-based
  - $\hat{\phi}(u, i)$  computed using feedback given by  $k$  **most similar users**

$$\mathcal{N}_i^{u,k} = \arg \max_{\mathcal{U}'} \sum_{\substack{v \in \mathcal{U}', v \neq u \\ \mathcal{U}' \subseteq \mathcal{U}_i, |\mathcal{U}'|=k}} sim(u, v)$$

- $\mathcal{U}_i = \{v \in \mathcal{U} \mid \phi(v, i) \text{ is defined on } \mathcal{D}\}$



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# Item recommendation

---

*What is the likelihood of an item  $i$  being liked by the user  $u$ ?*

---

<sup>1</sup>Simplified notation:  $\phi(u, i) \rightsquigarrow \phi_{ui}$ ,  $\mathcal{I}_u \cap \mathcal{I}_v \rightsquigarrow \mathcal{I}_{uv}$ ,  $\mathcal{U}_i \cap \mathcal{U}_j \rightsquigarrow \mathcal{U}_{ij}$



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$$\hat{\phi}_{ui} = \frac{\sum_{v \in \mathcal{N}_i^{u,k}} \text{sim}(u, v)}{k}$$

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assume that only (implicit) feedback  $\phi$  is available

- users and items represented by **sparse vectors**
  - cosine-vector similarity  $\text{sim}_{cv}$

---

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# Item recommendation – example

$sim_{cv}(i, j)$	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Titanic	1.0	0.87	0.67	0.82	0.67
Pulp Fiction	–	1.0	0.87	0.71	0.87
Iron Man	–	–	1.0	0.41	0.67
Forrest Gump	–	–	–	1.0	0.41
The Mummy	–	–	–	–	1.0

$sim_{cv}(u, v)$	Joe	Ann	Mary	Steve
Joe	1.0	0.75	0.75	0.87
Ann	–	1.0	0.75	0.58
Mary	–	–	1.0	0.58
Steve	–	–	–	1.0

## user-based<sup>1</sup>

- $\mathcal{N}_{Titanic}^{Steve,2} = \{Joe, Ann\}$ ,  $\hat{\phi}_{ST} = \frac{scv(S,J)+scv(S,M)}{2} = \frac{0.87+0.58}{2} = 0.725$
- $\mathcal{N}_{ForrestGump}^{Steve,2} = \{Ann, Mary\}$ ,  $\hat{\phi}_{ST} = \frac{scv(S,A)+scv(S,M)}{2} = \frac{0.58+0.58}{2} = 0.58$

## item-based

- $\mathcal{N}_{Steve}^{Titanic,2} = \{PulpFiction, IronMan\}$ ,  $\hat{\phi}_{ST} = \frac{scv(T,P)+scv(T,I)}{2} = \frac{0.87+0.67}{2} = 0.77$
- $\mathcal{N}_{Steve}^{ForrestGump,2} = \{PulpFiction, IronMan\}$ ,  $\hat{\phi}_{ST} = \frac{scv(F,P)+scv(F,I)}{2} = \frac{0.71+0.41}{2} = 0.56$

<sup>1</sup>  $s_{cv}$  – cosine–vector similarity



# Rating prediction

---

*How would the user rate an item?*



# Rating prediction

---

*How would the user rate an item?*

- user's/item's ratings are **biased**
  - optimistic, pessimistic users
  - items rated above or below average





# Rating prediction

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**mean-centered** rating prediction

- user-based

$$\hat{\phi}_{ui} = \bar{\phi}_u + \frac{\sum_{v \in \mathcal{N}_i^{u,k}} \text{sim}(u, v) \cdot (\phi_{vi} - \bar{\phi}_v)}{\sum_{v \in \mathcal{N}_i^{u,k}} |\text{sim}(u, v)|}$$

- $\bar{\phi}_u = \frac{\sum_{i \in \mathcal{I}_u} \phi(u, i)}{|\mathcal{I}_u|}$
- item-based

$$\hat{\phi}_{ui} = \bar{\phi}_i + \frac{\sum_{j \in \mathcal{N}_u^{i,k}} \text{sim}(i, j) \cdot (\phi_{uj} - \bar{\phi}_j)}{\sum_{v \in \mathcal{N}_u^{i,k}} |\text{sim}(i, j)|}$$

- $\bar{\phi}_i = \frac{\sum_{u \in \mathcal{U}_i} \phi(u, i)}{|\mathcal{U}_i|}$



# Pearson-correlation similarity

---

*What similarity measure to use?*

- $sim_{cv}$  doesn't take into account the mean and variances of ratings



# Pearson-correlation similarity

---

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**pearson-correlation** similarity

$$sim_{pc}(u, v) = \frac{\sum_{i \in \mathcal{I}_{uv}} (\phi_{ui} - \bar{\phi}_u)(\phi_{vi} - \bar{\phi}_v)}{\sqrt{\sum_{i \in \mathcal{I}_{uv}} (\phi_{ui} - \bar{\phi}_u)^2 \sum_{i \in \mathcal{I}_{uv}} (\phi_{vi} - \bar{\phi}_v)^2}}$$

$$sim_{pc}(i, j) = \frac{\sum_{u \in \mathcal{U}_{ij}} (\phi_{ui} - \bar{\phi}_i)(\phi_{uj} - \bar{\phi}_j)}{\sqrt{\sum_{u \in \mathcal{U}_{ij}} (\phi_{ui} - \bar{\phi}_i)^2 \sum_{u \in \mathcal{U}_{ij}} (\phi_{uj} - \bar{\phi}_j)^2}}$$



