NSWI166 Introduction to recommender systems and user preferences

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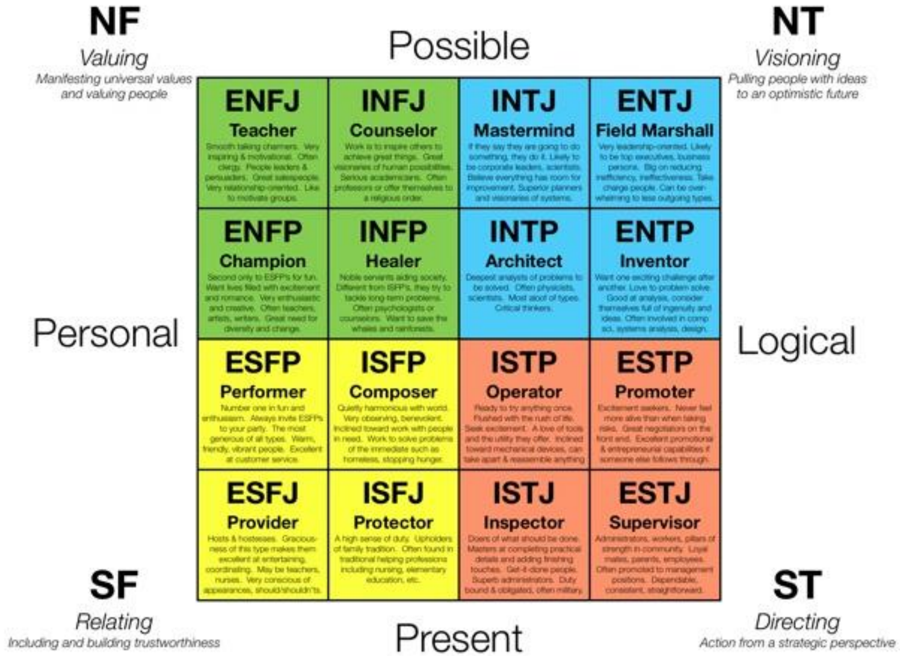
8/12 Challenge Response Framework

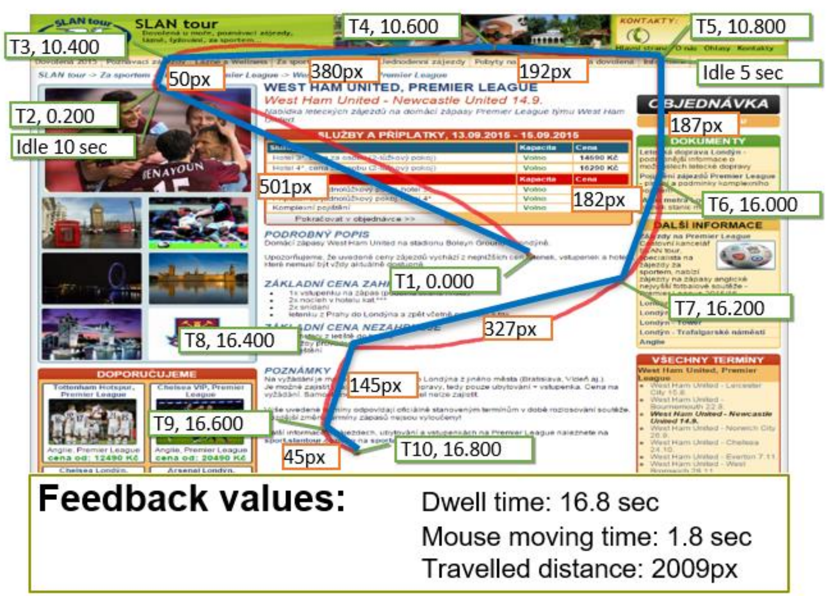
A general Framework for modeling and evaluation of model-quality

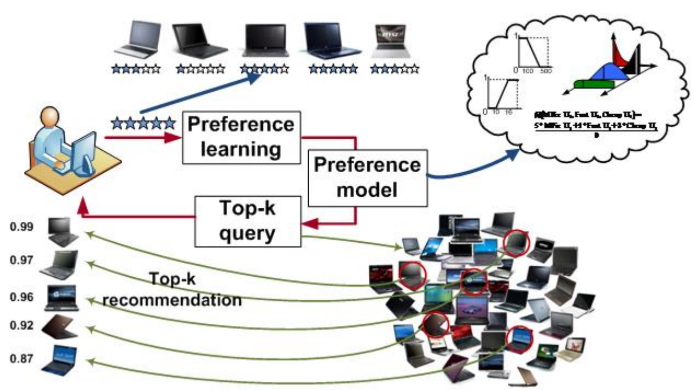
Nástin obsahu

- Dosavadní přehled témat – vše pro zákazníka
- Společný jmenovatel – modelování reality
- Další příklady modelování reality
- Potřeba abstraktního rámce pro modelování reality a vyhodnocení kvality modelu
  - Challenge - Response Framework, situations, reductions
- Model kvality e-shopu, +++

