NSWI166 Introduction to recommender systems and user preferences

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5/12 Linear Monotone Preference Model

Originally Information models with ordering NDBI037

Outline of this lecture

Information models and ordering

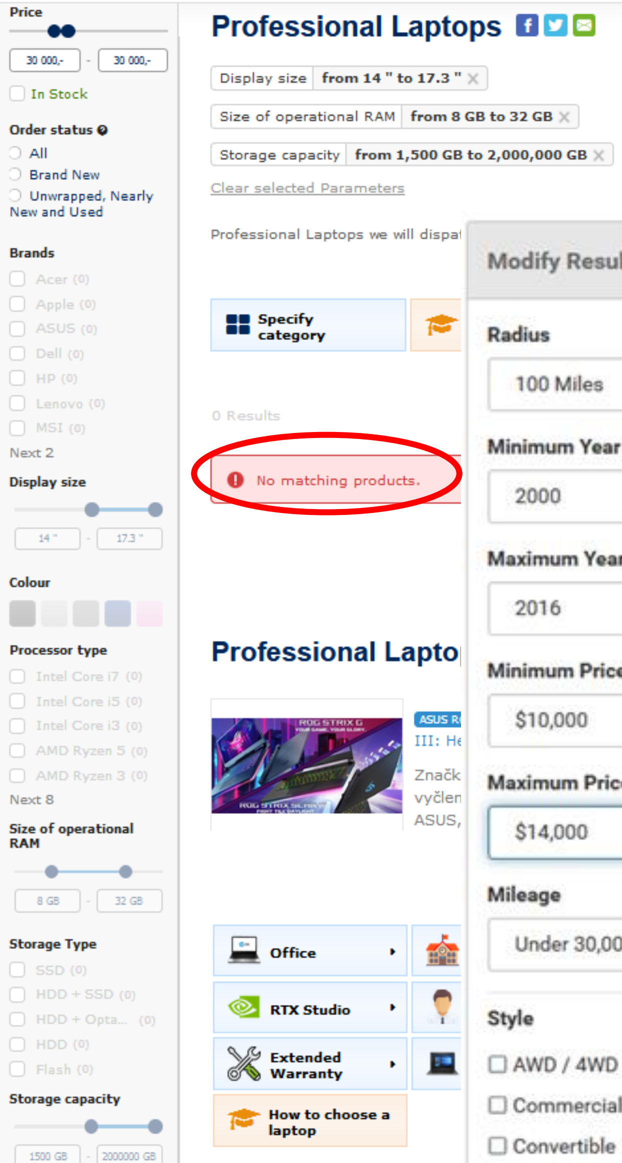
Various representation and presentations of ordering in data, information, knowledge +Linear Monotone Preference Model

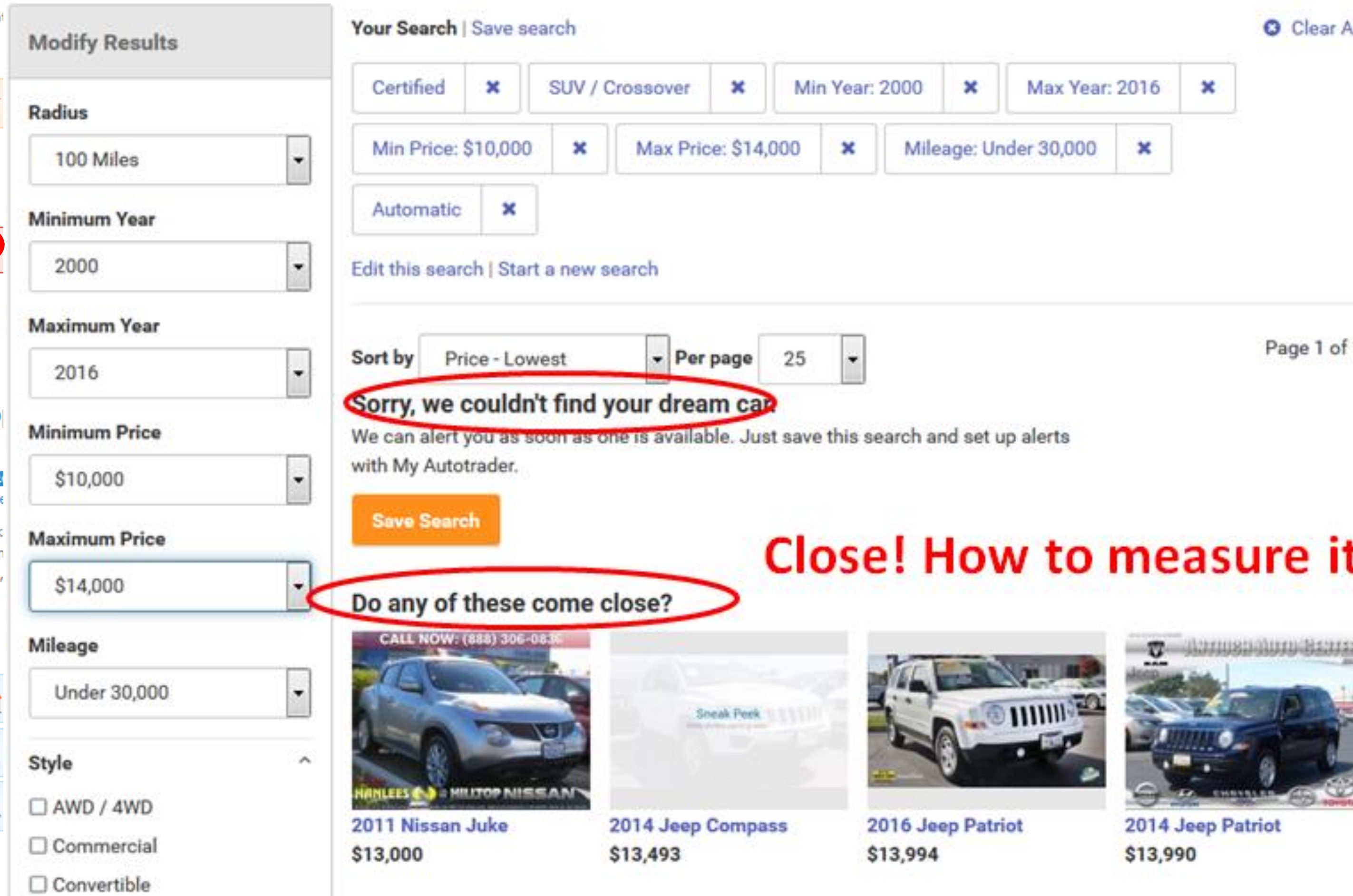
- User requirements – conflicting, multicriterial
- Ordering – human, intuitive, (self) explainable
- Decathlon
- Customer model – ideal values, choice
- Linear Monotone Preference Model
- Data cube, Preference cube, contour lines, top-k, …
- Examples, Conclusions

Motto: "The purpose of abstraction is not to be vague, but to create a new semantic level in which one can be absolutely precise."