# Rating prediction – example

$sim_{pc}(i, j)$	Titanic	Pulp Fiction	Iron Man	Forrest Gump	The Mummy
Titanic	1.0	-0.956	-0.815	NaN	-0.581
Pulp Fiction	-	1.0	0.948	NaN	0.621
Iron Man	-	-	1.0	NaN	0.243
Forrest Gump	-	-	-	1.0	NaN
The Mummy	-	-	-	-	1.0

NaN values are usually converted to zero (rare in case of enough data)

$sim_{pc}(u, v)$	Joe	Ann	Mary	Steve
Joe	1.0	-0.716	-0.762	-0.005
Ann	-	1.0	0.972	0.565
Mary	-	-	1.0	0.6
Steve	-	-	-	1.0

## user-based

- $\mathcal{U}_{Titanic} = \{Joe, Ann, Mary\}$ ,  $\mathcal{N}_{Titanic}^{Steve,2} = \{Mary, Ann\}$
- $\bar{\phi}_{Steve} = \frac{11}{3} = 3.67$ ,  $\bar{\phi}_{Mary} = \frac{12}{4} = 3$ ,  $\bar{\phi}_{Ann} = \frac{13}{4} = 3.25$
- $\hat{\phi}_{ST} = \bar{\phi}_S + \frac{s_{pc}(S,M) \cdot (\phi_{MT} - \bar{\phi}_M) + s_{pc}(S,A) \cdot (\phi_{AT} - \bar{\phi}_A)}{|s_{pc}(S,M)| + |s_{pc}(S,A)|} = 3.67 + \frac{0.6 \cdot (4-3) + 0.565 \cdot (5-3.25)}{0.6+0.565} = 1.36$

## item-based

- $\mathcal{I}_{Steve} = \{Pulp Fiction, Iron Man, The Mummy\}$ ,  $\mathcal{N}_{Steve}^{Titanic,2} = \{Iron Man, The Mummy\}$
- $\bar{\phi}_T = \frac{10}{3} = 3.34$ ,  $\bar{\phi}_I = \frac{11}{3} = 3.67$ ,  $\bar{\phi}_M = \frac{9}{3} = 3$
- $\hat{\phi}_{ST} = \bar{\phi}_T + \frac{s_{pc}(T,I) \cdot (\phi_{SI} - \bar{\phi}_I) + s_{pc}(T,M) \cdot (\phi_{SM} - \bar{\phi}_M)}{|s_{pc}(T,I)| + |s_{pc}(T,M)|} = 3.34 + \frac{-0.815 \cdot (4-3.67) - 0.581 \cdot (4-3)}{0.815+0.581} = 2.73$



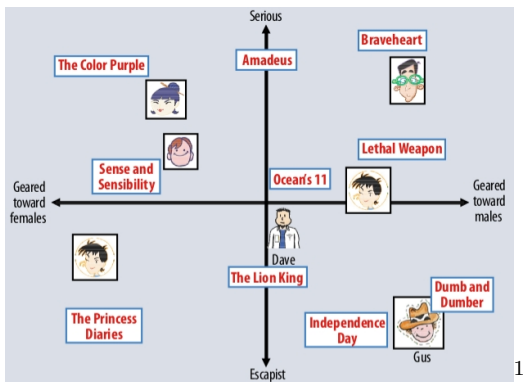
# Matrix factorization



# A latent space representation

Map users and items to a common latent space

- where dimensions or **factors** represent
  - items' **implicit properties**
  - users' **interest** in items' hidden properties



<sup>1</sup>The picture is taken from Y. Koren et al. (2009). *Matrix Factorization Techniques for Recommender Systems*. *Computer* 42 (8).

# Known factorization models (1/2)

---

$\phi$  represented as a user-item matrix  $\Phi^{n \times m}$

- $n$  users,  $m$  items

---

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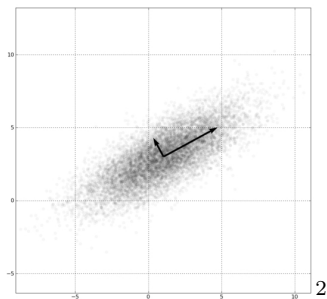
# Known factorization models (1/2)

$\phi$  represented as a user-item matrix  $\Phi^{n \times m}$

- $n$  users,  $m$  items

## Principal Component Analysis (PCA)

- transform data to a new coordinate system
  - variances by any projection of the data lies on coordinates in decreasing order



<sup>2</sup>The picture is taken from wikipedia.



## Singular Value Decomposition (SVD)

$$\Phi = W^{n \times k} \Sigma^{k \times k} H^{n \times k^T}$$

- $W^T W = I, H^T H = I$
- column vectors of  $W$  are orthonormal eigenvectors of  $\Phi \Phi^T$
- column vectors of  $H$  are orthonormal eigenvectors of  $\Phi^T \Phi$
- $\Sigma$  contains eigenvalues of  $W$  in descending order

---

<sup>1</sup>T.Raiko et al. (2007). Principal Component Analysis for Sparse High-Dimensional Data. Neural Information Processing, LNCS. 4984.

<sup>2</sup>A.K. Menon and Ch. Elkan (2011). Fast Algorithms for Approximating the Singular Value Decomposition. ACM Trans. Knowl. Discov. Data 5 (2).