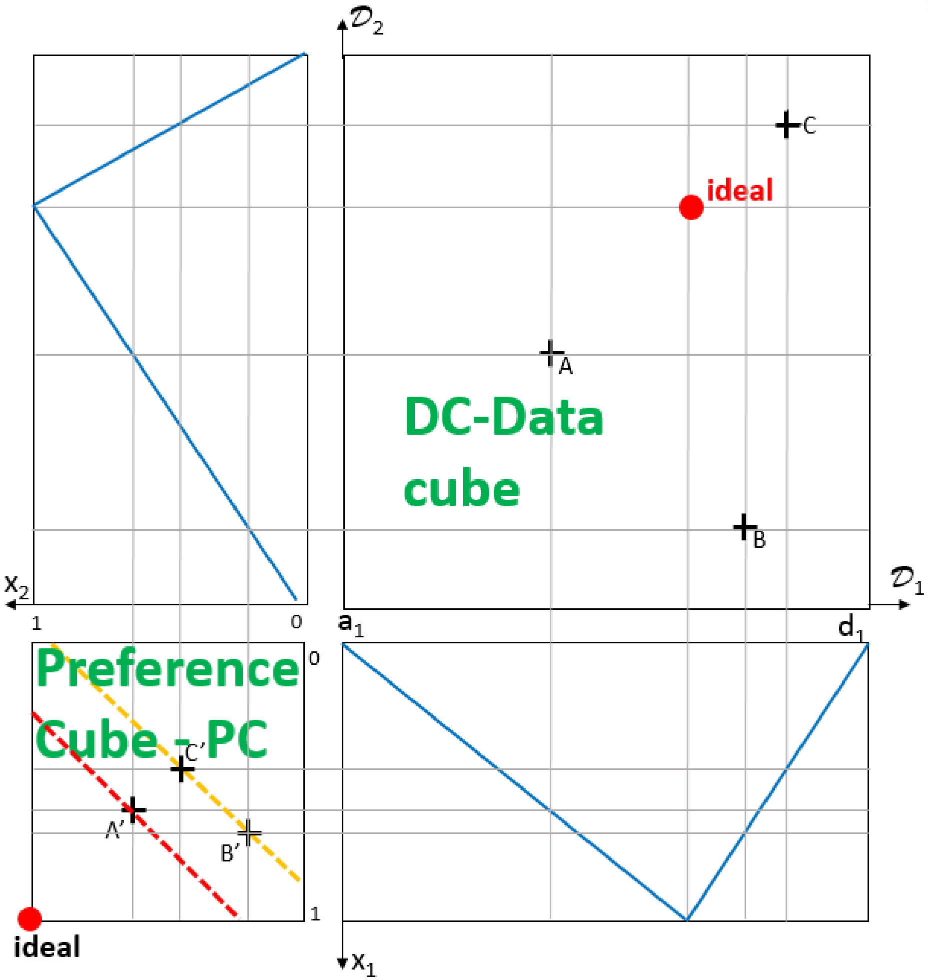
Motivation + L. Peska

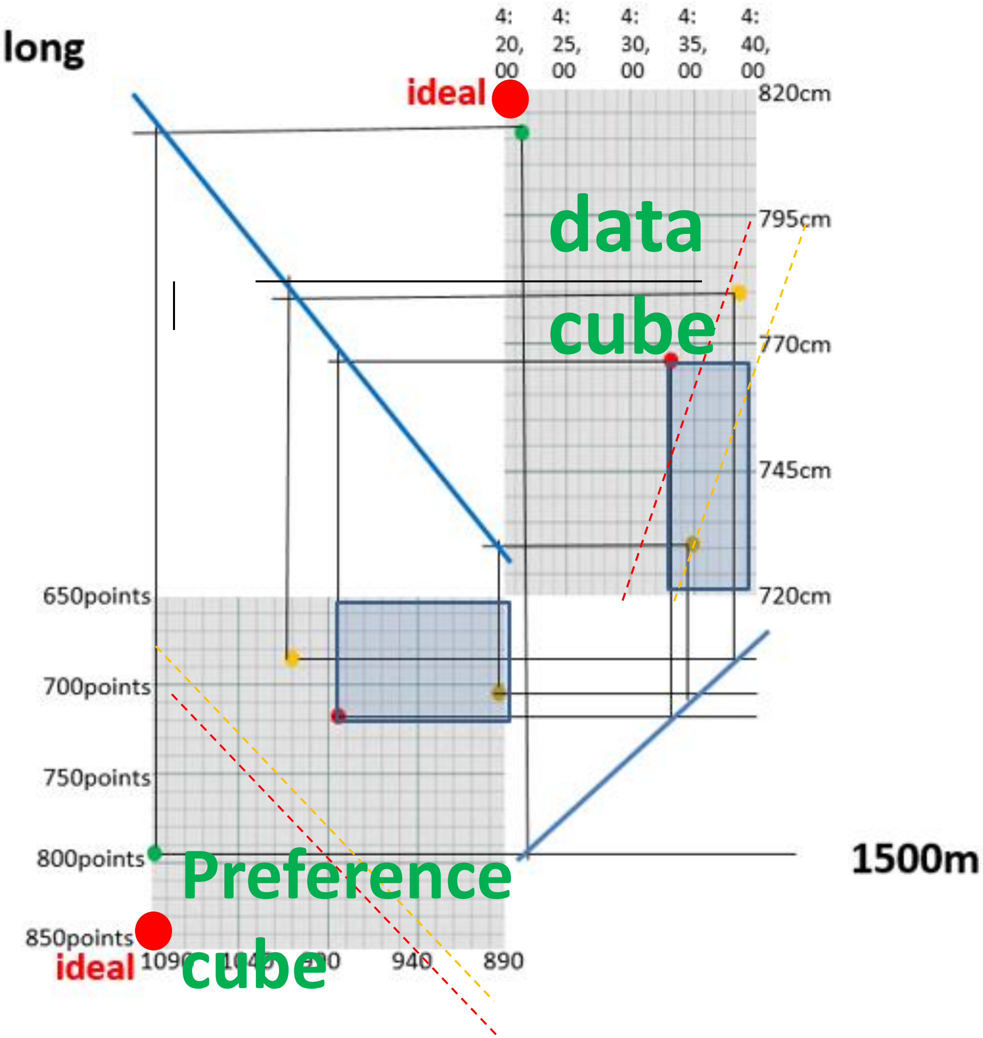




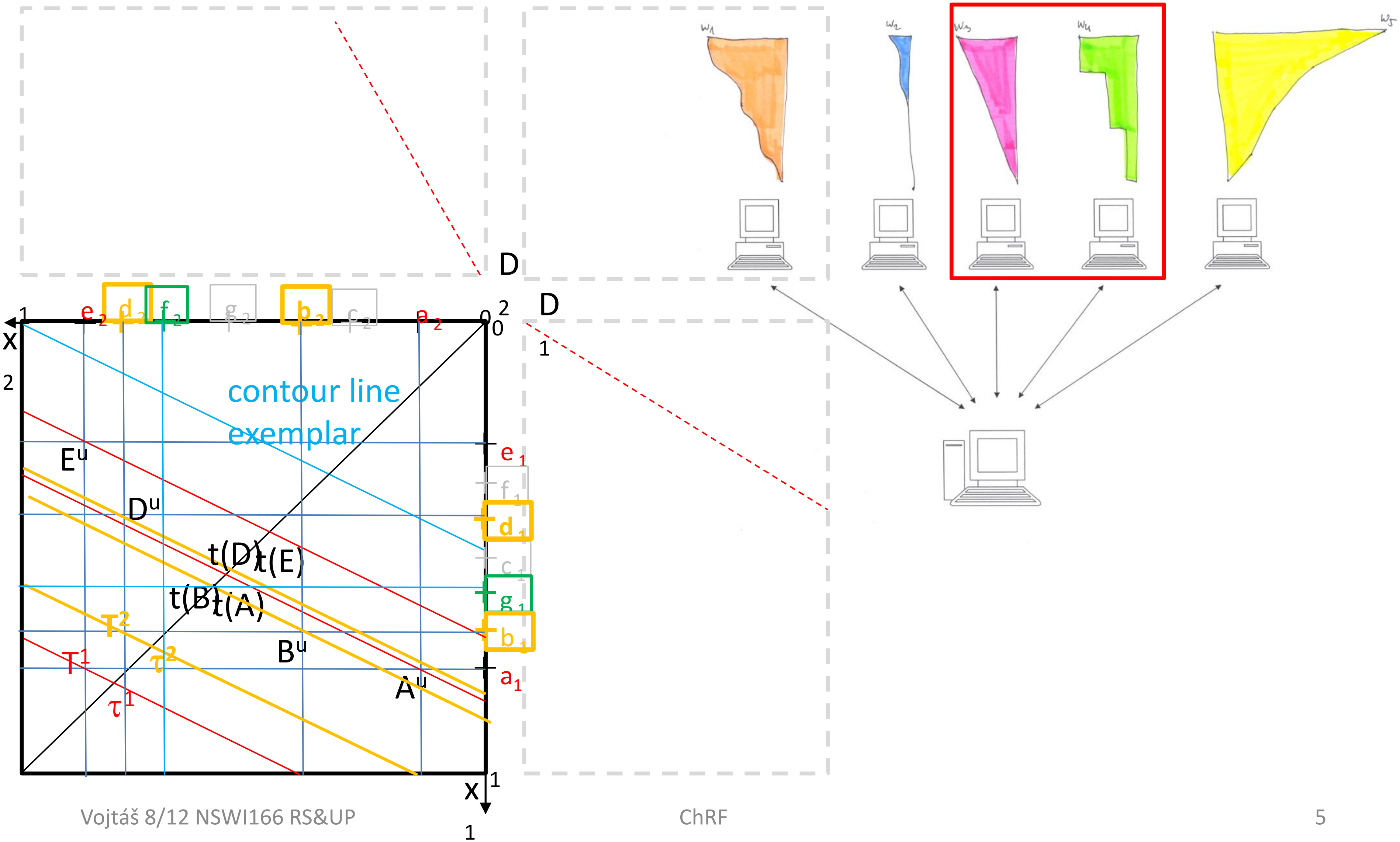


Topic 1 (5/12). Representation

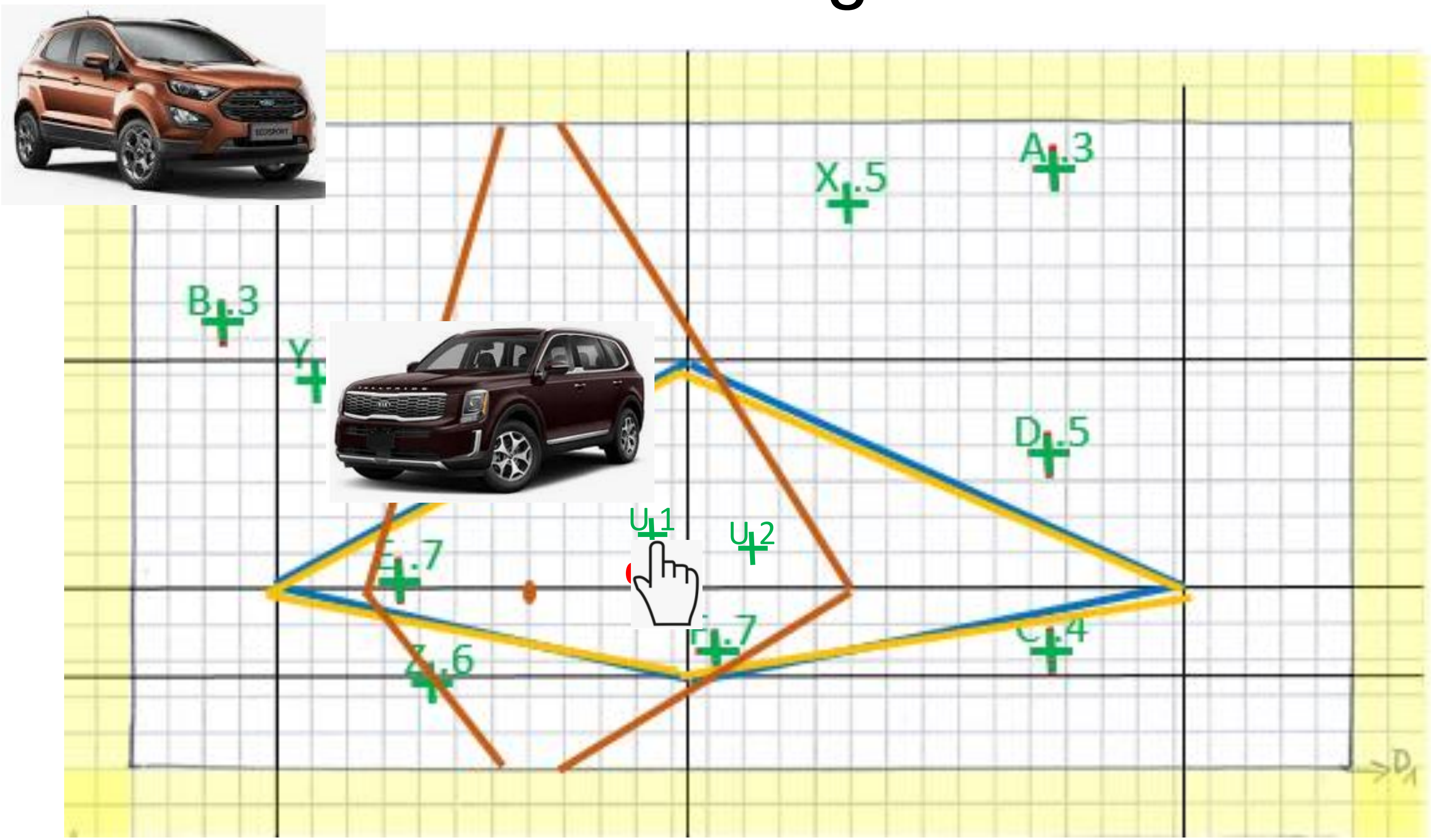




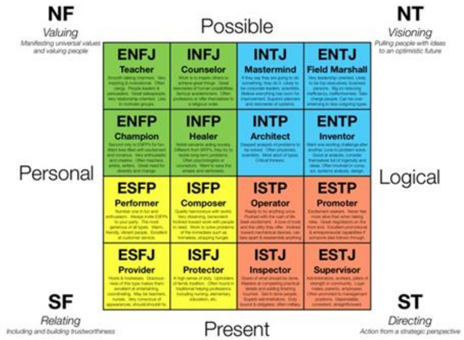
Topic 2 (6/12). Efficient top-k querying

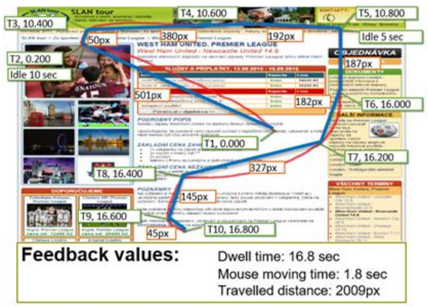


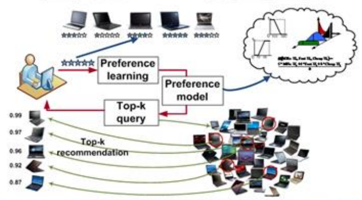
Topic 3 (7/12). Customer’s preference learning



all — Your best seen — ideal





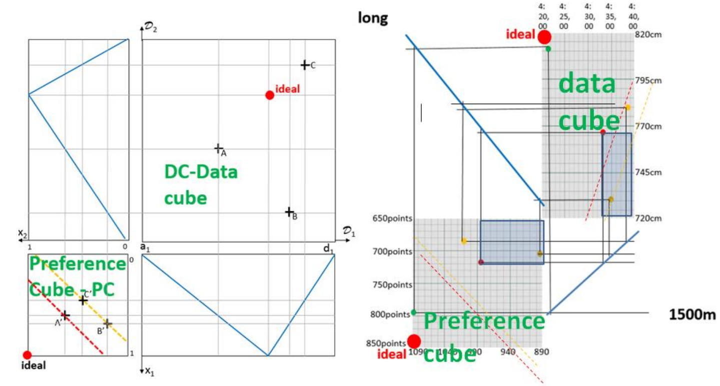


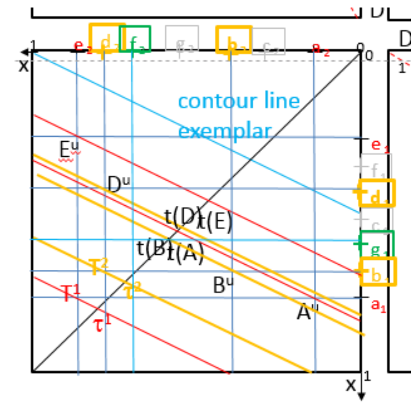
Google design thinking

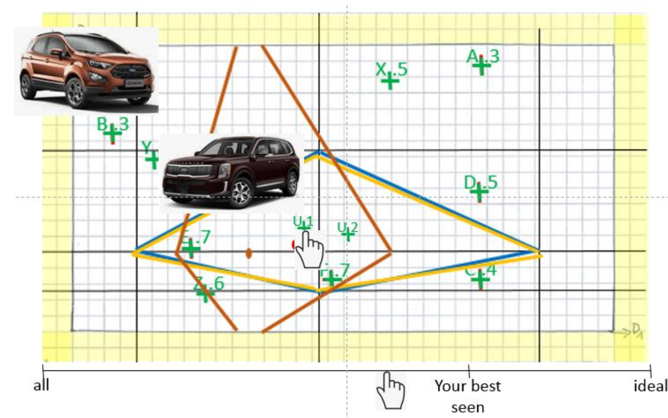
Empathy Expansive thinking Experimentation

or Lean Startup methodology?

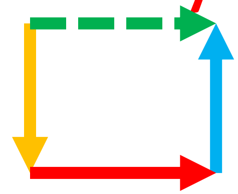
FROM REALITY TO MODEL

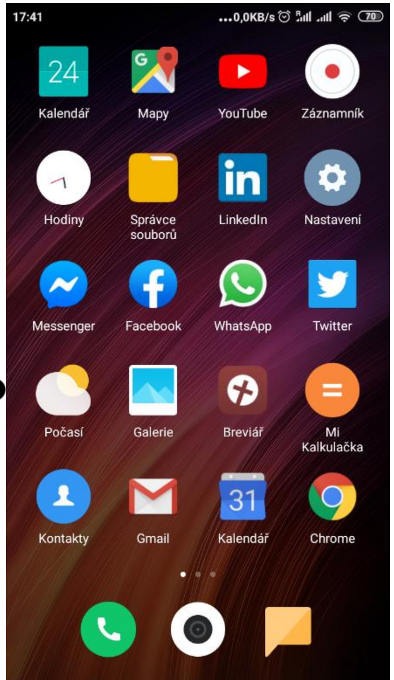


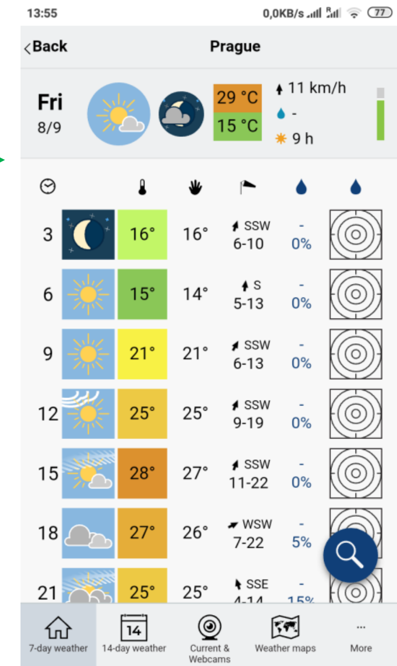


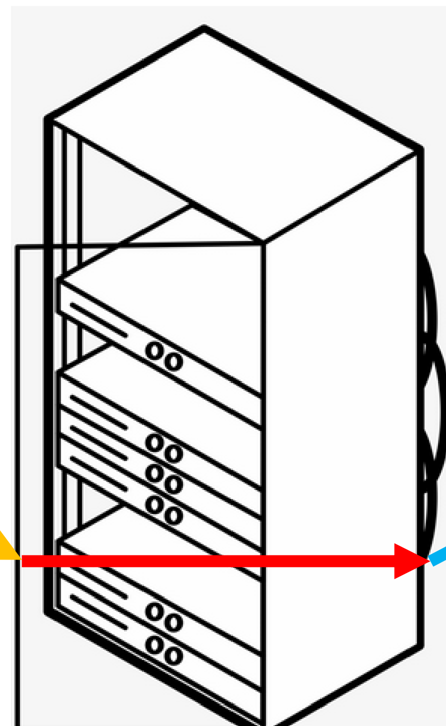


Weather forecast – reality to model reduction







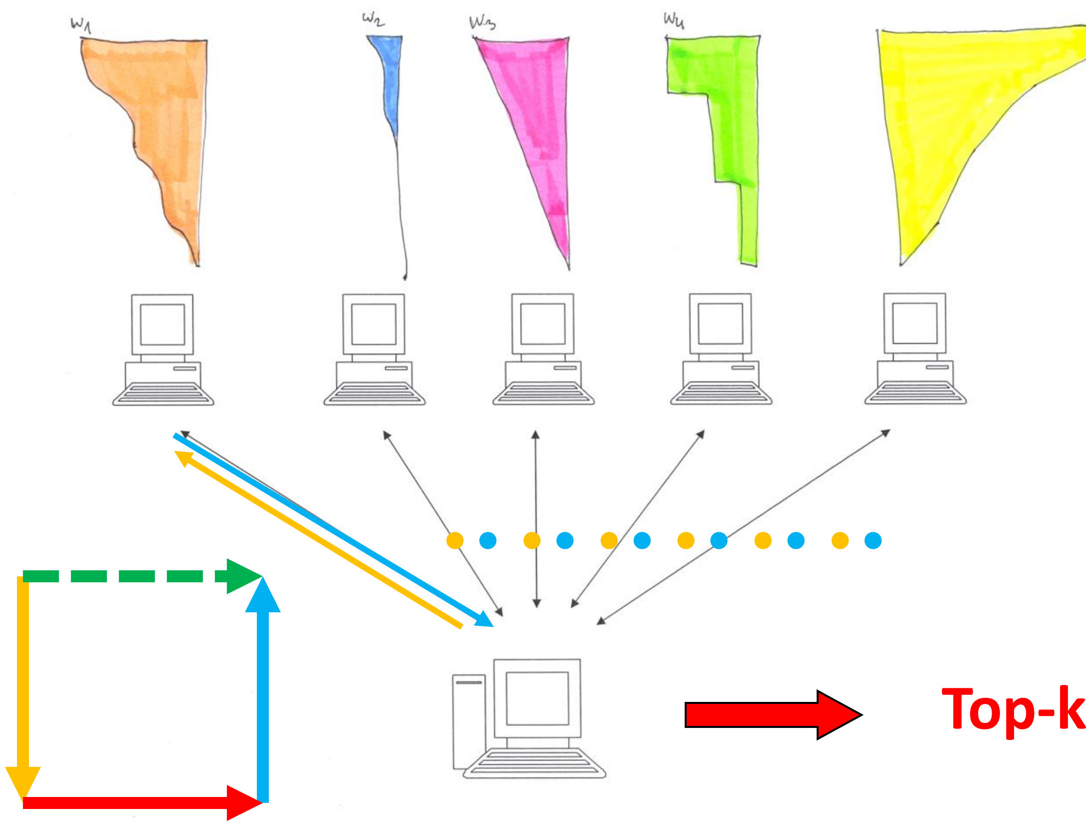


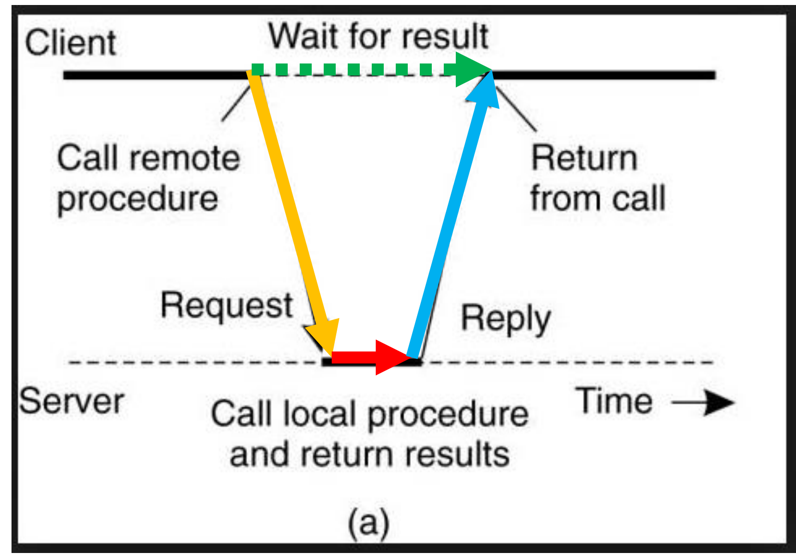
Problem instance: Prague, August 9th 2019, reduction

Computed forecast

Computed solution visual intuitive transformation / translation

Client-server / middleware – web service, ...