— Edsger W. Dijkstra, "The Humble Programmer" 1972 ACM Turing Lecture, see **Human-Centered Approach to Static-Analysis-Driven Developer Tools**

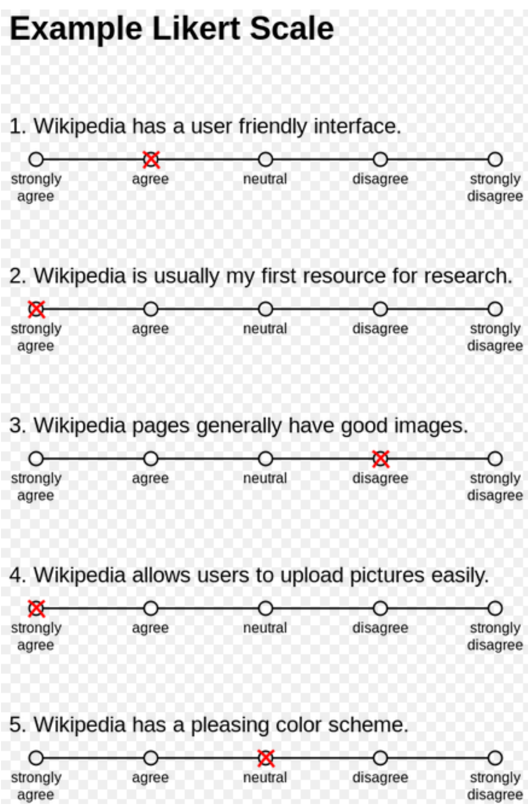
Conflicting requirements – No such product Do any of these come close?

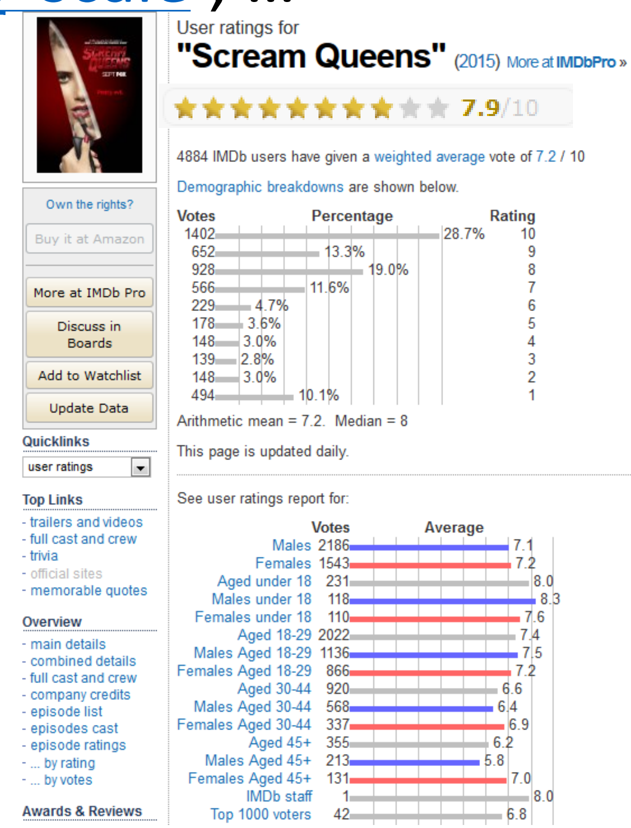




Ordering – human, intuitive, …scaled

- … Self explainable - Information technology - more and more about the people and for the people
- Ordering – in the language, Likert's scale https://en.wikipedia.org/wiki/Likert_scale , …



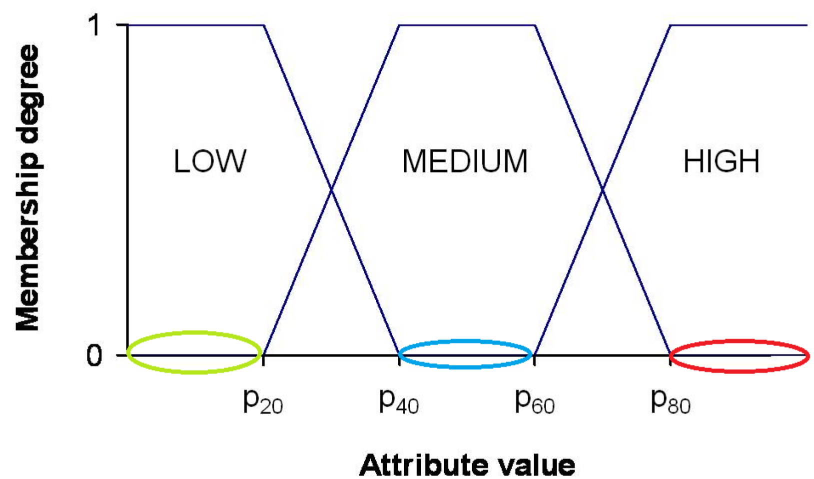


User preference – scaled + ideal values

- In competitions it is clear who is better = winner
- On e-shop it is not clear – users differ
- Producers know this - Marketing Segmentation
- „Faceted browsing“ – specifying „ideal values“
- Too many items, conflicting requirements

Preferential sets

- Preferential sets are variants of fuzzy sets
  - Fuzzy sets intended to model linguistic vagueness
  - Preferential sets model human some linear preferences
- Price - fully cheap, reasonable, expensive
- Linearly ordered domain low, medium, high
- Linguistic input is very rare – we usually have



  - Preference scale, e.g. [0, 1]