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- column vectors of  $H$  are orthonormal eigenvectors of  $\Phi^T \Phi$
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*PCA, SVD computed algebraically*

- $\Phi$  is a **big** and **sparse** matrix
  - approximations of PCA<sup>1</sup>, SVD<sup>2</sup>

---

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## MF – rating prediction (1/2)

---

recommendation task

- to find  $\hat{\phi} : \mathcal{U} \times \mathcal{I} \rightarrow \mathbb{R}$  such that  $acc(\hat{\phi}, \phi, \mathcal{T})$  is maximal



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  - $acc$  is the **expected** accuracy on  $\mathcal{T}$
  - training  $\hat{\phi}$  on  $\mathcal{D}$  such that the **empirical** loss  $err(\hat{\phi}, \phi, \mathcal{D})$  is minimal



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a simple, **approximative** MF **model**

- only  $W^{n \times k}$  and  $H^{m \times k}$
- $k$  – the number of factors

$$\Phi^{n \times m} \approx \hat{\Phi}^{n \times m} = WH^T$$

- predicted rating  $\hat{\phi}_{ui}$  of the user  $u$  for the item  $i$

$$\hat{\phi}_{ui} = w_u h_i^T$$



## MF – rating prediction (2/2)

---

the **loss** function  $err(\hat{\phi}, \phi, \mathcal{D})$

- squared loss

$$err(\hat{\phi}, \phi, \mathcal{D}) = \sum_{(u,i) \in \mathcal{D}} e_{ui}^2 = \sum_{(u,i) \in \mathcal{D}} (\phi_{ui} - \hat{\phi}_{ui})^2 = \sum_{(u,i) \in \mathcal{D}} (\phi_{ui} - w_u h_i^T)^2$$



## MF – rating prediction (2/2)

---

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the **objective function**

- **regularization** term  $\lambda \geq 0$  to prevent overfitting
  - penalizing the magnitudes of parameters

$$f(\hat{\phi}, \phi, \mathcal{D}) = \sum_{(u,i) \in \mathcal{D}} (\phi_{ui} - w_u h_i^T)^2 + \lambda(\|W\|^2 + \|H\|^2)$$



## MF – rating prediction (2/2)

---

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$$f(\hat{\phi}, \phi, \mathcal{D}) = \sum_{(u,i) \in \mathcal{D}} (\phi_{ui} - w_u h_i^T)^2 + \lambda(\|W\|^2 + \|H\|^2)$$

The task is to find parameters  $W$  and  $H$  such that, given  $\lambda$ , the objective function  $f(\hat{\phi}, \phi, \mathcal{D})$  is minimal.





# Gradient descent

---

*How to find a minimum of an “objective” function  $f(\Theta)$ ?*

- in case of MF,  $\Theta = W \cup H$ , and
- $f(\Theta)$  refers to the error of approximation of  $\Phi$  by  $WH^T$



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## Gradient descent

**input:**  $f, \alpha, \Sigma^2$ , *stopping criteria*

initialize  $\Theta \sim \mathcal{N}(0, \Sigma^2)$

**repeat**

$$\Theta \leftarrow \Theta - \alpha \frac{\partial f}{\partial \Theta}(\Theta)$$

**until** approximate minimum is reached

**return**  $\Theta$



# Gradient descent

---

How to find a minimum of an “objective” function  $f(\Theta)$ ?

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stopping criteria

- $|\Theta^{old} - \Theta| < \epsilon$
- maximum number of iterations reached
- a combination of both



# Stochastic gradient descent

---

if  $f$  can be written as

$$f(\Theta) = \sum_{i=1}^n f_i(\Theta)$$



# Stochastic gradient descent

---

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## Stochastic gradient descent (SGD)

**input:**  $f_i, \alpha, \Sigma^2$ , *stopping criteria*

initialize  $\Theta \sim \mathcal{N}(0, \Sigma^2)$

**repeat**

**for all**  $i$  in random order **do**

$$\Theta \leftarrow \Theta - \alpha \frac{\partial f_i}{\partial \Theta}(\Theta)$$

**end for**

**until** approximate minimum is reached

**return**  $\Theta$



**updating** parameters **iteratively** for each data point  $\phi_{ui}$  in the opposite direction of the **gradient** of the objective function at the given point until a **convergence** criterion is fulfilled.

- updating the vectors  $w_u$  and  $h_i$  for the data point  $(u, i) \in D$



**updating** parameters **iteratively** for each data point  $\phi_{ui}$  in the opposite direction of the **gradient** of the objective function at the given point until a **convergence** criterion is fulfilled.

- updating the vectors  $w_u$  and  $h_i$  for the data point  $(u, i) \in D$

$$\frac{\partial f}{\partial w_u}(u, i) = -2(e_{ui}h_i - \lambda w_u)$$

$$\frac{\partial f}{\partial h_i}(u, i) = -2(e_{ui}w_u - \lambda h_i)$$

$$w_u(u, i) \leftarrow w_u - \alpha \frac{\partial f}{\partial w_u}(u, i) = w_u + \alpha(e_{ui}h_i - \lambda w_u)$$

$$h_i(u, i) \leftarrow h_i - \alpha \frac{\partial f}{\partial h_i}(u, i) = h_i + \alpha(e_{ui}w_u - \lambda h_i)$$

where  $\alpha > 0$  is a **learning rate**.