What is the challenge? User expects top-k relevant answers.

Client can finish computation.

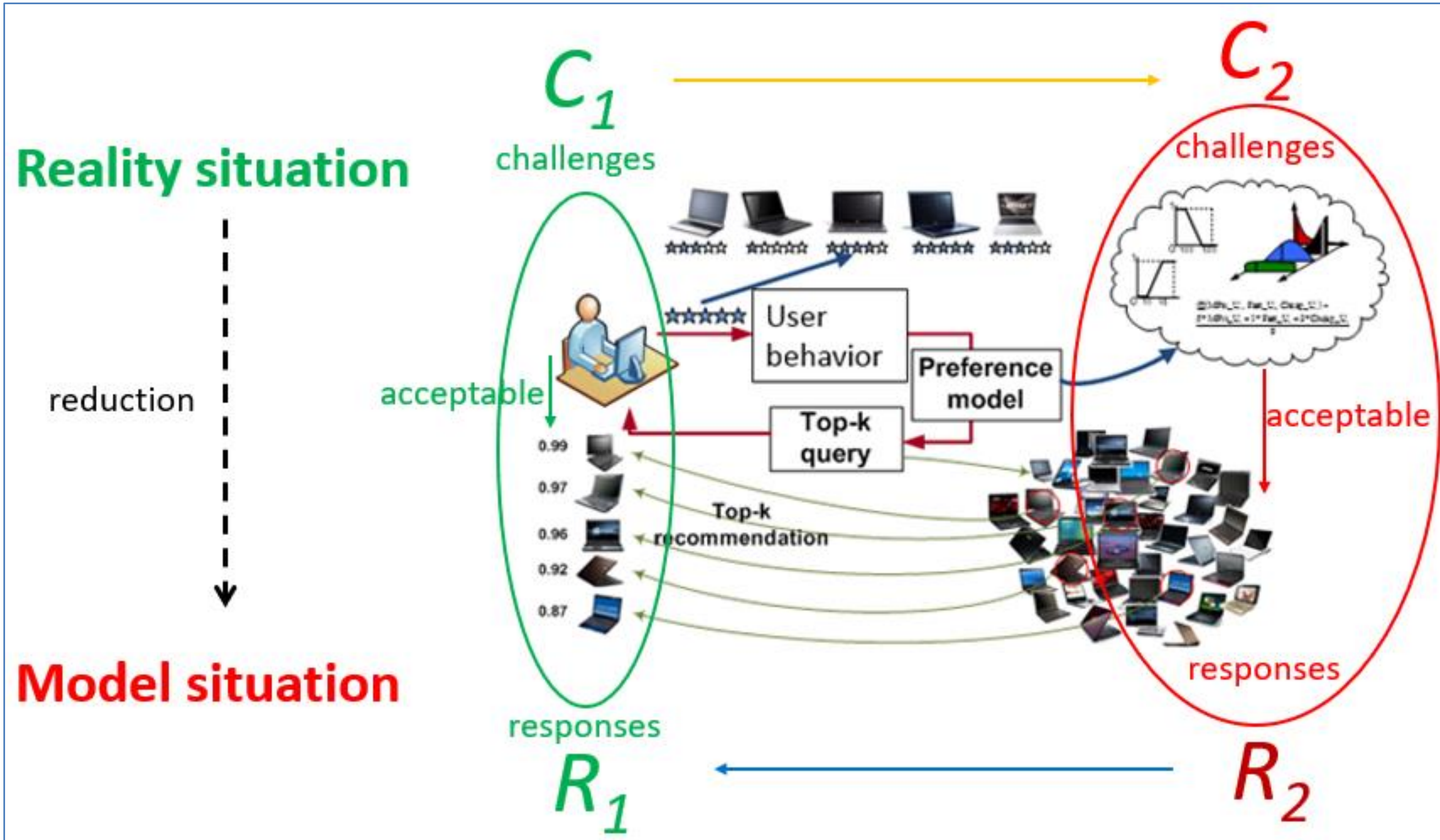
How can we meet the challenge?

Asking for/providing/accepting-using response

Reality → model, process, ...
One needs help → the other will provide assistance

$$
\begin{tikzcd}
\text{Application layer} & \text{input} \arrow[r, "\text{Code } \alpha"] \arrow[d] & \text{output} \\
\text{Compiled assembler} & \text{input} \arrow[r, "\alpha'"] \arrow[d] & \text{output} \arrow[u] \\
\text{Physical layer} & \text{input} \arrow[r, "\alpha''"] & \text{output} \arrow[u]
\end{tikzcd}
$$

Recommendation as reality to model reduction



Challenge-Response situation ChRS

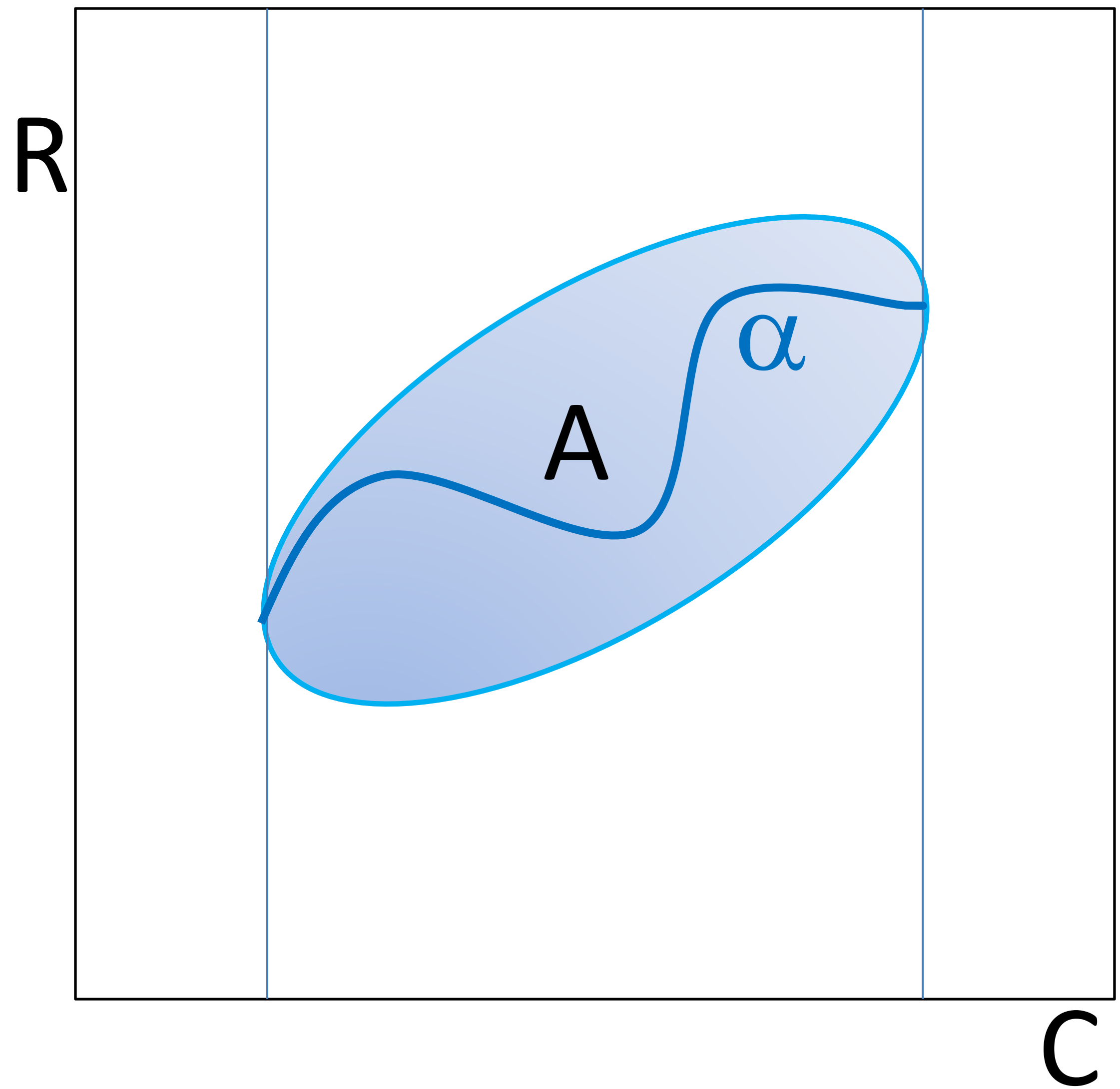
Let us have a set **C** of **challenges** and a set **R** **responses**.

A binary relation **A** $\subseteq$ C x R represents **acceptability** of a specific response r to a specific challenge c, denoted A(c, r).

Eventually A can be a program, process, then denoted α,

α can represent either

- an optimal response, or
- an algorithm



ChRF Challenge-Response Framework

$$\begin{tikzcd} C_1 \arrow[d, "\gamma^-"'] & \_A_1\_ & R_1 \\ C_2 & \_A_2\_ & R_2 \arrow[u, "\gamma^+"'] \end{tikzcd}$$

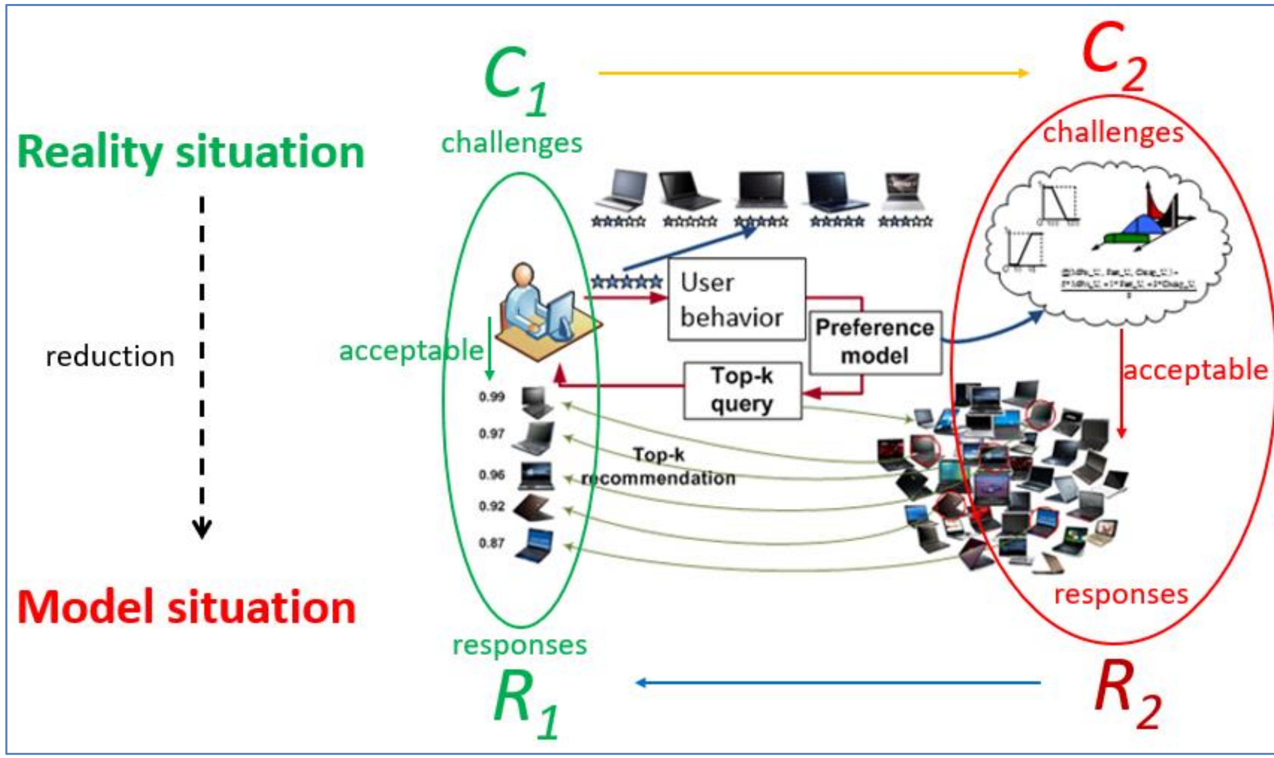
$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)$$