Decathlon data – scale-points, multicriterial

P	Athlete	Points		P	100m	P	Long	P	Shot	P	High	P	400m	P	110mh	P	Discus	P	Pole	P	Javelin	P	1500m
1	Šebrle CZE	9026		1	10,64	1	8.11	4	15.93	1	2.12	2	46,89	1	13,92	4	48.53	2	5.3	1	70.16	5	4.21,85
2	Nool EST	8604		4	10,66	2	7.8	3	15.83	5	2.12	8	47,70	3	13,99	1	47.92	4	5.2	6	68.15	1	4.21,98
3	Dvorak CZE	8527		2	10,73	3	7.69	9	15.67	7	2.06	1	47,79	4	14,22	3	46.74	11	5.1	2	66.94	12	4.26,13
4	Lobodin RUS	8465		17	10,76	8	7.49	1	15.33	12	2.03	9	48,01	7	14,30	5	46.73	10	5	3	66.66	10	4.27,65
5	Zsivoczky HUN	8173		3	10,84	6	7.39	8	15.16	4	2	14	48,67	10	14,30	7	46.61	12	5	14	63.93	11	4.31,69
6	Ambrosch AUT	8122		10	10,87	11	7.35	16	15.1	13	2	3	48,76	9	14,36	9	46.41	9	4.9	15	61.65	3	4.33,58
7	Kürtösi HUN	8099		14	10,89	4	7.32	10	14.85	2	1.97	17	48,76	2	14,46	11	44.07	1	4.8	5	60.57	4	4.35,97
8	Warners NED	8085		8	10,90	5	7.19	7	14.77	3	1.97	10	48,77	8	14,48	2	43.32	6	4.8	16	59.97	6	4.36,36
9	Hämäläinen FIN	8028		6	10,93	12	7.17	6	14.71	14	1.97	5	48,81	6	14,56	8	41.64	8	4.8	7	59.83	2	4.39,11
10	Jensen NOR	8004		9	10,99	7	7.16	17	14.67	15	1.97	4	48,91	12	14,61	12	41.56	3	4.7	10	58.51	16	4.40,22
11	Schönbeck GER	7891		13	10,99	9	7.11	5	14.6	6	1.94	7	49,07	14	14,70	15	41.3	7	4.7	17	58.23	8	4.42,47
12	Niklaus GER	7891		5	11,01	13	7.11	2	14.37	8	1.94	13	49,26	15	14,83	14	41.14	15	4.6	11	58.11	9	4.42,66
13	Tebbich AUT	7632		16	11,03	10	6.94	11	14.17	16	1.94	6	49,33	17	14,97	10	40.38	5	4.5	8	55.62	13	4.46,57
14	Llanos PUR	7613		7	11,08	15	6.93	13	13.78	9	1.91	16	49,90	11	15,06	13	40.18	13	4.5	4	54.56	15	4.47,22
15	SchnallingerAUT	7576		12	11,09	14	6.89	14	13.67	11	1.88	12	50,14	13	15,07	6	39.52	17	4.4	13	54.32	17	4.48,52
16	Walser AUT	7546		11	11,31	17	6.83	12	12.99	10	1.85	15	50,25	16	15,27	16	39.45	16	4.2	12	51.95	7	4.49,58
17	Walser AUT	7506		15	11,35	16	6.81	15	12.98	17	1.82	11	50,51	5	15,43	17	37.2	14	4.1	9	50.33	14	4.59,38

Decathlon points-commeasurable

P	Athlete	Points	P	100m	P	Long	P	Shot	P	High	P	400m	P	110mh	P	Discus	P	Pole	P	Javelin	P	1500m
1	Šebrle CZE	9026	1	942	1	1089	4	847	1	915	2	964	1	985	4	840	2	1004	1	892	5	799
2	Nool EST	8604	4	938	2	1010	3	841	5	915	8	924	3	976	1	827	4	972	6	861	1	798
3	Dvorak CZE	8527	2	922	3	982	9	831	7	859	1	919	4	946	3	803	11	941	2	843	12	770
4	Lobodin RUS	8465	17	915	8	932	1	810	12	831	9	909	7	936	5	803	10	910	3	839	10	760
5	Zsivoczky HUN	8173	3	897	6	908	8	800	4	803	14	877	10	936	7	800	12	910	14	797	11	734
6	Ambrosch AUT	8122	10	890	11	898	16	796	13	803	3	873	9	929	9	796	9	880	15	763	3	721
7	Kürtösi HUN	8099	14	885	4	891	10	780	2	776	17	873	2	916	11	748	1	849	5	746	4	706
8	Warners NED	8085	8	883	5	859	7	776	3	776	10	872	8	913	2	732	6	849	16	737	6	703
9	Hämäläinen FIN	8028	6	876	12	854	6	772	14	776	5	870	6	903	8	698	8	849	7	735	2	686
10	Jensen NOR	8004	9	863	7	853	17	769	15	776	4	866	12	897	12	696	3	819	10	715	16	679
11	Schönbeck GER	7891	13	863	9	840	5	765	6	749	7	858	14	886	15	691	7	819	17	711	8	665
12	Niklaus GER	7891	5	858	13	840	2	751	8	749	13	849	15	870	14	688	15	790	11	709	9	664
13	Tebbich AUT	7632	16	854	10	799	11	739	16	749	6	846	17	853	10	672	5	760	8	672	13	640
14	Llanos PUR	7613	7	843	15	797	13	715	9	723	16	819	11	842	13	668	13	760	4	656	15	636
15	Schnallinger AUT	7576	12	841	14	788	14	708	11	696	12	808	13	841	6	655	17	731	13	653	17	628
16	Walser AUT	7546	11	793	17	774	12	667	10	670	15	803	16	817	16	653	16	673	12	617	7	621
17	Walser AUT	7506	15	784	16	769	15	666	17	644	11	791	5	798	17	608	14	645	9	593	14	563

Better is rephrased by dominance

- Intuitively better (dominating) means better in all disciplines = Pareto ordering – it is a **partial ordering**! We need the winner!
- Lobodin • dominates both Nool • and Dvorak •. • and • are incomparable (restricted to 100m x shot data and points)

P	Athlete	Points
1	Šebrle CZE	9026
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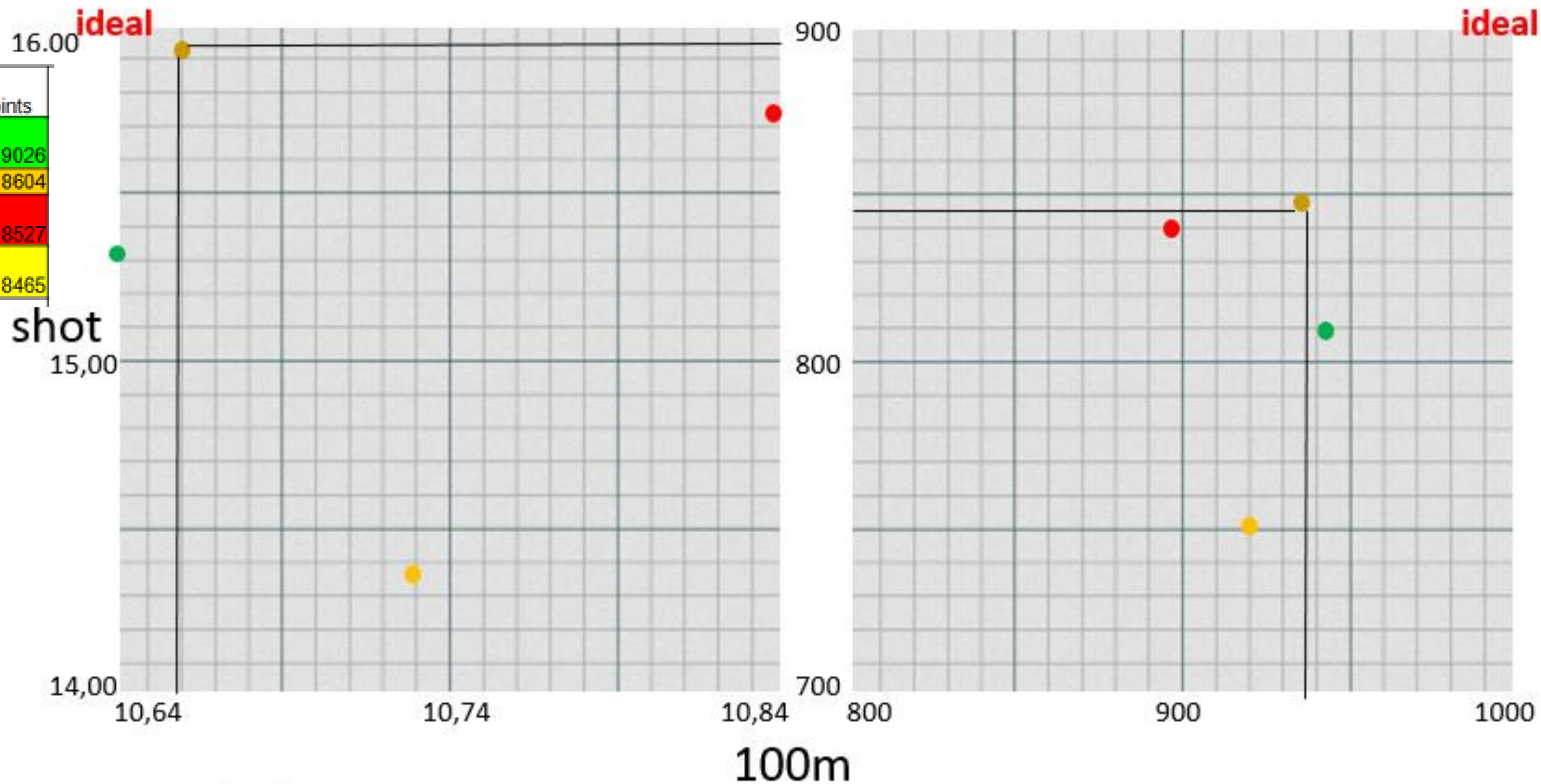