## MF with SGD – Algorithm

*Hyper-parameters:  $k$ ,  $iters$  (the max number of iteration),  $\alpha$ ,  $\lambda$ ,  $\Sigma^2$*

$W \leftarrow \mathcal{N}(0, \Sigma^2)$

$H \leftarrow \mathcal{N}(0, \Sigma^2)$

**for**  $iter \leftarrow 1, \dots, iters \cdot |\mathcal{D}|$  **do**

draw randomly  $(u, i)$  from  $\mathcal{D}$

$\hat{\phi}_{ui} \leftarrow 0$

**for**  $j \leftarrow 1, \dots, k$  **do**

$\hat{\phi}_{ui} \leftarrow \hat{\phi}_{ui} + W[u][j] \cdot H[i][j]$

**end for**

$e_{ui} = \phi_{ui} - \hat{\phi}_{ui}$

**for**  $j \leftarrow 1, \dots, k$  **do**

$W[u][j] \leftarrow W[u][j] + \alpha * (e_{ui} * H[i][j] - \lambda * W[u][j])$

$H[i][j] \leftarrow H[i][j] + \alpha * (e_{ui} * W[u][j] - \lambda * H[i][j])$

**end for**

**end for**

**return**  $\{W, H\}$





## MF with SGD – Example<sup>2</sup>

Let's have the following hyper-parameters:

$K = 2$ ,  $\alpha = 0.1$ ,  $\lambda = 0.15$ ,  $iter = 150$ ,  $\sigma^2 = 0.01$

$$\Phi = \begin{array}{|c|c|c|c|c|} \hline 1 & 4 & 5 & & 3 \\ \hline 5 & 1 & & 5 & 2 \\ \hline 4 & 1 & 2 & 5 & \\ \hline & 3 & 4 & & 4 \\ \hline \end{array}$$

Results are:

$$W = \begin{array}{|c|c|} \hline 1.1995242 & 1.1637173 \\ \hline 1.8714619 & -0.02266505 \\ \hline 2.3267753 & 0.27602595 \\ \hline 2.033842 & 0.539499 \\ \hline \end{array}$$

$$H^T = \begin{array}{|c|c|c|c|c|} \hline 1.6261001 & 1.1259034 & 2.131041 & 2.2285593 & 1.6074764 \\ \hline -0.40649664 & 0.7055319 & 1.0405376 & 0.39400166 & 0.49699315 \\ \hline \end{array}$$

Results<sup>1</sup> are:

$$\hat{\Phi} = \begin{array}{|c|c|c|c|c|} \hline 1.477499 & 2.171588 & 3.767126 & 3.131717 & 2.506566 \\ \hline 3.052397 & 2.091094 & 3.964578 & 4.161733 & 2.997066 \\ \hline 3.671365 & 2.814469 & 5.245668 & 5.294111 & 3.877419 \\ \hline 3.087926 & 2.670543 & 4.895569 & 4.745101 & 3.537480 \\ \hline \end{array}$$

<sup>1</sup>Note, that these hyper-parameters are just picked up in an ad-hoc manner. One should search for the “best” hyper-parameter combinations using e.g. grid-search (a brute-force approach).

<sup>2</sup>Thanks to my colleague Thai-Nghe Nguyen for computing an example.



## baseline estimate

- user-item bias

$$b_{ui} = \mu + b'_u + b''_i$$

- $\mu$  – average rating across the whole  $\mathcal{D}$
- $b', b''$  – vectors of user and item biases, respectively



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## prediction

$$\hat{\phi}_{ui} = \mu + b'_u + b''_i + w_u h_i$$



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## prediction

$$\hat{\phi}_{ui} = \mu + b'_u + b''_i + w_u h_i$$

## objective function to minimize

$$f(\phi, \hat{\phi}, \mathcal{D}) = \sum_{(u,i) \in \mathcal{D}} (\phi_{ui} - \mu - b'_u - b''_i - w_u h_i)^2 + \lambda(\|W\|^2 + \|H\|^2 + b'^2 + b''^2)$$



# Biased MF with SGD

---

similar to unbiased MF

- initialize average and biases

$$\mu = \frac{\sum_{(u,i) \in \mathcal{D}}}{|\mathcal{D}|}$$

$$b' \leftarrow (\bar{\phi}_{u_1}, \dots, \bar{\phi}_{u_n})$$

$$b'' \leftarrow (\bar{\phi}_{i_1}, \dots, \bar{\phi}_{i_m})$$



# Biased MF with SGD

similar to unbiased MF

- initialize average and biases

$$\mu = \frac{\sum_{(u,i) \in \mathcal{D}}}{|\mathcal{D}|}$$

$$b' \leftarrow (\bar{\phi}_{u_1}, \dots, \bar{\phi}_{u_n})$$

$$b'' \leftarrow (\bar{\phi}_{i_1}, \dots, \bar{\phi}_{i_m})$$

- update average and biases

$$\mu \leftarrow \mu - \frac{\partial f}{\partial \mu}(u, i) = \mu + \alpha e_{ui}$$

$$b' \leftarrow b' - \frac{\partial f}{\partial b'}(u, i) = b' + \alpha(e_{ui} - \lambda b')$$

$$b'' \leftarrow b'' - \frac{\partial f}{\partial b''}(u, i) = b'' + \alpha(e_{ui} - \lambda b'')$$



## MF – item recommendation

---

to predict a personalized **ranking score**<sup>1</sup>  $\hat{\phi}_{ui}$

- how the item  $i$  is preferred to other items for the user  $u$
- to find  $W$  and  $H$  such that  $\hat{\Phi} = WH^T$

$$\hat{\phi}_{ui} = w_u h_i^T$$

---

<sup>1</sup>S. Rendle et al. (2009). BPR: Bayesian Personalized Ranking from Implicit Feedback. 25th Conference on Uncertainty in Artificial Intelligence.