$$A_2(\gamma^-(c_1), r_2)$$

$$\Downarrow$$

$$A_1(c_1, \gamma^+(r_2))$$

Problems with $\Rightarrow$



Challenge-Response reduction ChRR

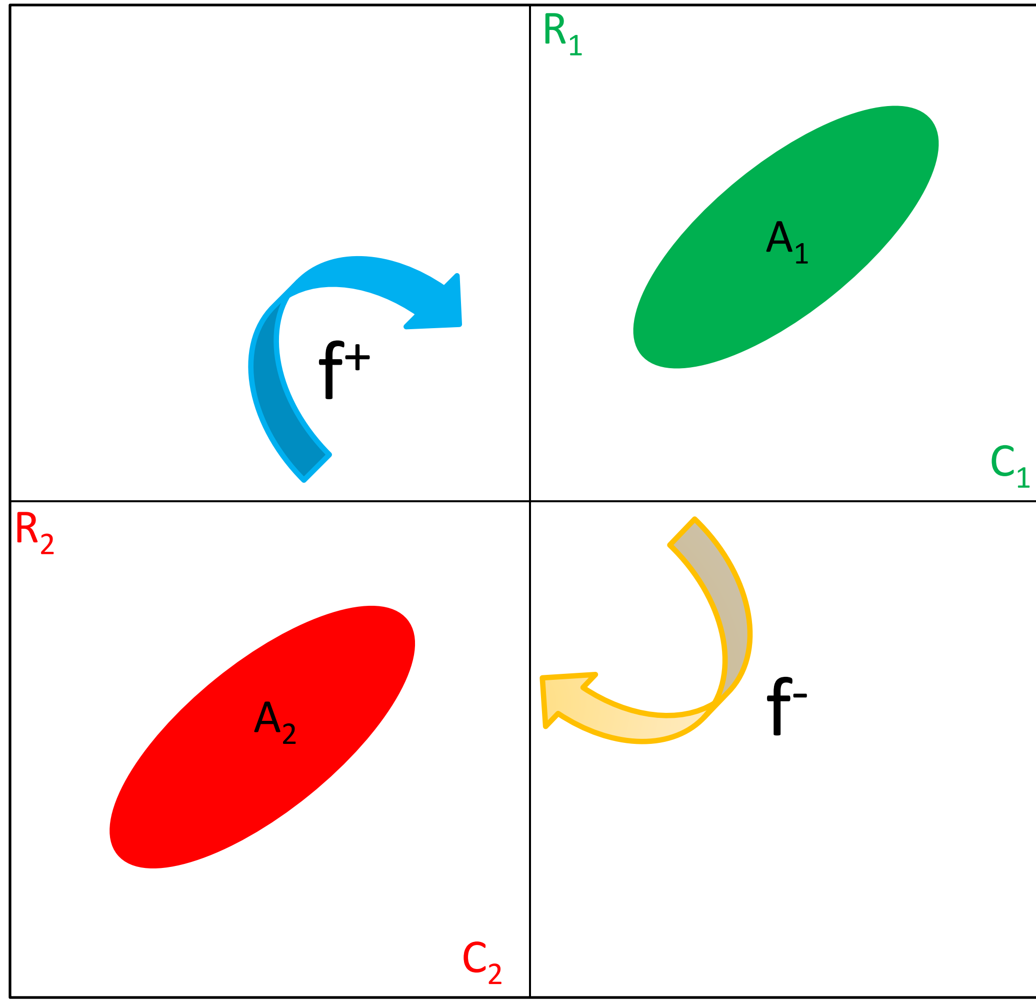
$$\begin{tikzcd}
C_1 \arrow[d, "f^-"'] & \underline{A_1} \arrow[d, dashed] & R_1 \\
C_2 & \underline{A_2} & R_2 \arrow[u, "f^+"]
\end{tikzcd}$$

$(\forall c_1 \in C_1)(\forall r_2 \in R_2)$

IF $A_2(f^-(c_1), r_2)$

Then

$A_1(c_1, f^+(r_2))$



ChRF = ChRS + ChRR formally

Challenge Response Situation ***ChRS*** S = (C, R, A) consists of a set C of challenges, set of responses R and an acceptability relation $A \subseteq C \times R$ (can be preferential, probabilistic, ...)

For an $c \in C$, $r \in R$ we read A(c, r) as "r is an acceptable response (in some degree) for challenge c", or also "response r meets challenge c"

Challenge Response Reduction ***ChRR*** of a situation $S_1 = (C_1, R_1, A_1)$ to a situation $S_2 = (C_2, R_2, A_2)$ consists of a pair of functions (f^-, f^+) such that

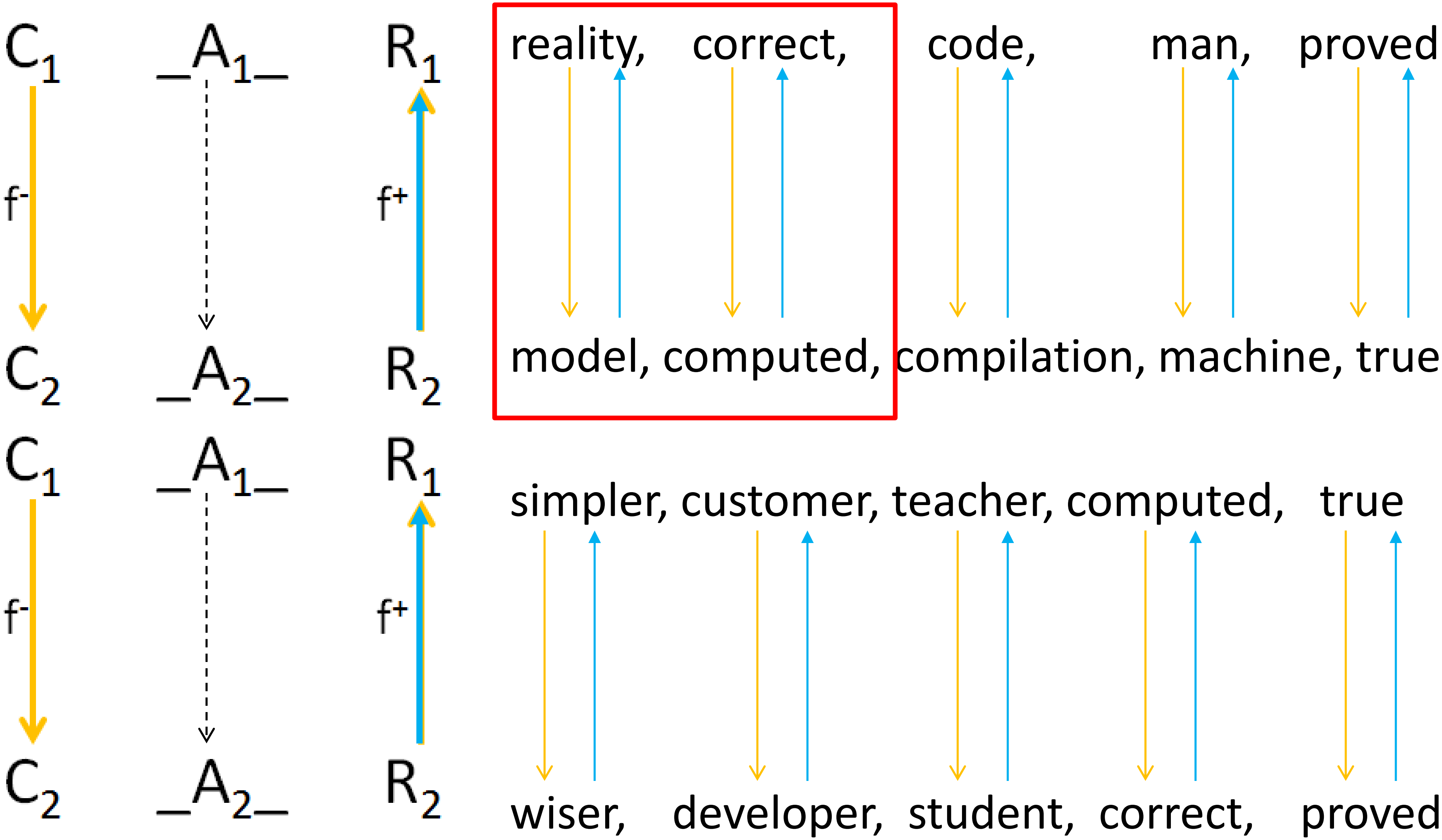
$$f^-: C_1 \to C_2 \,,$$

$$f^+: R_2 \to R_1, \text{ such that following holds:}$$

$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)(A_2(f^-(c_1), r_2) \Rightarrow A_1(c_1, f^+(r_2))) \qquad (*)$$

We will deal later with mathematical problem of "0 → *", so far, we can understand it procedurally

Various CRR instances



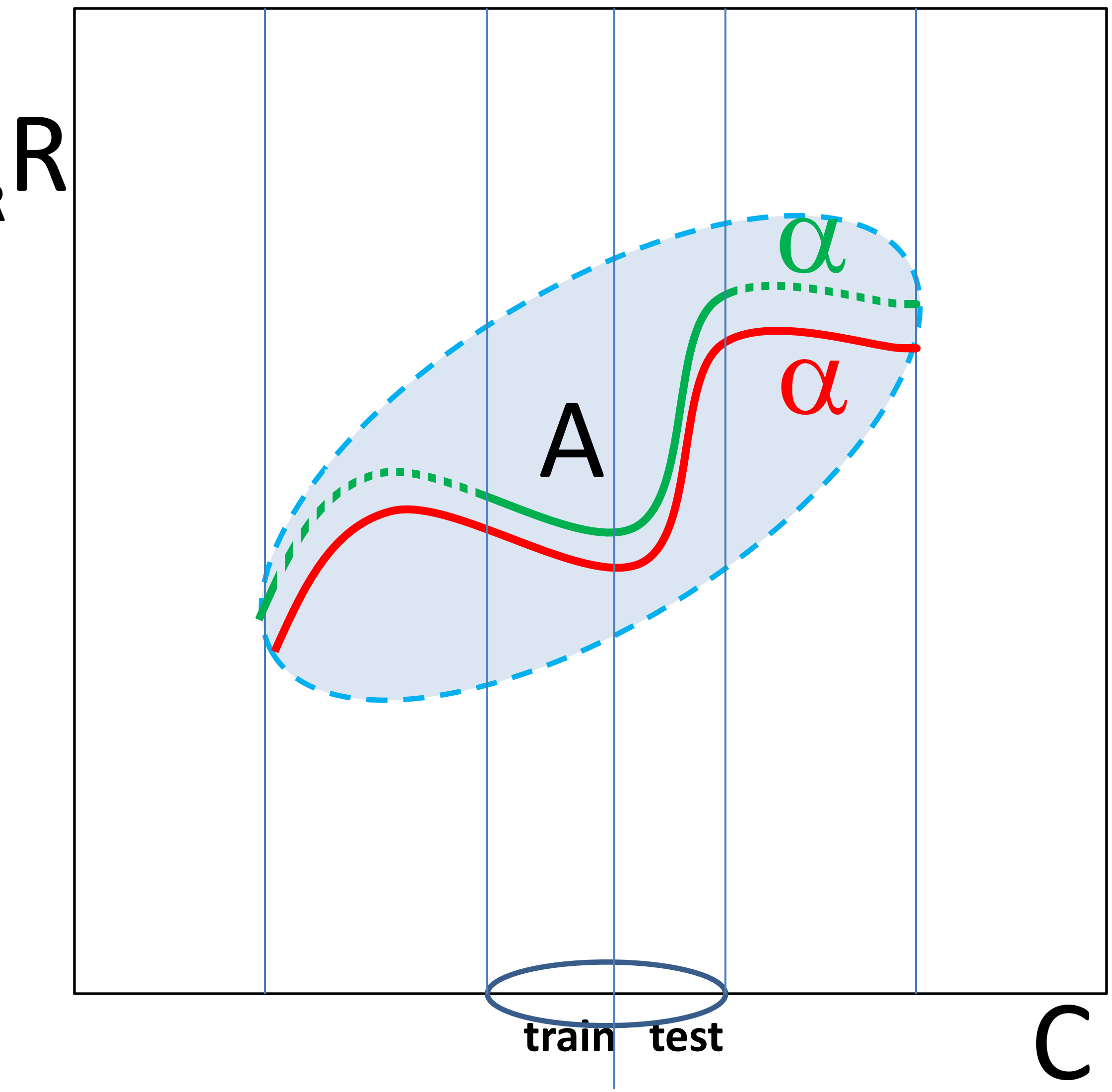
Partial Challenge-Response situation for learning

In learning set of

challenges C (vector of independent variables $\underline{x}$) and the set of responses R (dependent variable y) are not know in total – we know only train and test data (role of extra element nar will be discussed later).

Binary acceptability relation $A \subseteq C \times R$ (selector α) is also known only on train+test).

The task is to find α, as good as possible



Learning from data ($\underline{x}$, y) gives ($\underline{x}$, $\hat{y}$)

T-train, V-validate

$\underline{x}_T, y_T$ _RMSE_ $\underline{x}_V, y_V$

id= f^- id=f^+

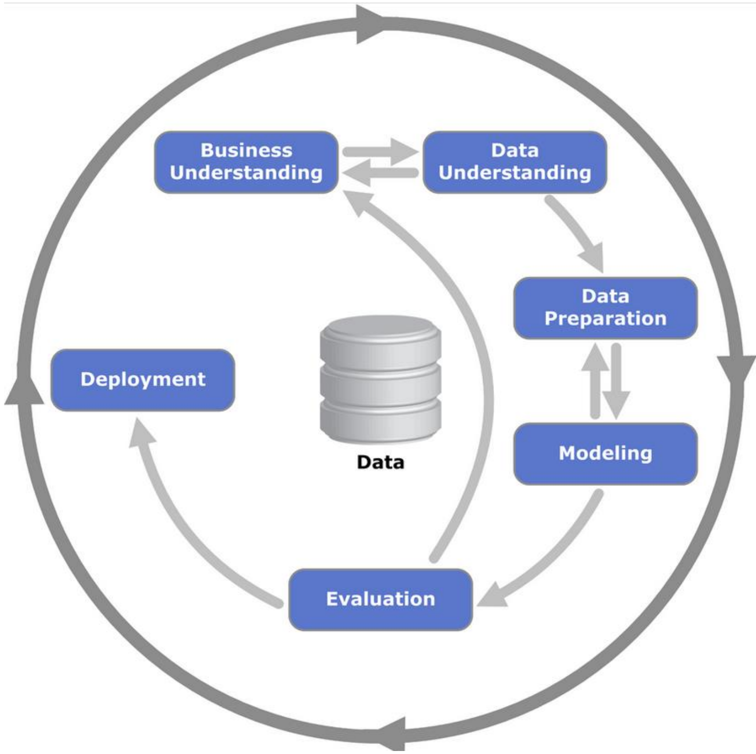
$\underline{x}_T, y_T$ model $\hat{y}_T$