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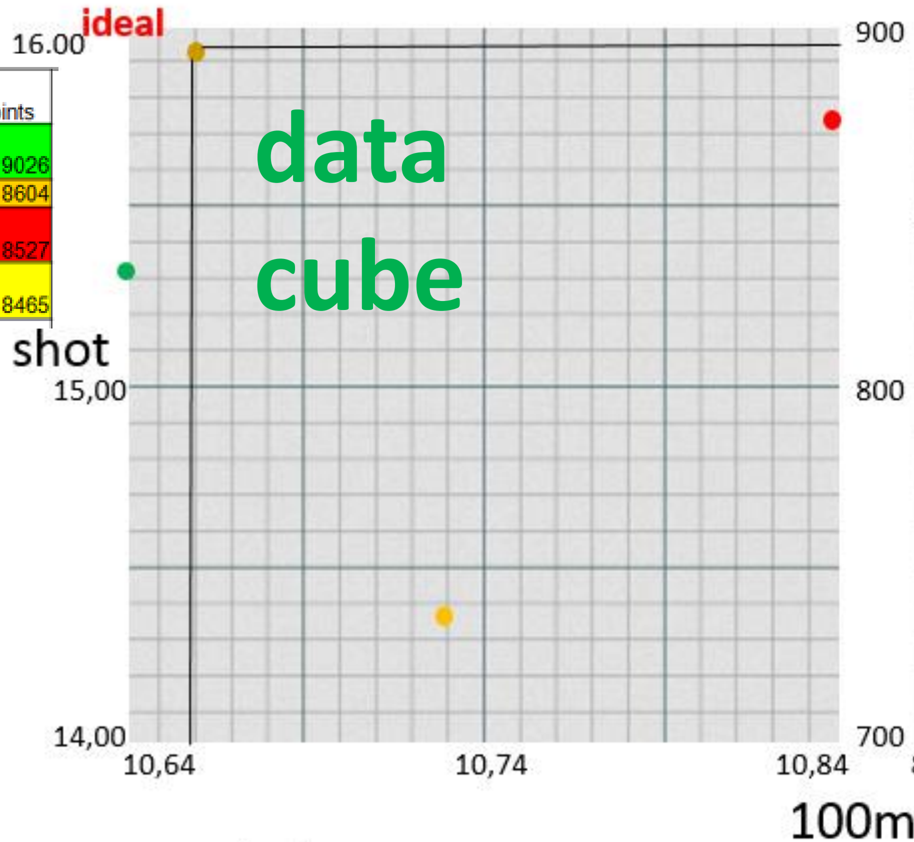