# MF – item recommendation

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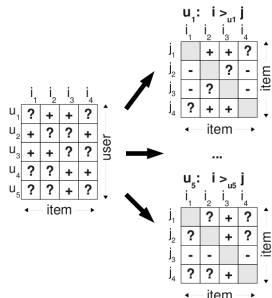
- how the item  $i$  is preferred to other items for the user  $u$
- to find  $W$  and  $H$  such that  $\hat{\Phi} = WH^T$

$$\hat{\phi}_{ui} = w_u h_i^T$$

problem: positive feedback only

- **pairwise ranking data**

$$\mathcal{D}_p = \{(u, i, j) \in \mathcal{D} \mid i \in \mathcal{I}_u \wedge j \in \mathcal{I} \setminus \mathcal{I}_u\}$$



<sup>1</sup>S. Rendle et al. (2009). BPR: Bayesian Personalized Ranking from Implicit Feedback. 25th Conference on Uncertainty in Artificial Intelligence.



## Bayesian formulation of the problem

- $\succ$  – the unknown preference structure (ordering)
  - we use the derived pairwise ranking data  $\mathcal{D}_p$
- $\Theta$  – parameters of an arbitrary prediction model
  - in case of MF,  $\Theta = W \cup H$

$$p(\Theta | \succ) \propto p(\succ | \Theta)p(\Theta)$$



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$$p(\Theta | \succ) \propto p(\succ | \Theta)p(\Theta)$$

## prior probability

- assume independence of parameters
- assume,  $\Theta \sim N(0, \frac{1}{\lambda}I)$

$$p(\Theta) = \prod_{\theta \in \Theta} \sqrt{\frac{\lambda}{2\pi}} e^{-\frac{1}{2}\lambda\theta^2}$$



## likelihood

- assume users' feedbacks are independent
- assume, ordering of each pair is independent

$$p(\gamma | \Theta) = \prod_{u \in \mathcal{U}} p(\gamma_u | \Theta) = \prod_{(u,i,j) \in \mathcal{D}_p} p(i \succ_u j | \Theta)$$



# MF – Bayesian Personalized Ranking (2/3)

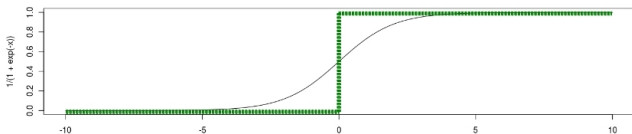
## likelihood

- assume users' feedbacks are independent
- assume, ordering of each pair is independent

$$p(\succ | \Theta) = \prod_{u \in \mathcal{U}} p(\succ_u | \Theta) = \prod_{(u,i,j) \in \mathcal{D}_p} p(i \succ_u j | \Theta)$$

- using the ranking scores  $\hat{\phi}$

$$p(i \succ_u j | \Theta) = p(\hat{\phi}_{ui} - \hat{\phi}_{uj} > 0) = \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) = \frac{1}{1 + e^{-(\hat{\phi}_{ui} - \hat{\phi}_{uj})}}$$



# MF – Bayesian Personalized Ranking (3/3)

---

maximum **a posteriori estimation** of  $\Theta$

$$\arg \max_{\Theta} p(\Theta, \gamma) =$$



# MF – Bayesian Personalized Ranking (3/3)

---

maximum **a posteriori estimation** of  $\Theta$

$$\arg \max_{\Theta} p(\Theta, \gamma) =$$

$$\arg \max_{\Theta} p(\gamma | \Theta)p(\Theta) =$$



## MF – Bayesian Personalized Ranking (3/3)

---

maximum **a posteriori estimation** of  $\Theta$

$$\arg \max_{\Theta} p(\Theta, \gamma) =$$

$$\arg \max_{\Theta} p(\gamma | \Theta)p(\Theta) =$$

$$\arg \max_{\Theta} \ln p(\gamma | \Theta)p(\Theta) =$$



## MF – Bayesian Personalized Ranking (3/3)

maximum **a posteriori** estimation of  $\Theta$

$$\arg \max_{\Theta} p(\Theta, \gamma) =$$

$$\arg \max_{\Theta} p(\gamma | \Theta)p(\Theta) =$$

$$\arg \max_{\Theta} \ln p(\gamma | \Theta)p(\Theta) =$$

$$\arg \max_{\Theta} \ln \prod_{(u,i,j) \in \mathcal{D}_p} \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) \sqrt{\frac{\lambda}{2\pi}} e^{-\frac{1}{2}\lambda\theta^2}$$





# MF – Bayesian Personalized Ranking (3/3)

maximum a posteriori estimation of  $\Theta$

$$\arg \max_{\Theta} p(\Theta, \gamma) =$$

$$\arg \max_{\Theta} p(\gamma | \Theta) p(\Theta) =$$

$$\arg \max_{\Theta} \ln p(\gamma | \Theta) p(\Theta) =$$

$$\arg \max_{\Theta} \ln \prod_{(u,i,j) \in \mathcal{D}_p} \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) \sqrt{\frac{\lambda}{2\pi}} e^{-\frac{1}{2}\lambda\theta^2}$$

$$\arg \max_{\Theta} \underbrace{\sum_{(u,i,j) \in \mathcal{D}_p} \ln \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) - \lambda \|\Theta\|^2}_{\text{BPR-OPT}}$$