$\underline{x}_V$ _α_2_ $\hat{y}_V$

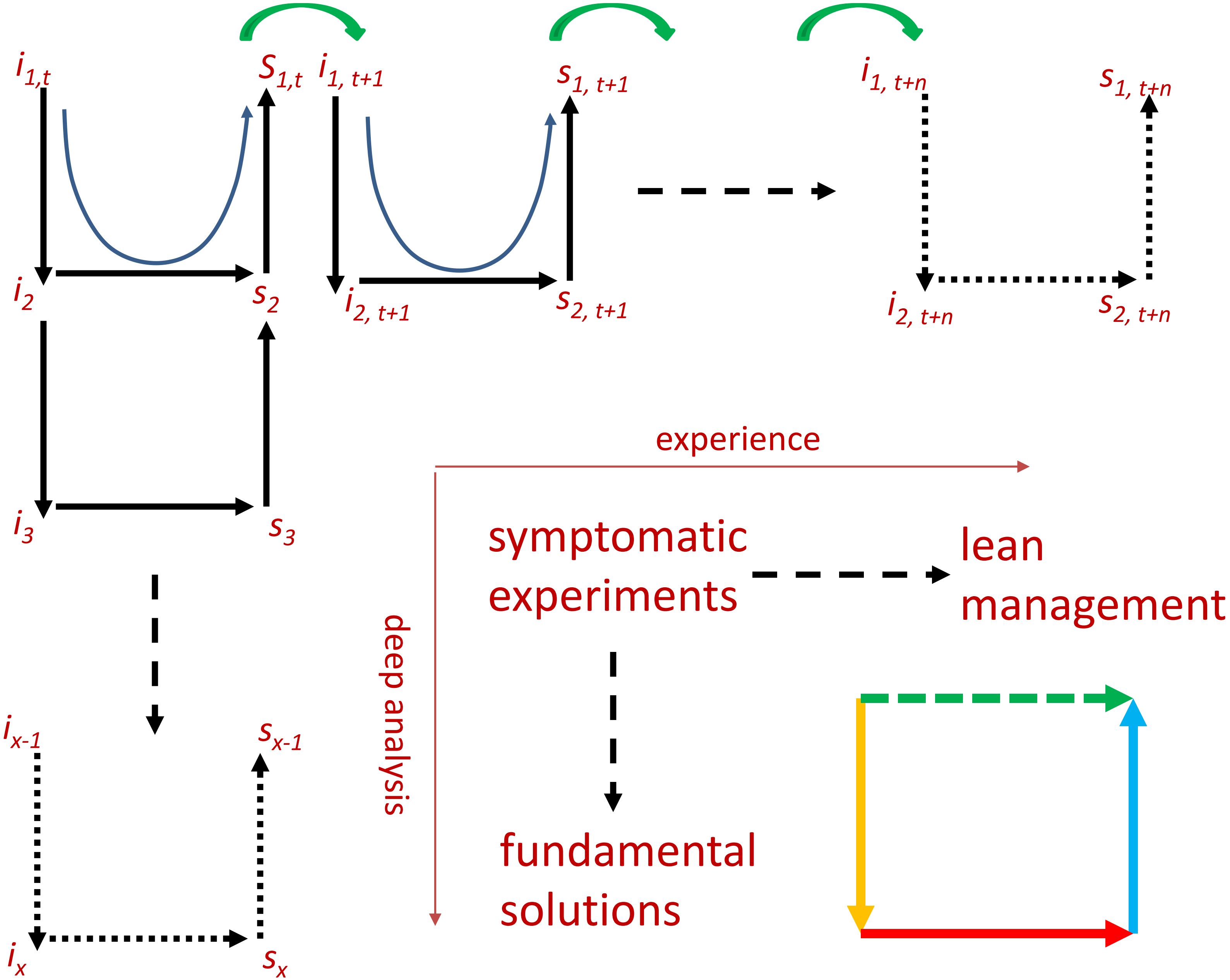
Whenever cross validation gives satisfactory results, then $\alpha_2(\underline{x}_{valid}) = \hat{y}_{Valid}$ is sent to be compared with y_{valid}. RMSE(y_{valid}, $\hat{y}_{Valid}$) can be considered the truth degree of this ChRR

Complex model

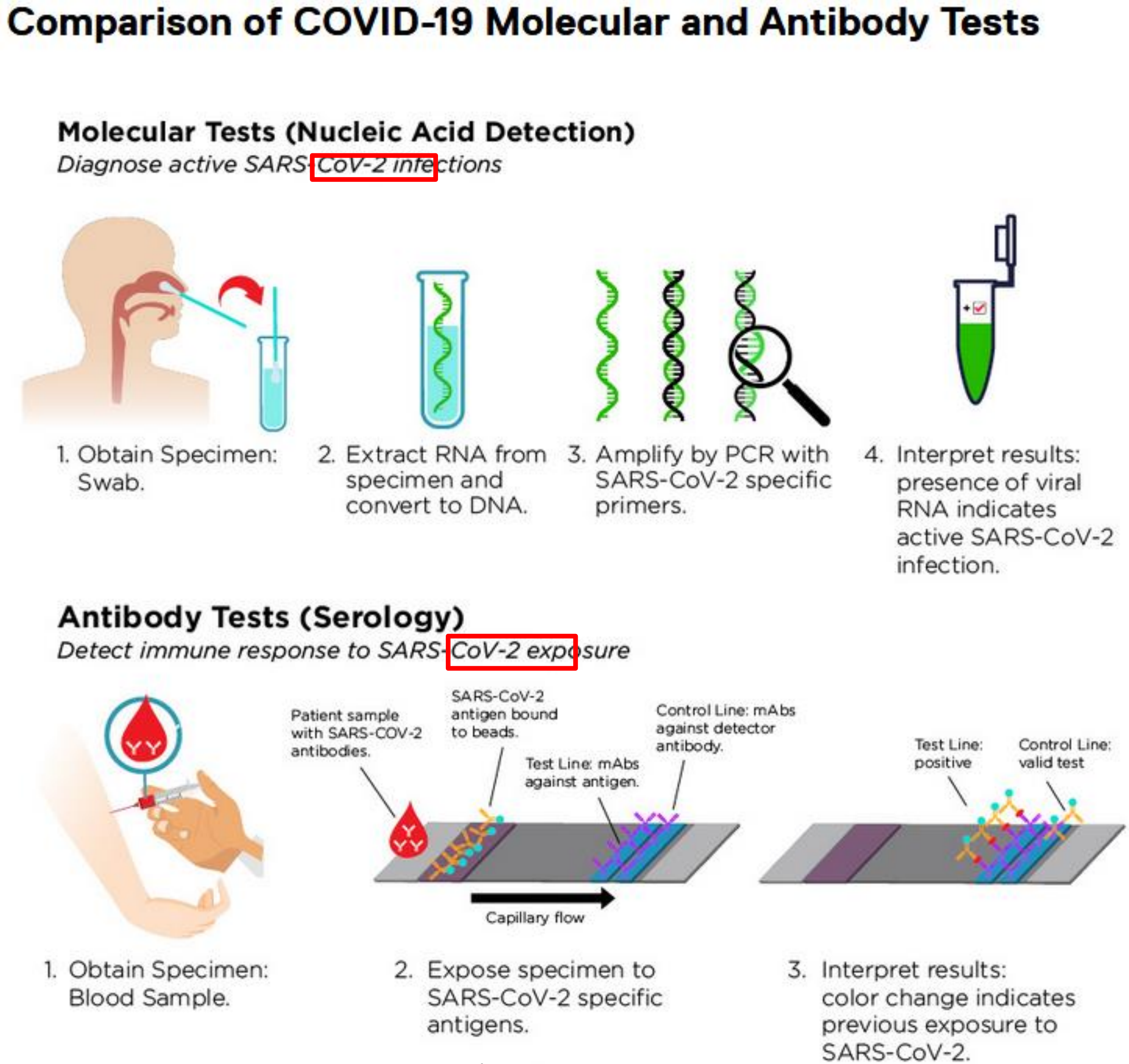
CRISP-DM



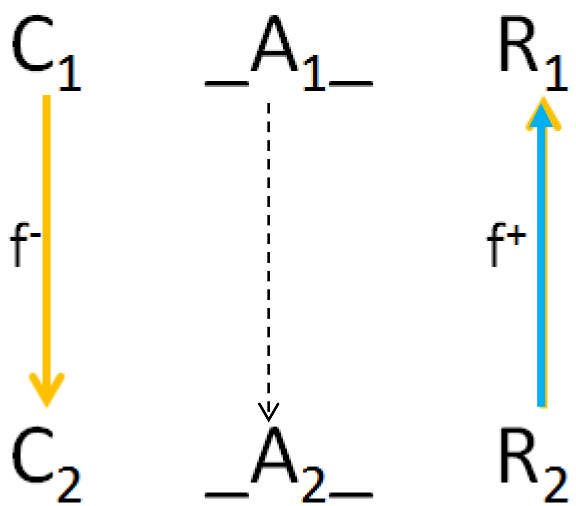
Learning and management strategies

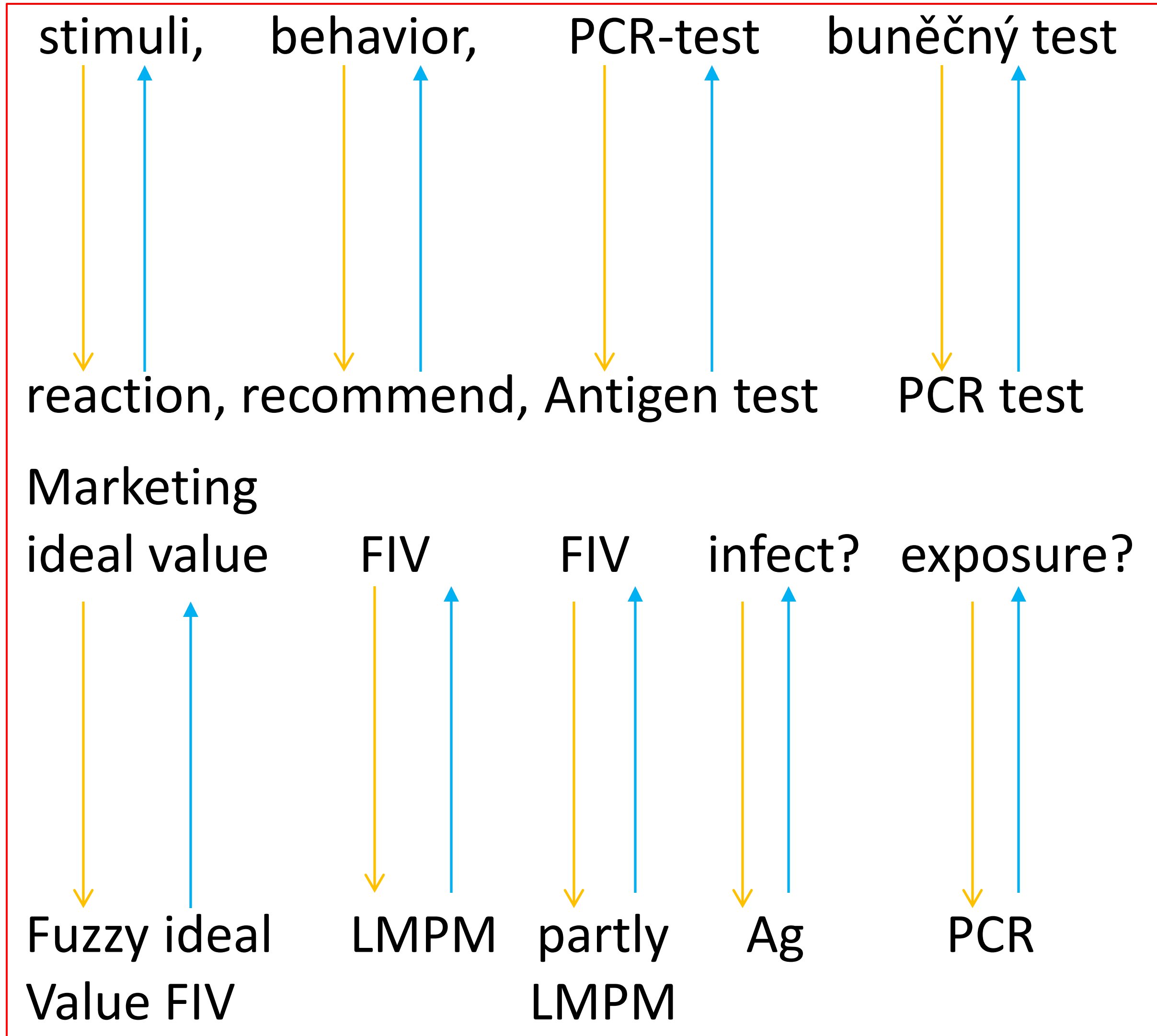


What is reality?
Some tests represent reality



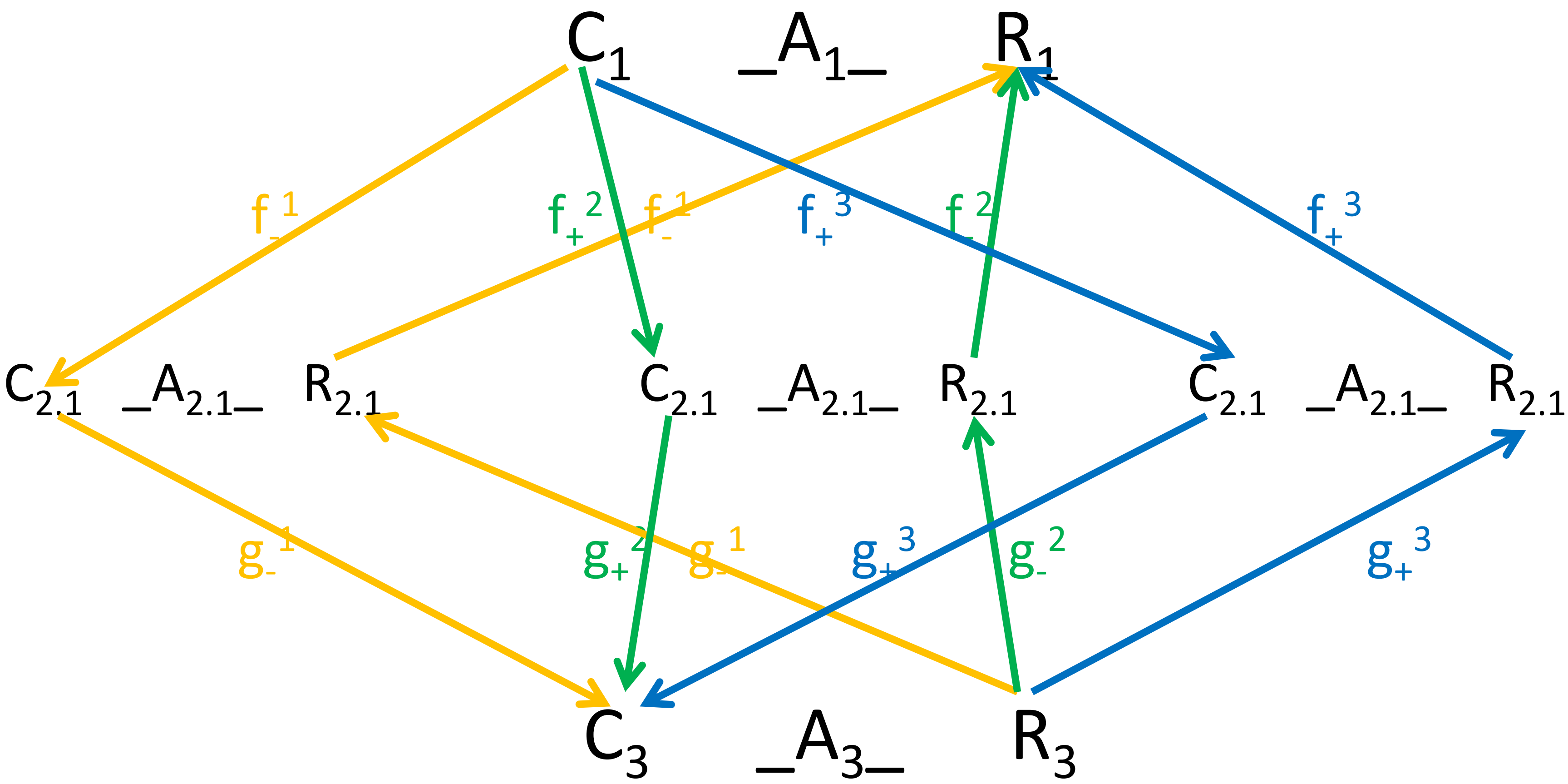
Various CRR instances





query	answer
Input	output
Item	attributes
object	owner
speech	asci record
Image	recognition