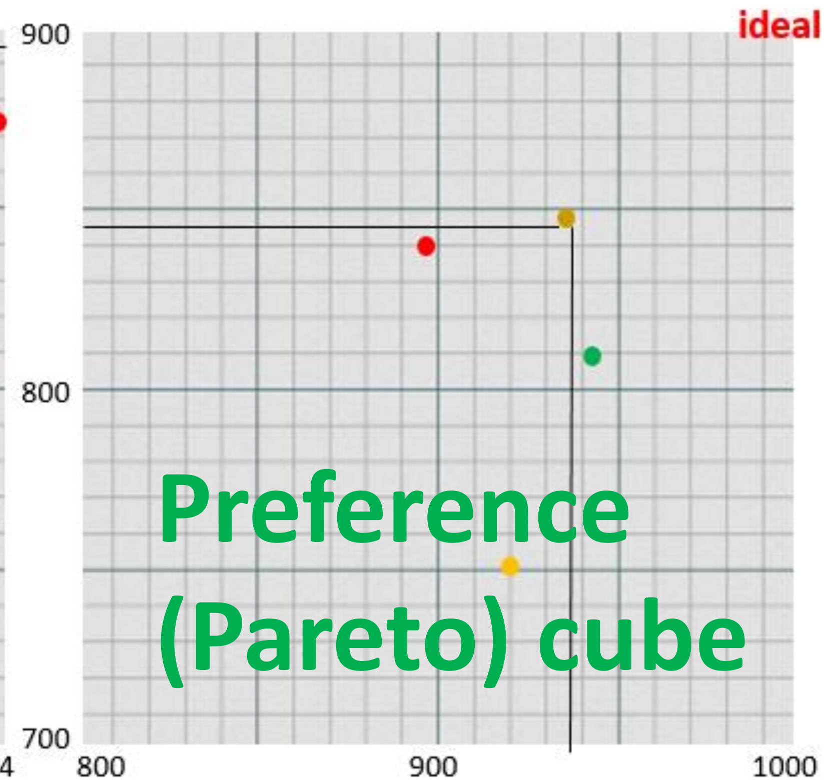

Sum of points makes Decathlon linear

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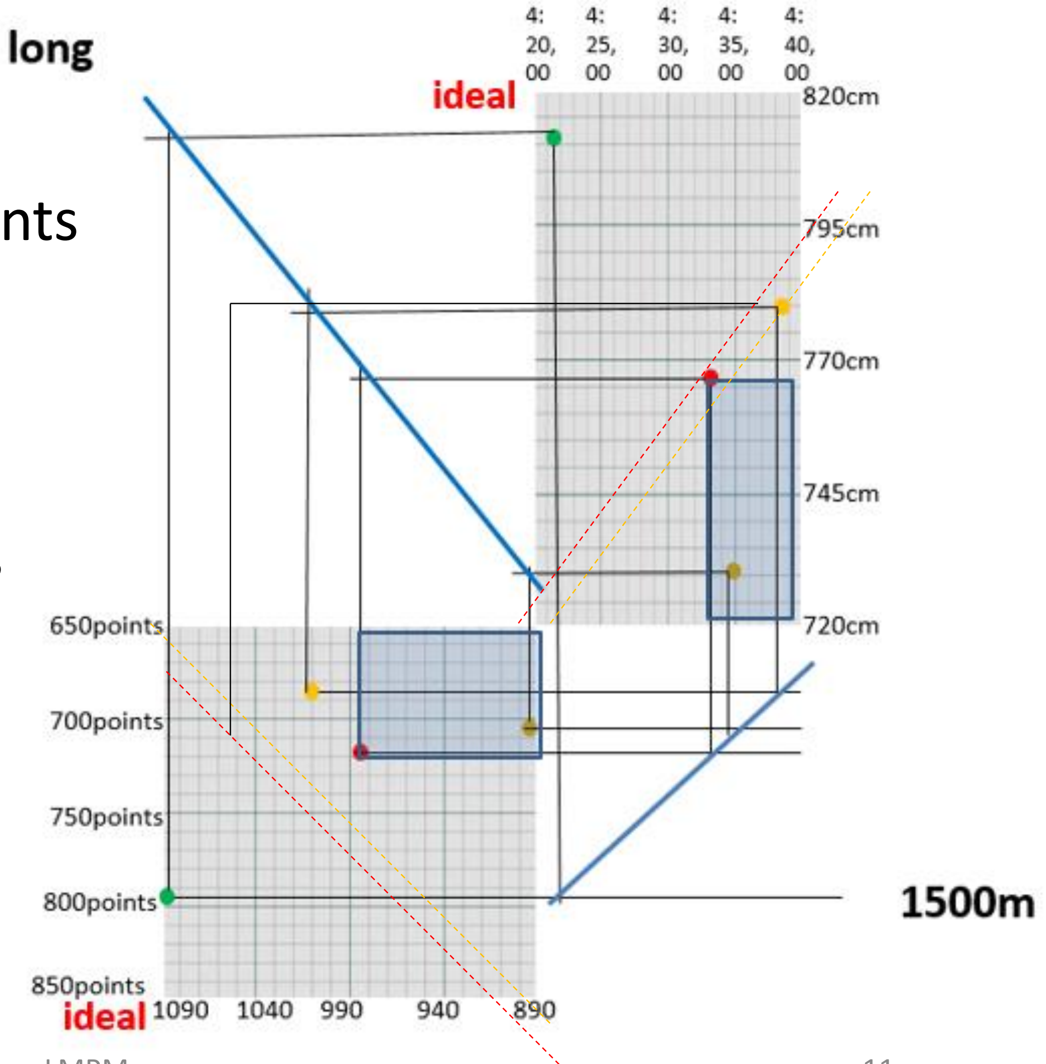
- Data cube (upper right) – **point function** transforms achievements to preference cube (lower left)
- • dominates •
- • and • are incomparable
- Sum (aggregation) of points in these two events
  - • = 1703 points
  - • = 1696 points
- PC contour line connects points with same result in Pareto cube
- Contour line can be propagated to data cube