# Finding parameters for BPR-OPT

---

Stochastic gradient ascent

$$\frac{\partial BPR - OPT}{\partial \Theta} \propto \sum_{(u,i,j) \in \mathcal{D}_p} \frac{e^{-(\hat{\phi}_{ui} - \hat{\phi}_{uj})}}{1 + e^{-(\hat{\phi}_{ui} - \hat{\phi}_{uj})}} \frac{\partial}{\partial \Theta} (\hat{\phi}_{ui} - \hat{\phi}_{uj}) - \lambda \Theta$$



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$$\frac{\partial}{\partial \theta} (\hat{\phi}_{ui} - \hat{\phi}_{uj}) = \begin{cases} (h_i - h_j) & \text{if } \theta = w_u \\ w_u & \text{if } \theta = h_i \\ -w_u & \text{if } \theta = h_j \\ 0 & \text{else} \end{cases}$$



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## LearnBPR

**input:**  $f_i, \alpha, \Sigma^2$ , *stopping criteria*

initialize  $\Theta \sim \mathcal{N}(0, \Sigma^2)$

**repeat**

draw  $(u, i, j) \in \mathcal{D}_p$  randomly

$\Theta \leftarrow \Theta + \alpha \frac{\partial BPR - OPT}{\partial \Theta}(\Theta)$

**until** approximate maximum is reached

**return**  $\Theta$



## Area under the ROC curve (AUC)

- probability that the ranking of a randomly drawn pair is correct

$$AUC = \sum_{u \in \mathcal{U}} AUC(u) = \frac{1}{|\mathcal{U}|} \frac{1}{|\mathcal{I}_u| |\mathcal{I} \setminus \mathcal{I}_u|} \sum_{(u,i,j) \in \mathcal{D}_p} \delta(\hat{\phi}_{ui} \succ \hat{\phi}_{uj})$$

- $\delta(\hat{\phi}_{ui} \succ \hat{\phi}_{uj}) = 1$  if  $\hat{\phi}_{ui} \succ \hat{\phi}_{uj}$ , and 0, else



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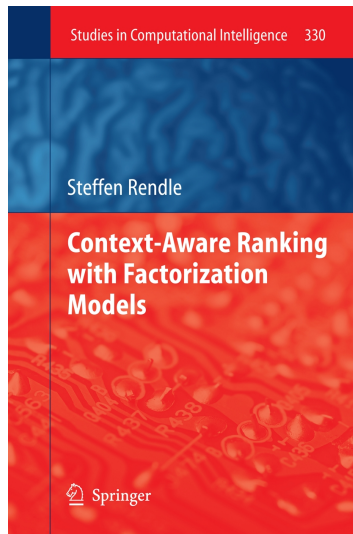
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$$BPR - OPT = \sum_{(u,i,j) \in \mathcal{D}_p} \ln \sigma(\hat{\phi}_{ui} - \hat{\phi}_{uj}) - \lambda \|\Theta\|^2$$

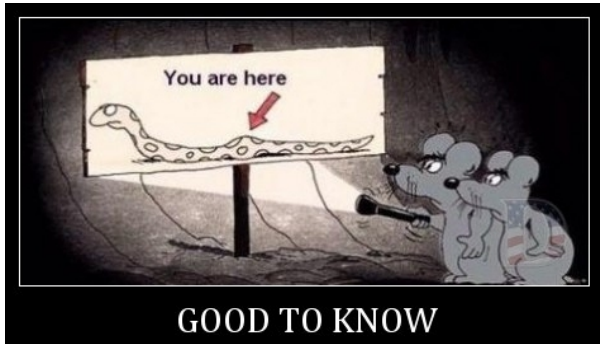


# More info on ranking with factorization models





## Issues worth to mention



**GOOD TO KNOW**

# The cold-start problem

---

arises when not enough collaborative information is available

- new user or new item

---

<sup>1</sup>Z. Gantner et al. (2010). Learning Attribute-to-Feature Mappings for Cold-Start Recommendations. 10th IEEE International Conference on Data Mining.



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possible solutions

- recommend popular items, “predict” global average, ...

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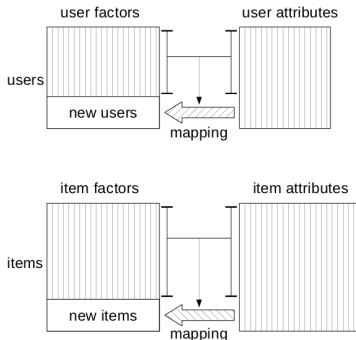
# The cold-start problem

arises when not enough collaborative information is available

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possible solutions

- recommend popular items, “predict” global average, ...
- utilize item attributes<sup>1</sup>



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# Context-aware recommendation

---

**Context** is any additional information, besides  $\chi^{user}$ ,  $\chi^{item}$ ,  $\phi$  and  $\kappa$ , that is relevant for the recommendation<sup>1</sup>

- time, location, companion (when, where and with whom the user wants to watch some movie)