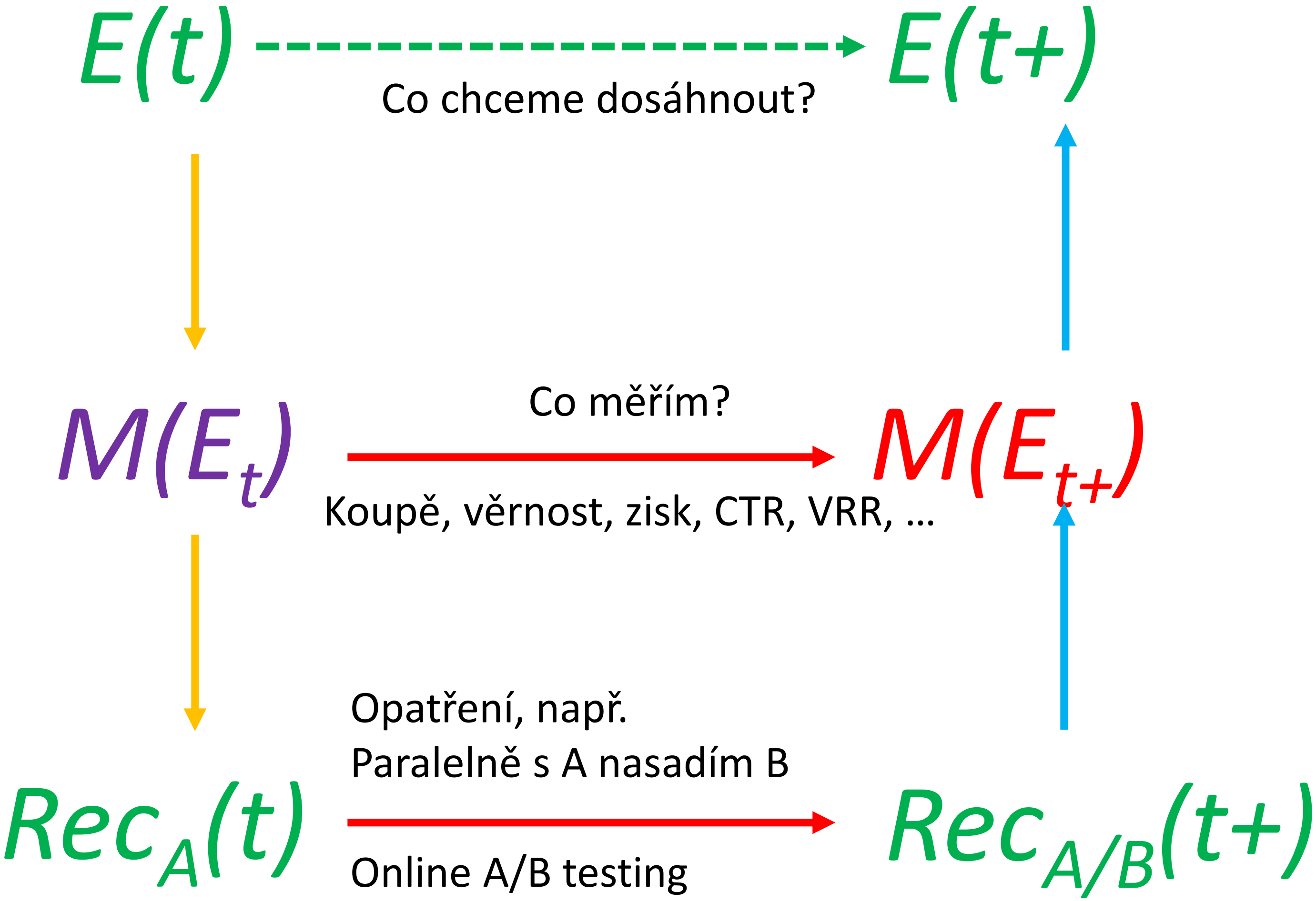
Cartesian closed category



Upper level - Several models, chose the best / different aspects

Lower level - ensemble learning, integration/ ?transfer learning

Globální vyhodnocení e-shopu E

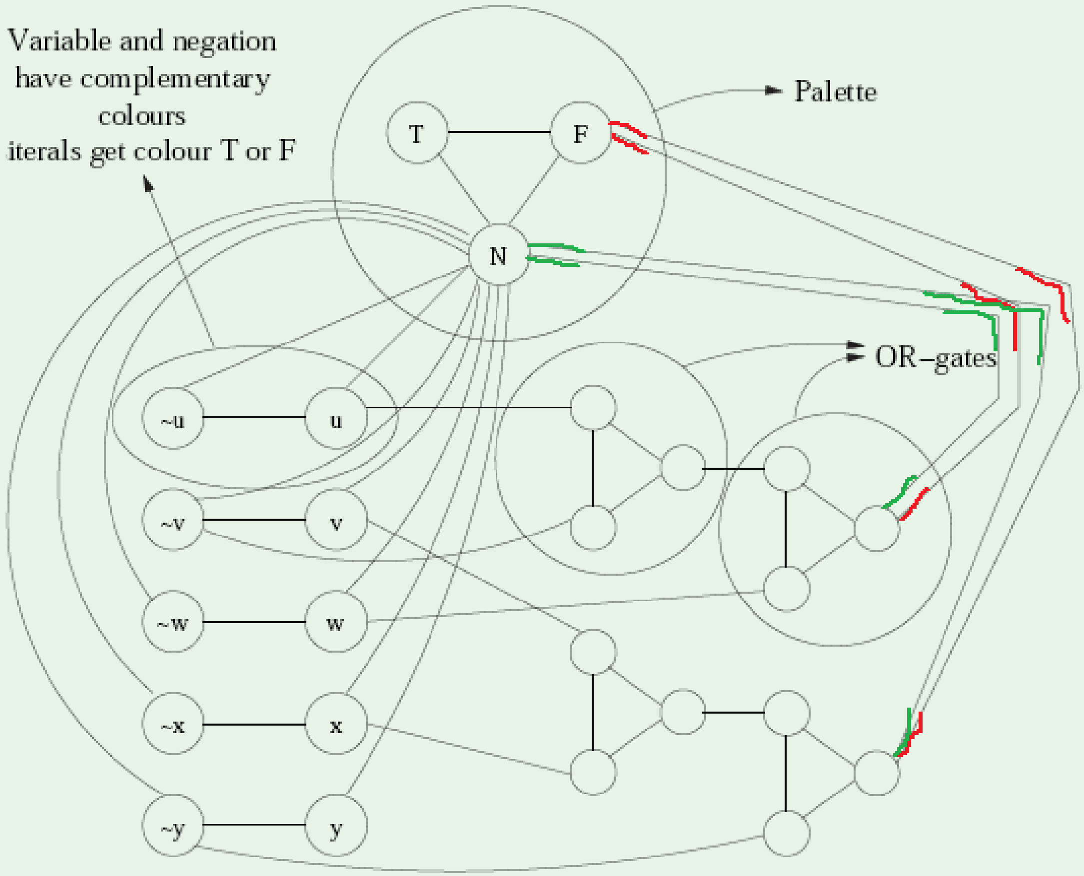


Further material

- 3 SAT reduction to 3 COL
- Mathematical problem of Challenge Response Framework – CRFm
- CRF – situations and reductions
  - Definitions
  - Decision (yes-no) and search problems
  - NAR – no acceptable responses
  - Theorem – one implication suffices
- Metrics

Example

$$\varphi = (u \vee \neg v \vee w) \wedge (v \vee x \vee \neg y)$$



Example CRR of 3SAT to 3COL

Start with 3-SAT formula φ with n variables $x_1, \dots, x_n$ and m clauses $K_1, \dots, K_m$. Create graph G_φ such that

- G_φ is 3-colorable iff φ is satisfiable (decision problem) and
- 3SAT is CR reducible to 3COL , up to this we need to establish truth assignment for $x_1, \dots, x_n$ via colors for some nodes in G.

create triangle with node True, False, Base

for each variable x_i two nodes v_i and $\underline{v_i}$ connected in a triangle with common Base

If graph is 3-colored, either v_i or $\underline{v_i}$ gets the same color as True. Interpret this as a truth assignment to v_i

For each clause $K_j = (a \vee b \vee c)$, create a small gadget graph

gadget graph connects to nodes corresponding to a, b, c,

implements OR connect output node of gadget to both False and Base

Correctness of $3SAT^{dec}$ reduction to $3COL^{dec}$

φ is satisfiable implies G_φ is 3-colorable

- if x_i is assigned True, color v_i True and $\underline{v}_i$ False
- for each clause $K_j = (a \vee b \vee c)$ at least one of a, b, c is colored True. OR-gadget for K_j can be 3-colored such that output is True.

G_φ is 3-colorable implies is φ satisfiable

- if v_i is colored True then set x_i to be True, this is a legal truth

assignment

- consider any clause $K_j = (a \vee b \vee c)$. it cannot be that all a, b, c are False. If so, output of OR-gadget for K_j has to be colored False but output is connected to Base and False!

CRF situations and reductions "0 → * is true"

Challenge Response Situation S = (C, R, A) consists of a set C of challenges, set of responses R and an acceptability relation $A \subseteq C \times R$ (can be preferential)

For an $c \in C$, $r \in R$ we read A(c, r) as "r is an acceptable response (in some degree) for challenge c", or also "response r meets challenge c"

We assume, that each set R contains also a special element "nar" representing "there is no acceptable response". We assume:

$$A(c, nar) \text{ is equivalent to } (\forall r \in R \setminus \{nar\})(\neg A(c, r)) \qquad (*^{nar})$$

R\{nar} are meaningful responses, "nar" is like logical "not" in decision problems

Example CRR of 3SAT to 3COL with “nar”

Assume we have situation $3SAT^{nar}$ ($3SAT^{dec}$ variant with R_1 = {true, false} is constructed analogously, with same $f^-(\varphi)$) with

C_1 is the set of all 3CNF clauses,

R_1 is the set of all truth assignments of variables + nar_1 and

$A_1(\varphi, v)$ if $v|= \varphi$, here $|=$ says that in the world v is true (xor nar)

And situation $3COL^{nar}$, where

C_2 is the sets of all graphs G(V, E),

R_2 is the set of all 3-vertex coloring + nar_2 and

$A_2(G, c)$ if c is a proper coloring of vertices of G (xor nar)

CR reduction consists of a pair of functions (f^-, f^+) such that

$f^-(\varphi) = G_\varphi$, see illustration

$f^+(c) = v_c$, also described in illustration

$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)(A_2(f^-(c_1), r_2) \Rightarrow A_1(c_1, f^+(r_2)) \qquad (*)$$

Note that $A_2(f^-(c_1), nar_2) \Rightarrow A_1(c_1, nar_1)$ is equivalent to

$\neg\ A_2(f^-(c_1), nar_2) \Leftarrow \neg A_1(c_1, nar_1)$ and this is equivalent to

$$(\exists r_1 \in R_1 \setminus \{nar_1\})(A_1(c_1, r_1)) \Rightarrow (\exists r_2 \in R_2 \setminus \{nar_2\})(A_2(f^-(c_1), r_2)) \qquad (**)$$

Challenge-Response situation "nar"

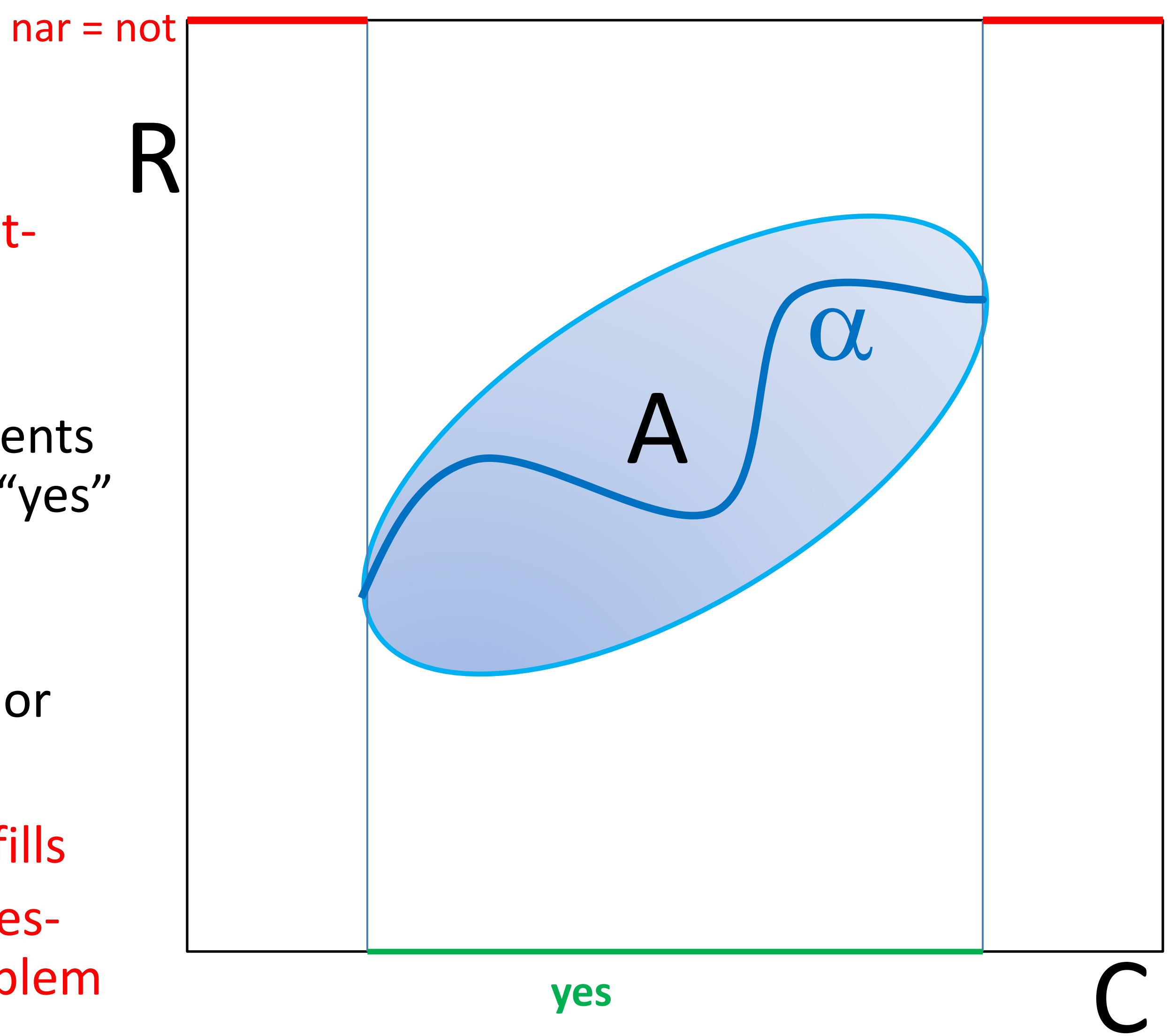
Let us have a set C of challenges and a set R responses (with an extra element nar = "no acceptable response in R).