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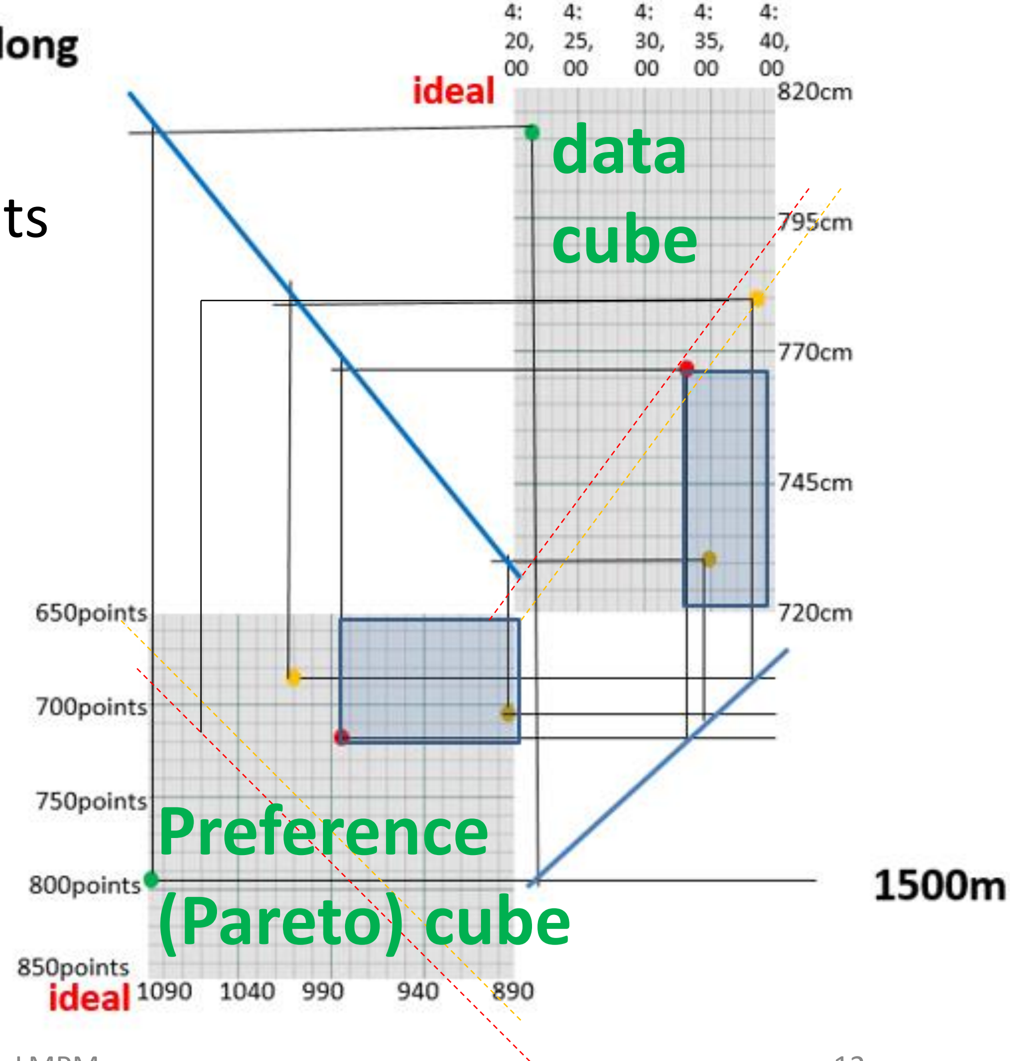
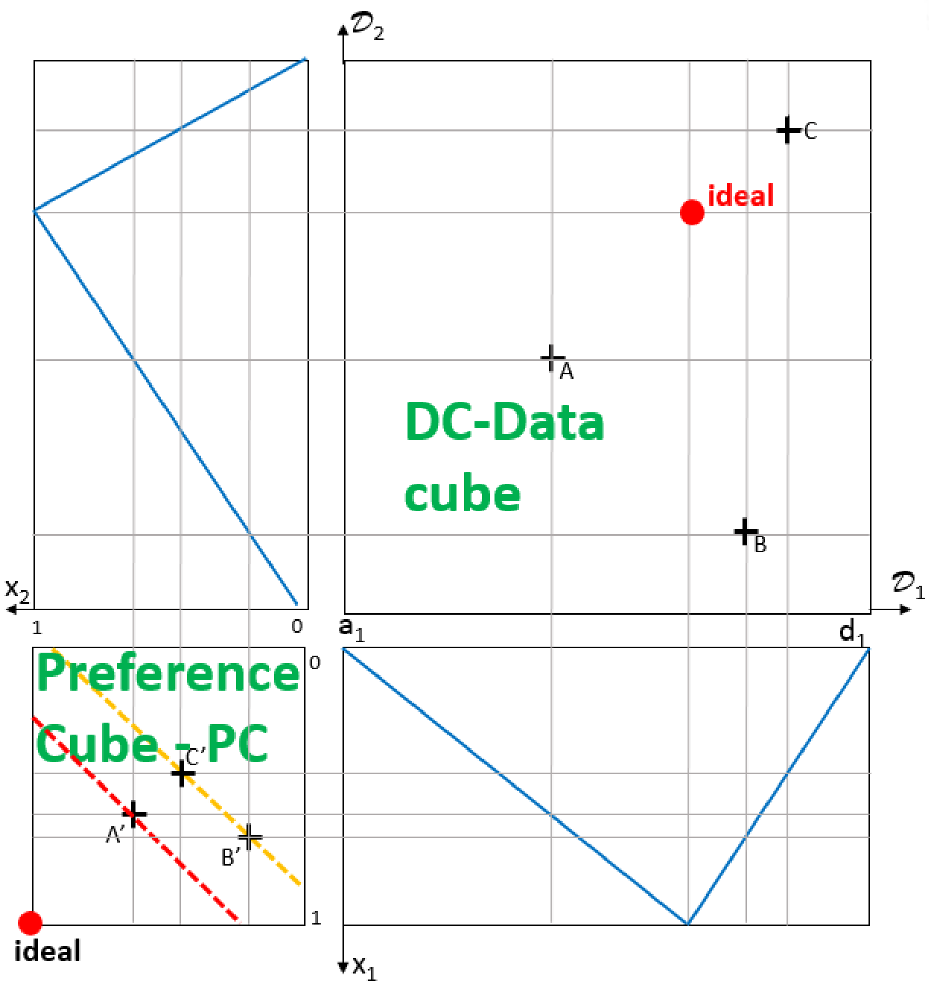
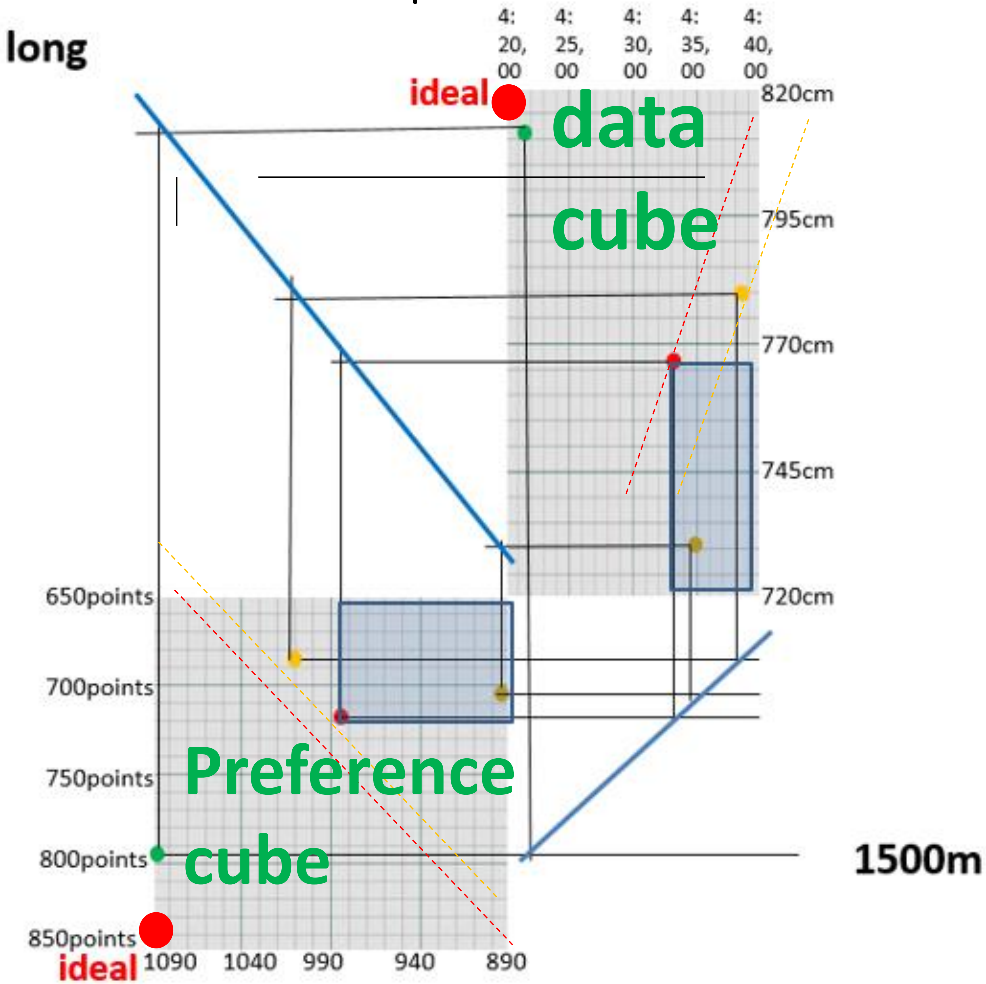


Decathlon like preference model for information ordering in web e-shops
DC-Athletes → items , Ordering made linear by sum → aggregation
Preference scale linear point system → [0,1] preference degree
Single authority decides winner → each user separately has/can have own preferences
PC-Multicriterial Pareto partial ordering of preference degree vectors
Decathlon like preference model – all parts linear – **linear monotone preference model - LMPM**