---

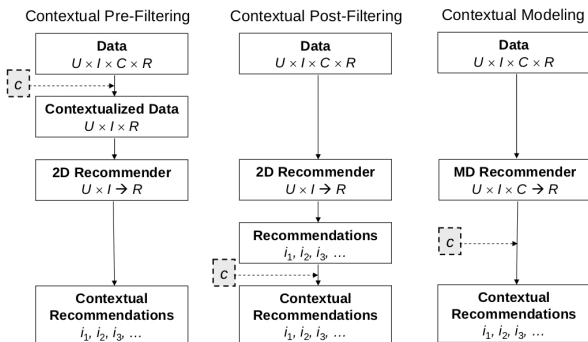
<sup>1</sup>Picture from *G. Adomavicius and A. Tuzhilin: Context-Aware Recommender Systems. Tutorial on the 2nd ACM International Conference on Recommender Systems, 2008.*  
<http://ids.csom.umn.edu/faculty/gedas/talks/RecSys2008-tutorial.pdf>



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## experiments



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- **offline**
  - no interaction with real users, need to simulate user behaviour
  - low cost, short time
  - answers only a few questions, e.g. the predictive power of techniques





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  - observing test subjects' behaviour in the system
  - questionnaires
  - expensive, small scale,



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- **online evaluation**

- redirect a small part of the traffic to an alternative recommendation engine
- risky – we can lose some customers
- good to do after an offline testing of a recommendation engine shows good results



## Evaluating RS (2/3)

---

### **properties** of a recommender system

- user preference
  - Which one from different RS users prefer more?



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- trust
  - What is the users’ trust in recommendation?





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  - How surprising the recommendations are? (e.g. a new movie with the user's favourite actor can be novel but not surprising)



## Evaluating RS (3/3)

---

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- adaptivity
  - How does a RS adapt to changes in the item collection?
- scalability
  - How scalable a RS is?



## The MyMediaLite library





## MyMediaLite

- is lightweight, multi-purpose library
- is mainly a library, meant to be used by other applications
- is free software (under the terms of the GNU General Public License)
- was developed by Zeno Gantner, Steffen Rendle, and Christoph Freudenthaler at University of Hildesheim



<http://ismll.de/mymedialite>

## major

- **scalable** implementations of many state-of-the-art recommendation methods
- evaluation framework for **reproducible** research
- **ready to be used**: command line tools, not programming necessary



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## using for

- rating prediction
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- evaluation framework for **reproducible** research
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## using for

- rating prediction
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## next features

- usable from C#, Python, Ruby, F#
- Java ports available
- written in C#, runs on Mono
- regular releases (ca. 1 every 2 months)



## State-of-the-art recommendation methods in MyMediaLite:

- kNN variants
- Online-Updating Regularized Kernel Matrix Factorization [Rendle and Schmidt-Thieme, RecSys 2009]
- SocialMF [Jamali and Ester, RecSys 2010] Freudenthaler at University of Hildesheim
- Asymmetric Factor Models (AFM) [Paterek, KDD Cup 2007]
- SVD++ [Koren, KDD 2008]
- Weighted Regularized Matrix Factorization (WR-MF) [Hu and Koren, ICDM 2008], [Pan et al., ICDM 2008]
- BPR-MF [Rendle et al., UAI 2009]



e.g. MovieLens, Netflix

user ID	item ID	rating	timestamp
196	242	3	881250949
186	302	3	891717742
22	377	1	878887116
244	51	2	880606923



e.g. MovieLens, Netflix

user ID	item ID	rating	timestamp
196	242	3	881250949
186	302	3	891717742
22	377	1	878887116
244	51	2	880606923

## Remarks

- user and item IDs can be (almost) arbitrary strings
- separator: whitespace, tab, comma, ::
- alternative date and time format: yyyy-mm-dd
- rating and date and time fields are optional
- import script; Unix tools, Perl, Python . . .



## Getting Help

- rating prediction --help

## Data sets

- rating prediction --training-file=u1.base  
--test-file=u1.test

## Recommender Options

- rating prediction --training-file=u.data  
--test-ratio=0.2

## Fixing the Random Seed

- rating prediction ... --random-seed=1

## Choosing a Recommender (algorithm)

- rating prediction ... --recommender=UserAverage
- rating prediction ... --recommender=UserItemBaseline





## Iterative Recommenders

- rating prediction  
... --recommender=BiasedMatrixFactorization  
--find-iter=1 --max-iter=30

## Recommender Options (Hyperparameters)

- rating prediction  
... --recommender-options='num factors=5'
- rating prediction ...  
--recommender-options='num\_factors=5 reg=0.05''

## SVD++

- rating prediction ... --recommender=SVDPlusPlus  
--recommender-options='num factors=5 reg=0.1  
learn rate=0.01''



### input data

- `user_id item_id rating`

1	1	5
1	2	3
1	3	4
1	4	3
1	5	3
1	7	4

where `user_id` and `item_id` are integers referring to users and items, respectively, and `rating` is a floating-point number expressing how much a user likes an item

- **separator:** either spaces, tabs, or commas
- only three columns, all additional columns will be ignored



## Example: rating prediction

---

### input data

- `user_id item_id rating`

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- **separator:** either spaces, tabs, or commas
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### usage of the rating prediction program

```
rating_prediction --training-file=TRAINING_FILE  
--test-file=TEST_FILE --recommender=METHOD [OPTIONS]
```