a binary acceptability relation $A \subseteq C \times R$ represents search problem (and "yes" decision). A selector

α can represent either

- an optimal response, or
- an algorithm

$\{(c, nar): c \in C \setminus dom(A)\}$ fills "no" decision ...and corresponding decision problem



CRF situations and reductions "nar"

Challenge Response Reduction of a situation $S_1 = (C_1, R_1, A_1)$ to a situation $S_2 = (C_2, R_2, A_2)$ consists of a pair of functions (f^-, f^+) such that

$f^-: C_1 \to C_2$,

$f^+: R_2 \to R_1$, such that $f^+(nar_2) = nar_1$, $f^+(r_2) = nar_1$ implies $r_2 = nar_2$ and following holds:

$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)(A_2(f^-(c_1), r_2) \Rightarrow A_1(c_1, f^+(r_2)) \qquad (*)$$

Note that $A_2(f^-(c_1), nar_2) \Rightarrow A_1(c_1, nar_1)$ is equivalent to

$\neg A_2(f^-(c_1), nar_2) \Leftarrow \neg A_1(c_1, nar_1)$ and this to

$(\exists r_1 \in R_1 \setminus \{nar_1\})(A_1(c_1, r_1)) \Rightarrow (\exists r_2 \in R_2 \setminus \{nar_2\})(A_2(f^-(c_1), r_2))$

Hence "nar" prevents fake reductions

Challenge-Response reduction "nar"

$$
\begin{tikzcd}
C_1 \arrow[d, "f^-"'] & \_A_1\_ & R_1 \\
C_2 & \_A_2\_ & R_2 \arrow[u, "f^+"']
\end{tikzcd}
$$

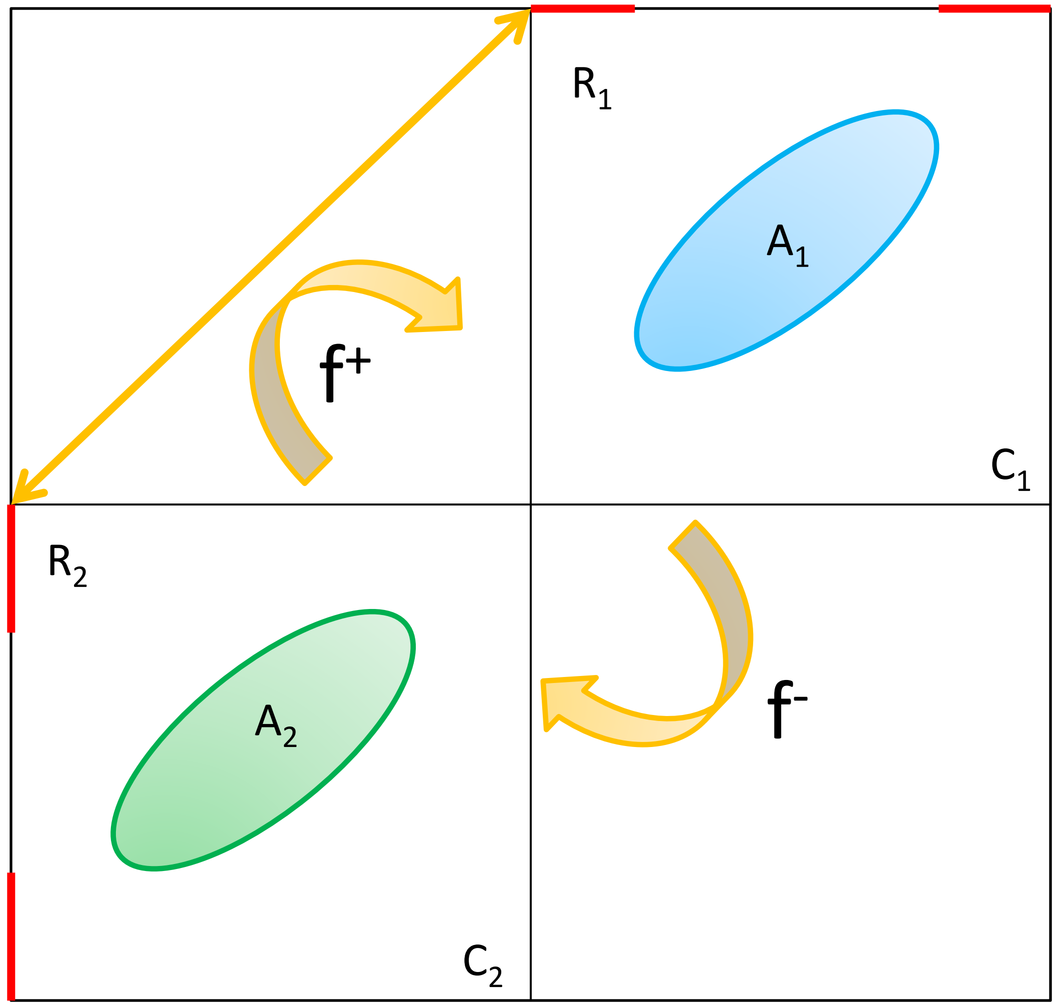
$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)$$

$$A_2(f^-(c_1), r_2)$$

$$\Downarrow$$

$$A_1(c_1, f^+(r_2))\ \&$$

$f^+(nar_2) = nar_1$



Correctness of 3SAT^{nar} reduction to 3COL^{nar}

Denote $G_\varphi(V_\varphi, E_\varphi) = f^-(\varphi)$, assume $A_2(G_\varphi, c)$ and c: $V_\varphi \rightarrow$ {True, False, Base}, hence $c \neq nar_2$, is a proper 3-coloring. Let us construct $f^+(c)$ as follows: if v_i is colored True then set $f^+(c) = v_c(x_i)$ to be True, this is a legal truth assignment and $v_c \models \varphi$

- consider any clause $K_j = (a \vee b \vee c)$. it cannot be that all a, b, c are False. If so, output of OR-gadget for K_j has to be colored False but output is connected to Base and False!

To show $A_2(G_\varphi, nar_2)$ implies $A_1(\varphi, nar_1)$, by (**) it is sufficient to show that if $v \in R_1$ is a truth assignment with $A_1(\varphi, v)$ then there is a c a proper 3-coloring of $G_\varphi = f^-(\varphi)$, hence $A_2(G_\varphi, c)$

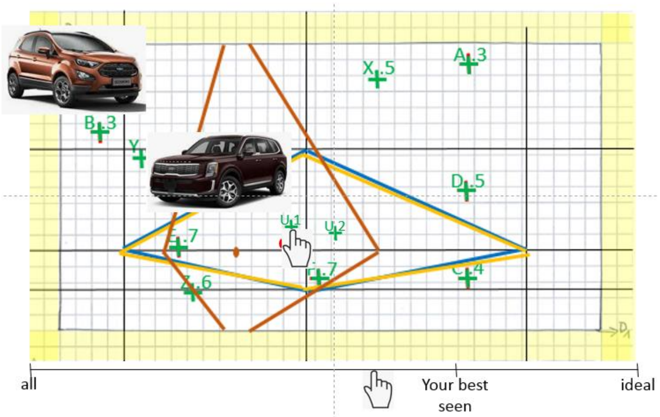
Indeed

- if variable x_i is assigned True by v, color v_i True and $\underline{v_i}$ False by c
- for each clause $K_j = (a \vee b \vee c)$ at least one of a, b, c is colored True. OR-gadget for K_j can be 3-colored such that output is True.

Graphical solution and visualization in LMPM

Human visual perception

$$\begin{array}{ccc} \text{input} & \xrightarrow{\text{User study}} & \text{output} \\ \downarrow & & \uparrow \\ \text{input} & \xrightarrow{\alpha'} & \text{output} \\ \downarrow & & \uparrow \\ \text{input} & \xrightarrow{\alpha''} & \text{output} \end{array}$$



Analytical layer

What is the role of "nar" in practical applications?

Database as a mediator by AHV

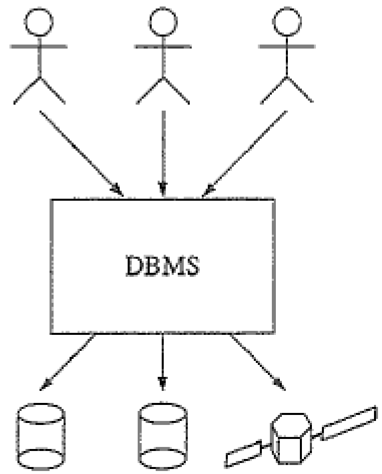
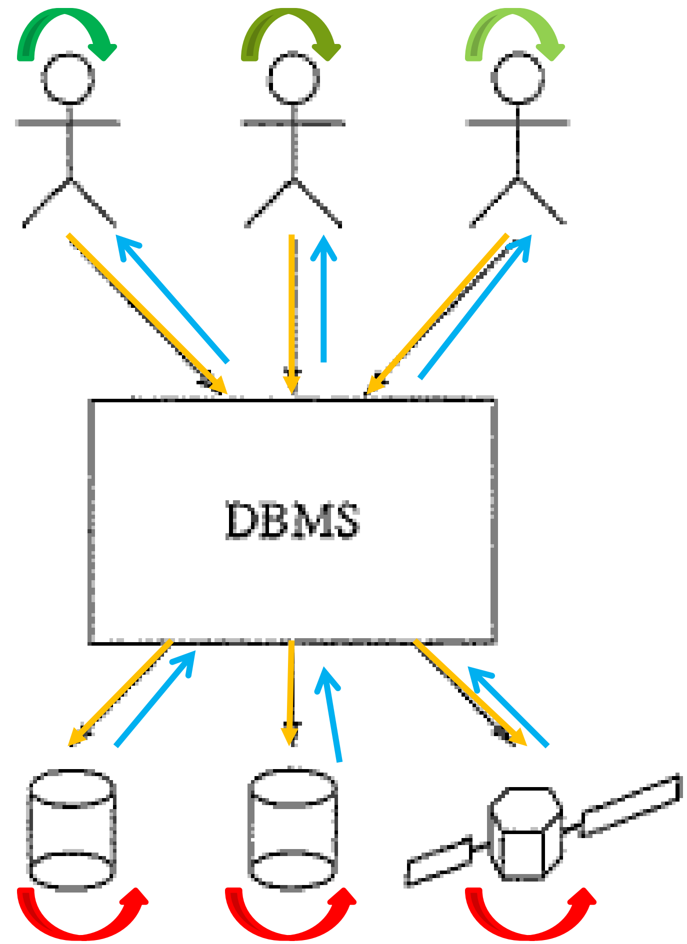