Linear **Monotone** Preference Model-LMPM

- Decathlon – "**single user**" **IAAF** rules order athletes
  - Disciplines $\mathcal{A}_1,\dots,\mathcal{A}_{10}$; domains $\mathcal{D}_1,\dots,\mathcal{D}_{10}$; ideal (field / track)
  - $\mathcal{A}_i$ point function $\mathbf{f_i} : \mathcal{D}_i \rightarrow N$ makes results commeasurable
    - Winner - overall IAAF achievement is obtained via sum
      $\Sigma\{\mathbf{f_i}(\text{athleteID}.\mathcal{A}_i): i = 1, \dots, 10\}$
- Retail, e-shop – **set of users U**, $\text{LMPM}^{\mathbf{u}}$ orders items
  - Attributes $\mathcal{A}_1,\dots,\mathcal{A}_m$; domains $\mathcal{D}_1,\dots,\mathcal{D}_m$; ideal points can be for each user different
  - Degree of preference for $\mathcal{A}_i$ and user $\mathbf{u} \in U$ $\mathbf{f_i^u}: \mathcal{D}_i \rightarrow [0, 1]$ – hardly made commeasurable in response time
    - Winner, top-k, overall degree of preference - aggregation
      $r^{f,t}(\text{objectID}) = \mathbf{t^u}\{\mathbf{f_i^u}(\text{objectID}.\mathcal{A}_i): i = 1, \dots, m\}$

  Here $\mathbf{t^u}: [0, 1]^m \rightarrow [0, 1]$, $\mathbf{t^u}(0,\dots,0) = 0$, $\mathbf{t^u}(1,\dots,1) = 1$,
  $\mathbf{t^u}$ **monotone**(linear) - preserves Pareto ordering,

Let us build LMPM step by step

Data model: attributes $\mathcal{A}_1, \mathcal{A}_2$; domains $\mathcal{D}_1, \mathcal{D}_2$; only 2-dimensional – makes paper drawing easier

triangular degree of preference of $\mathcal{A}_j$, a value from $\mathcal{D}_j$ (local preference) is given by an ideal point i_j and function f_j

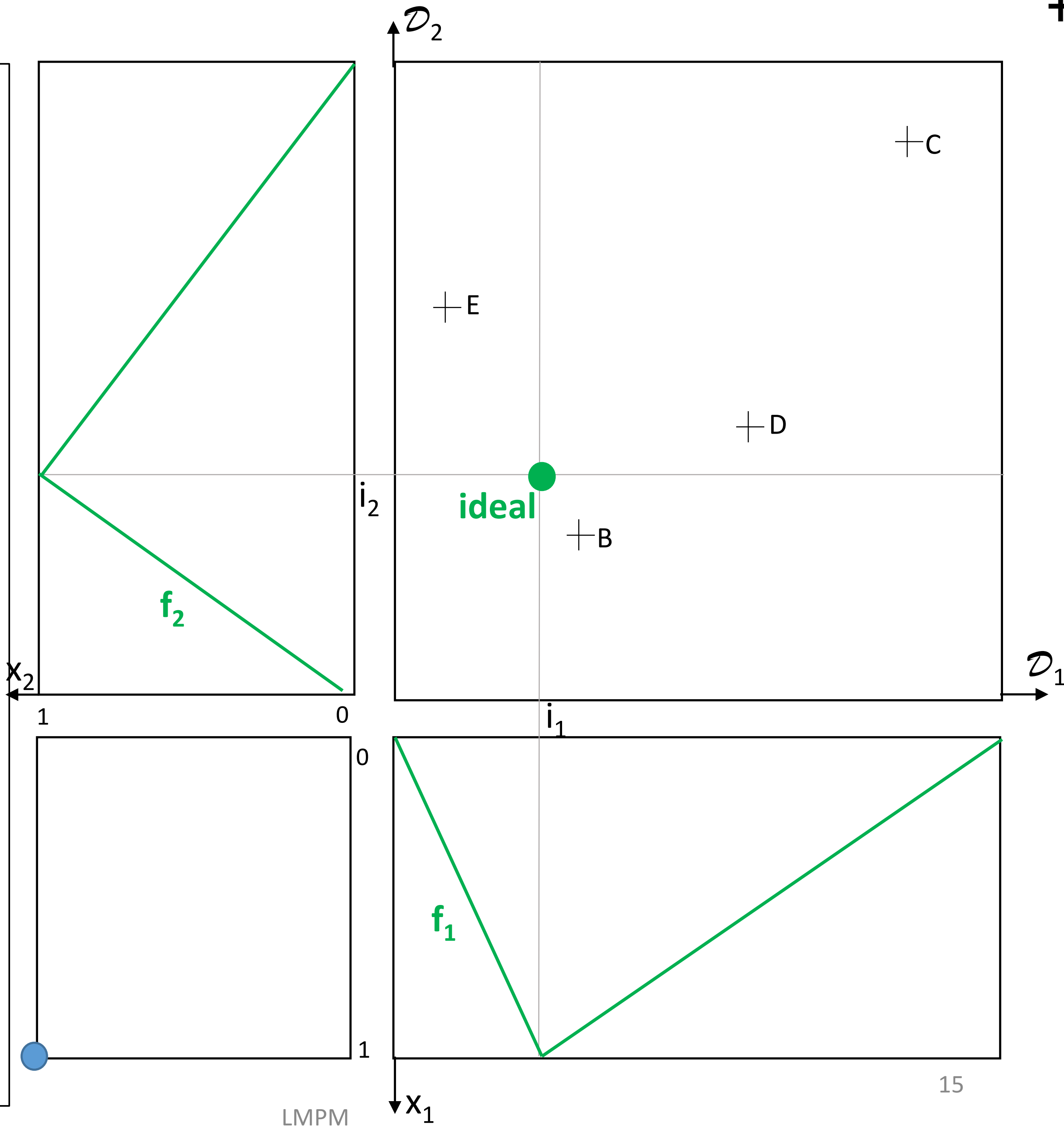
$f_j(x_j) = 0$, $x_j = \min \mathcal{D}_j$