# Example: simple and advanced recommender

---

## simple recommender

- run: `rating_prediction --training-file=u1.base --test-file=u1.test --recommender=UserAverage`
- output: `UserAverage training_time 00:00:00.000098 RMSE 1.063 MAE 0.85019 testing_time 00:00:00.032326`



# Example: simple and advanced recommender

---

## simple recommender

- run: `rating_prediction --training-file=u1.base --test-file=u1.test --recommender=UserAverage`
- output: `UserAverage training_time 00:00:00.000098 RMSE 1.063 MAE 0.85019 testing_time 00:00:00.032326`

## advanced recommender

- run: `rating_prediction --training-file=u1.base --test-file=u1.test --recommender=BiasedMatrixFactorization`
- output: `BiasedMatrixFactorization num_factors=10 regularization=0.015 learn_rate=0.01 num_iter=30 init_mean=0 init_stdev=0.1 training_time 00:00:03.3575780 RMSE 0.96108 MAE 0.75124 testing_time 00:00:00.0159740`



## Example: hyperparameter search

---

- run: `rating_prediction --training-file=u1.base  
--test-file=u1.test  
--recommender=BiasedMatrixFactorization  
--recomender-options="num_factors=20 num_iter=0"  
--max-iter=25 --num-iter=0`



## Example: hyperparameter search

- run: `rating_prediction --training-file=u1.base --test-file=u1.test --recommender=BiasedMatrixFactorization --recomender-options="num_factors=20 num_iter=0" --max-iter=25 --num-iter=0`
- output:

```
...
RMSE 1.17083 MAE 0.96918 iteration 0      RMSE 0.95578 MAE 0.75501 iteration 13
RMSE 1.01383 MAE 0.8143 iteration 1      RMSE 0.95573 MAE 0.75467 iteration 14
RMSE 0.98742 MAE 0.78742 iteration 2      RMSE 0.95611 MAE 0.75467 iteration 15
RMSE 0.97672 MAE 0.77668 iteration 3      RMSE 0.9569 MAE 0.75499 iteration 16
RMSE 0.9709 MAE 0.77078 iteration 4      RMSE 0.95802 MAE 0.75551 iteration 17
RMSE 0.96723 MAE 0.76702 iteration 5      RMSE 0.95942 MAE 0.75623 iteration 18
RMSE 0.96466 MAE 0.76442 iteration 6      RMSE 0.96102 MAE 0.7571 iteration 19
RMSE 0.96269 MAE 0.76241 iteration 7      RMSE 0.96277 MAE 0.75806 iteration 20
RMSE 0.96104 MAE 0.76069 iteration 8      RMSE 0.96463 MAE 0.75909 iteration 21
RMSE 0.95958 MAE 0.75917 iteration 9      RMSE 0.96656 MAE 0.76017 iteration 22
RMSE 0.95825 MAE 0.75783 iteration 10     RMSE 0.96852 MAE 0.7613 iteration 23
RMSE 0.95711 MAE 0.75667 iteration 11     RMSE 0.9705 MAE 0.76246 iteration 24
RMSE 0.95626 MAE 0.75569 iteration 12     RMSE 0.97247 MAE 0.76364 iteration 25
```



# Why use MyMediaLite?

---

- simple
- free
- scalable
- well-documented
- well-tested





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## possibility of using extra features

- Item Recommendation Tool (very similar usage like `rating_prediction`)
- `--cross-validation=K`
- `--chronological-split=2012-01-01`
- `--online-evaluation`
- `--save-model=FILE --load-model=FILE`
- `--measure=RMSE --epsilon=0.001`
- ...



# Summary



# Types of RS (1/2)

---

## Knowledge-based

- pros: no cold-start, deterministic
- cons: knowledge-engineering needed, static

## Content-based

- pros: no collaborative information needed
- cons: content is needed, cold-start for new users, no serendipity

## Collaborative-filtering

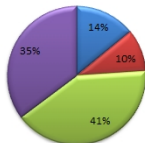
- pros: no user nor item attributes needed, serendipity
- cons: cold-start for new users and items



# Types of RS (2/2)

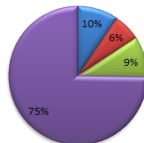
## RecSys 2010 - 2012

■ Content-based ■ Knowledge-Based ■ Collaborative filtering ■ other



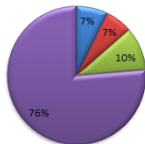
## WWW 2010 - 2012

■ Content-based ■ Knowledge-Based ■ Collaborative filtering ■ other



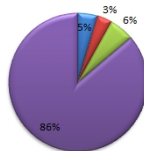
## UMAP 2010 - 2012

■ Content-based ■ Knowledge-Based ■ Collaborative filtering ■ other



## WSDM 2010 - 2012

■ Content-based ■ Knowledge-Based ■ Collaborative filtering ■ other





*That's all Folks!*

## Many thanks go to

---

- **Štefan Pero** for his great help
- **Zeno Gantner** for providing materials and help regarding MyMediaLite
- **Artus Krohn-Grimberghe** for a picture from his PhD defense presentation
- all my colleagues and friends from ICS, UPJŠ and the ISMLL, UHI as well as other institutes for helping me to understand these things ;)

...also,

- all the people providing their materials (funny pictures, graphs, leaderboards, ...) on the web

...and, last but not least

- **YOU for your attention!**

# Questions?



Tomas.Horvath@upjs.sk

<http://www.ics.upjs.sk/~horvath>