Figure 1.1: Database as mediator between humans and data



Query _correct_ answer

 # reduction is correct

Query _computed_ answer

AHV three levels of DB architecture

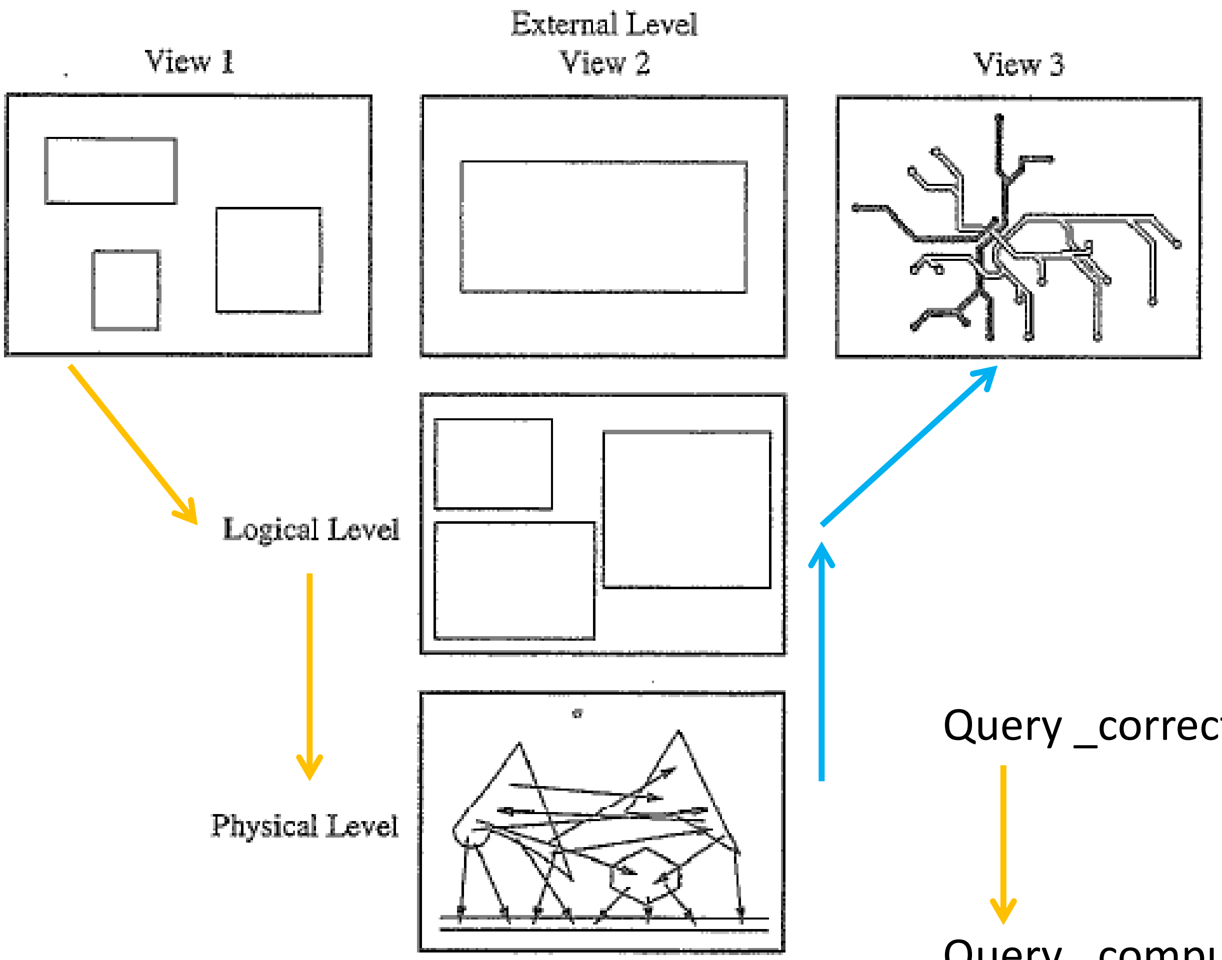


Figure 1.2: Three-level architecture of database systems

Query _correct_ answer

Query _computed_ answer

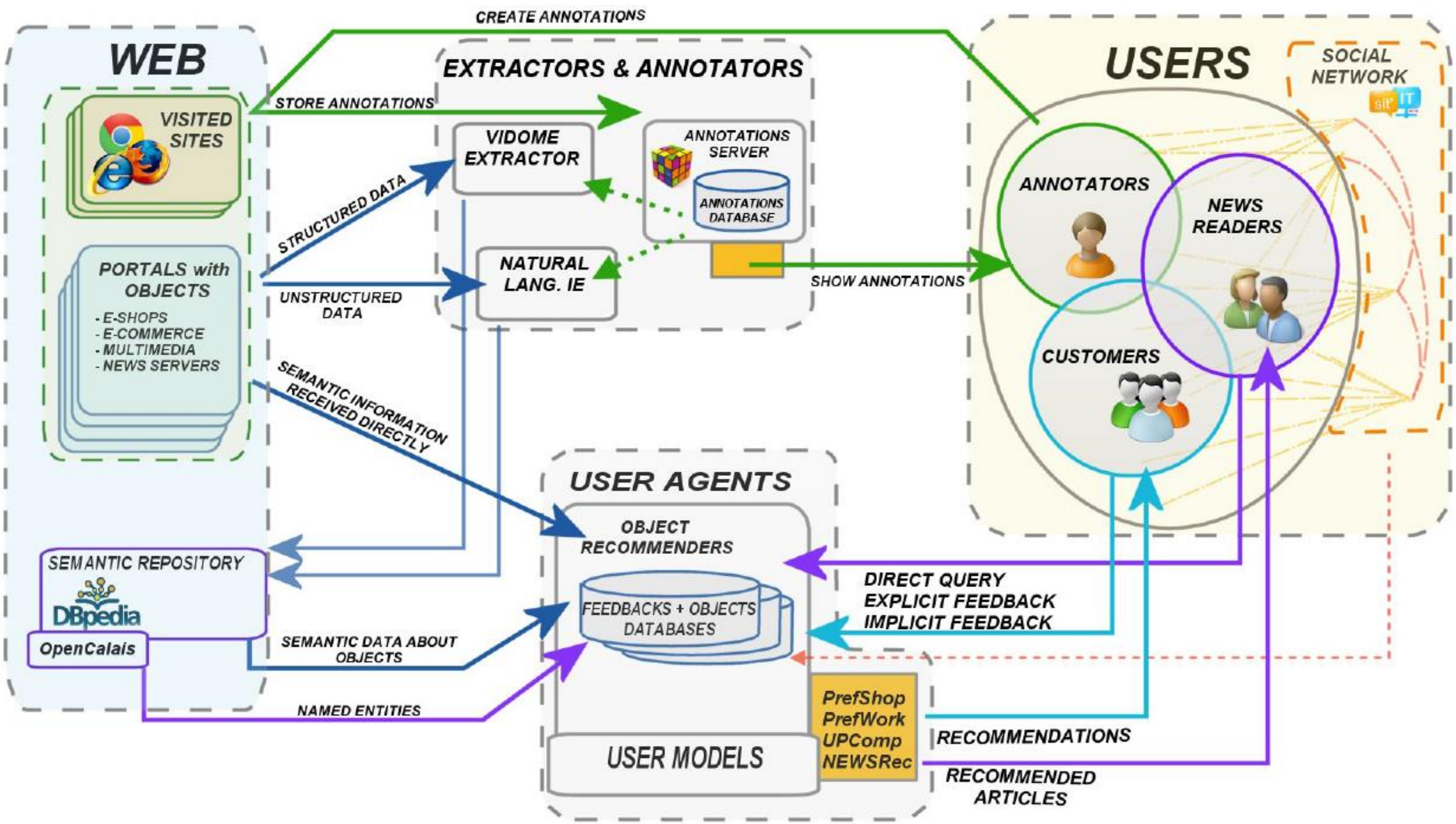
Computer architecture

$$\begin{tikzcd} \text{Application layer} & \text{input} \arrow[r, "\text{Code } \alpha"] \arrow[d] & \text{output} \\ \text{Compiled assembler} & \text{input} \arrow[r, "\alpha'"] \arrow[d] & \text{output} \arrow[u] \\ \text{Physical layer} & \text{input} \arrow[r, "\alpha''"] & \text{output} \arrow[u] \end{tikzcd}$$

 # reduction is correct means “if α'' is computed then (implies) α is correct” is a true statement – implication can be true also in case “False implies *” - fake reduction

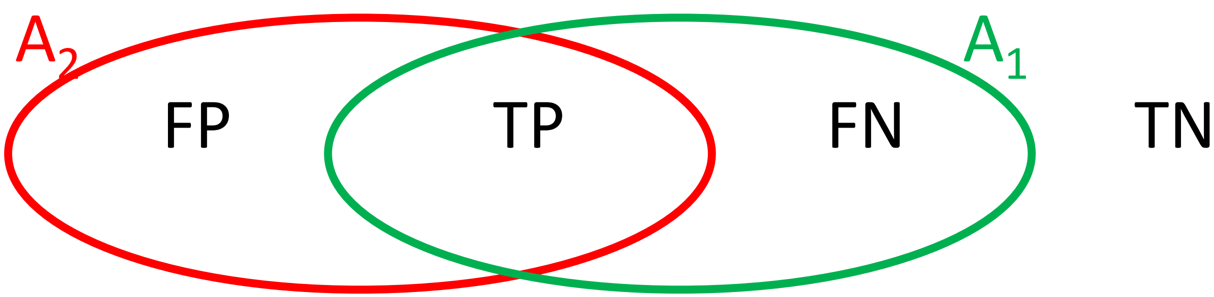
In computer science we have to be careful – **consequence should have a true witness**

SemPre – aggregated measures - evaluating success of semantization and recommendation



ChRF as a general epistemic reasoning method?

- What is the "truth value" **$A_2(r(i_1), s_2)$ => $A_1(i_1, t(s_2))$?**
- A_1 - target, hypothesis, event, reality, deployment,
  A_2 - source, model, evidence, test, experiment,
- A_1 - declarative, correct, semantics, truth, tautology
  A_2 - procedural, computed, syntax , proof
- Preferential logic; Hájek's comparative notion of truth; Bayes; Hájek's observational logic, 4ft, IR; user studies; formal proofs



$$\frac{(A_2, b), (A_1 \leftarrow A_2 , r)}{(A_1, C_{\rightarrow}(b, r))} \qquad Pr(A_1|A_2) = \frac{Pr(A_2|A_1)*Pr(A_1)}{Pr(A_2)} \qquad \frac{\#\ \text{true positive}}{\#\ \text{all}}$$

https://en.wikipedia.org/wiki/Precision_and_recall

		True condition				
	Total population	Condition positive	Condition negative	Prevalence $= \frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	
Predicted condition	Predicted condition positive	**True positive**, Power	**False positive**, Type I error	Positive predictive value (PPV), Precision = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Predicted condition positive}}$	False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Predicted condition positive}}$	
	Predicted condition negative	**False negative**, Type II error	**True negative**	False omission rate (FOR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Predicted condition negative}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Predicted condition negative}}$	
		True positive rate (TPR), Recall, Sensitivity, probability of detection $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False positive rate (FPR), Fall-out, probability of false alarm $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) $= \frac{\text{LR+}}{\text{LR−}}$	F_1 score = $2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$
		False negative rate (FNR), Miss rate $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$	Specificity (SPC), Selectivity, True negative rate (TNR) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$	Negative likelihood ratio (LR−) $= \frac{\text{FNR}}{\text{TNR}}$		

ChRR motivated problems “challenges”

- Specify A_1
- Learn, find, code A_2 , α_2 ,
- Learn, find, code f^-, f^+, …
- What is the right “truth value” of $A_1 \Rightarrow A_2$
  - math. logic (fuzzy connectives)
  - probabilistic measures P/R , RMSE, …
  - offline A/B testing measures, business metric, …
- U-process $\downarrow \rightarrow \uparrow$
- Iterative coupling of enriched / reccurent $\downarrow \rightarrow \uparrow$

ChRR → many valued logic → preferential Datalog + Data domain calculi

- Integration of search and decision complexity problems, of deduction and induction
- Only one implication!
- No equivalence! Still correct …
- No nontrivial fulfillment of $\Rightarrow$
- Cartesian closed category $\Downarrow$
- Complexity strength as that of search and decision problem
- Acceptability can be function, algorithm
- Not necessary 100%, various metric

$$
\begin{tikzcd}
C_1 \arrow[d, "f^-"] & \_A_1\_ \arrow[d, Rightarrow] & R_1 \\
C_2 & \_A_2\_ & R_2 \arrow[u, "f^+"']
\end{tikzcd}
$$

$$(\forall c_1 \in C_1)(\forall r_2 \in R_2)$$
$$A_2(f^-(c_1), r_2)$$
$$\Downarrow$$
$$A_1(c_1, f^+(r_2))$$
$$f^+(nar_2) = nar_1$$

Theoretical problems

$$\begin{tikzcd} C_1 \arrow[d, "f^-"'] & \_A_1\_ & R_1 \\ C_2 & \_A_2\_ & R_2 \arrow[u, "f^+"] \end{tikzcd}$$

$$\begin{tikzcd} C_1 \arrow[d, "f^-"'] & \_A_1\_ & R_1 \\ C_2 & \_A_2\_ & R_2 \arrow[u, "f^+"] \end{tikzcd}$$

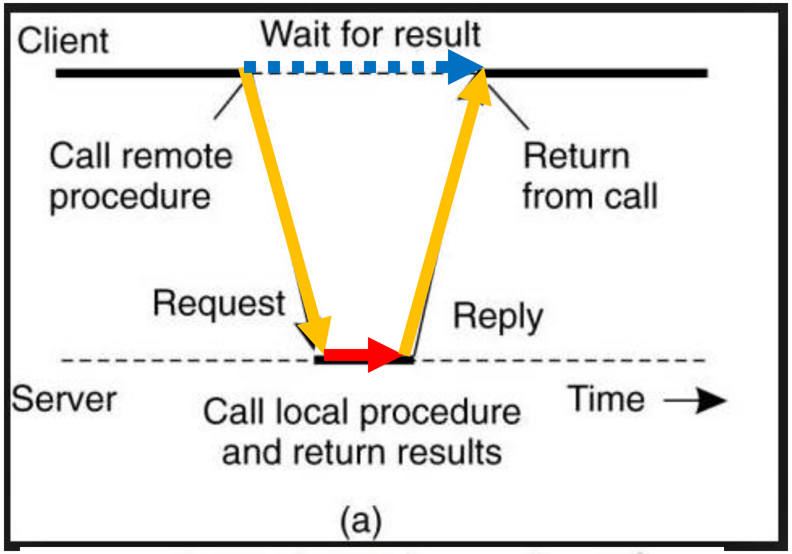
what does it mean there is a CRR reduction between two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ and all possible graph theoretic questions

$\mathbb{C} = \{(S_1, S_2)$: where S_1 and S_2 are finite CR situations, $S_i = (C_i, R_i, A_i)\}$,

$\mathbb{R} = \{(f_-, f_+)$: pairs of finite functions$\}$

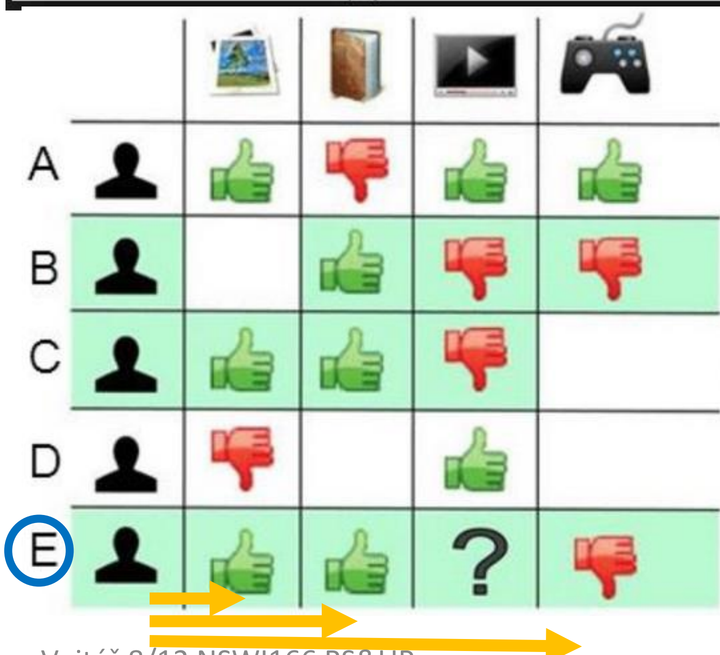
$\mathbb{A}$ checks whether (f_-, f_+) form a CR -reduction from S_1 to S_2

ChRR in decision support, client server



Problem situations

- **client** - **server**
- **Manager - decision support** expert/tool
- Customer - **recommender system** of e-shop



Collaborative filtering model + history of users' behavior data