$f_j(x) = (x - x_j)/(i_j - x_j)$ if $x_j \leq x \leq i_j$

$f_j(i_j) = 1$

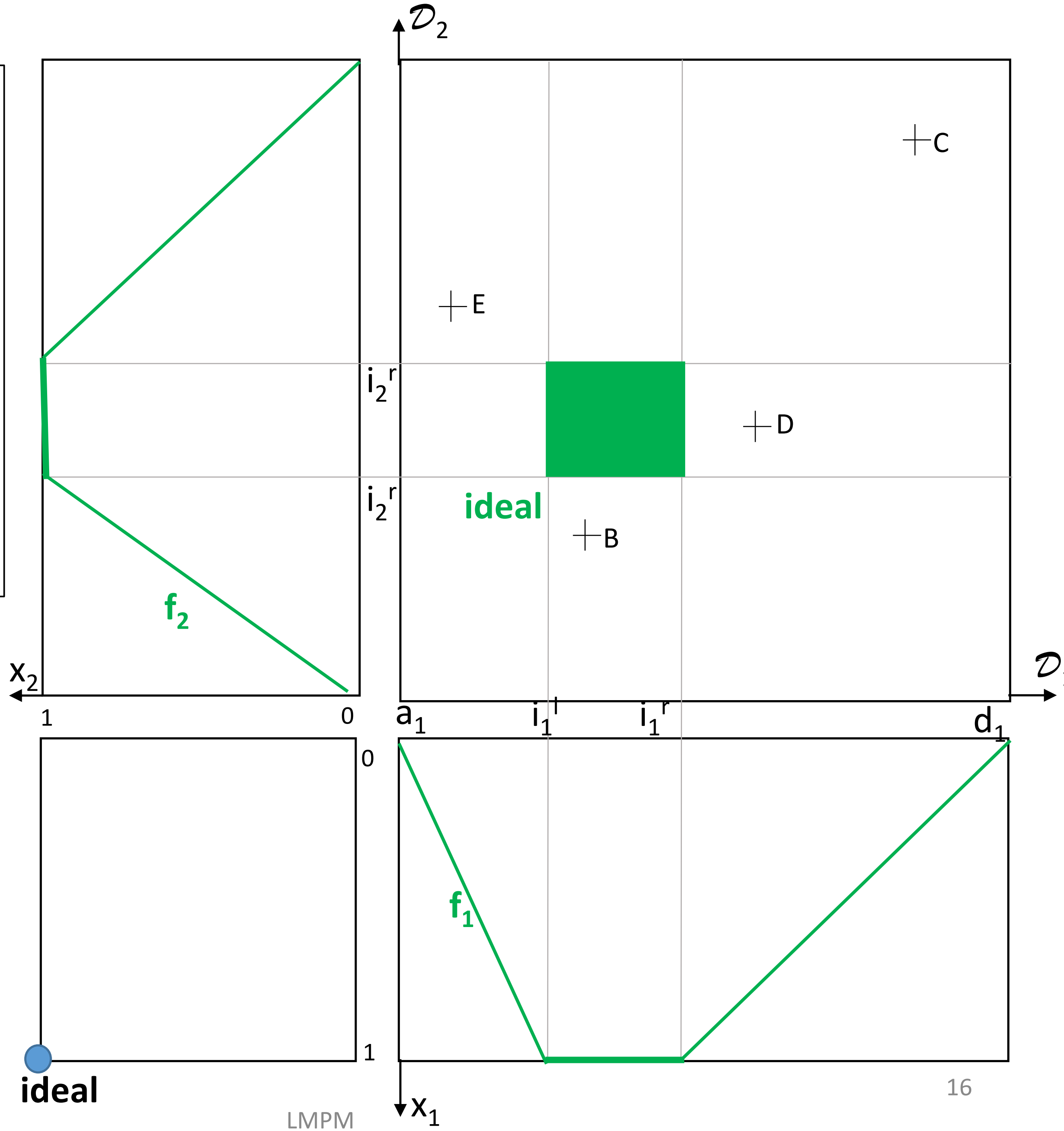
$f_j(x) = (y_j - x)/(y_j - i_j)$ if $i_j \leq x \leq y_j$

$f_j(y_j) = 0$, $y_j = \max \mathcal{D}_i$,

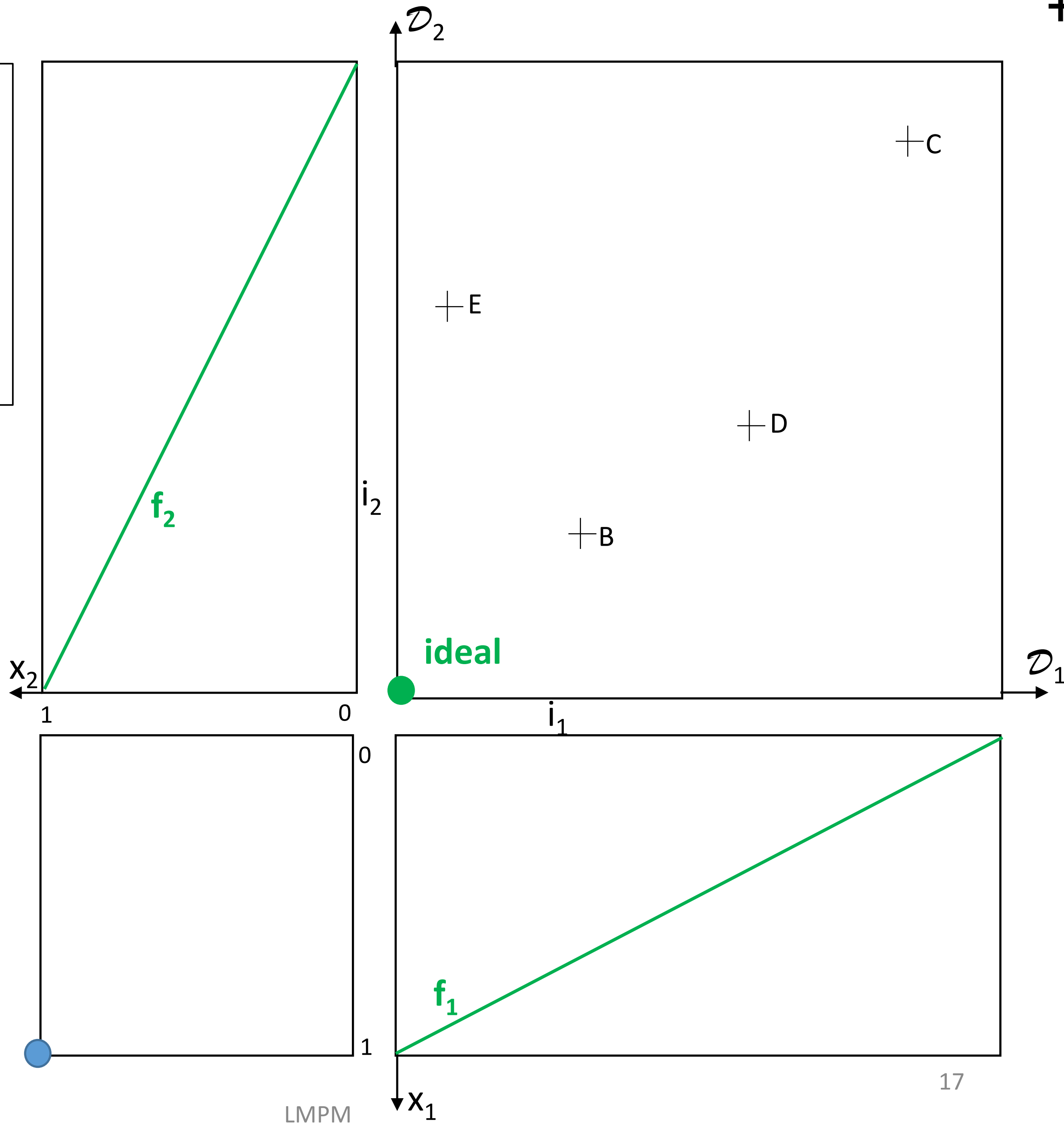


Similarly

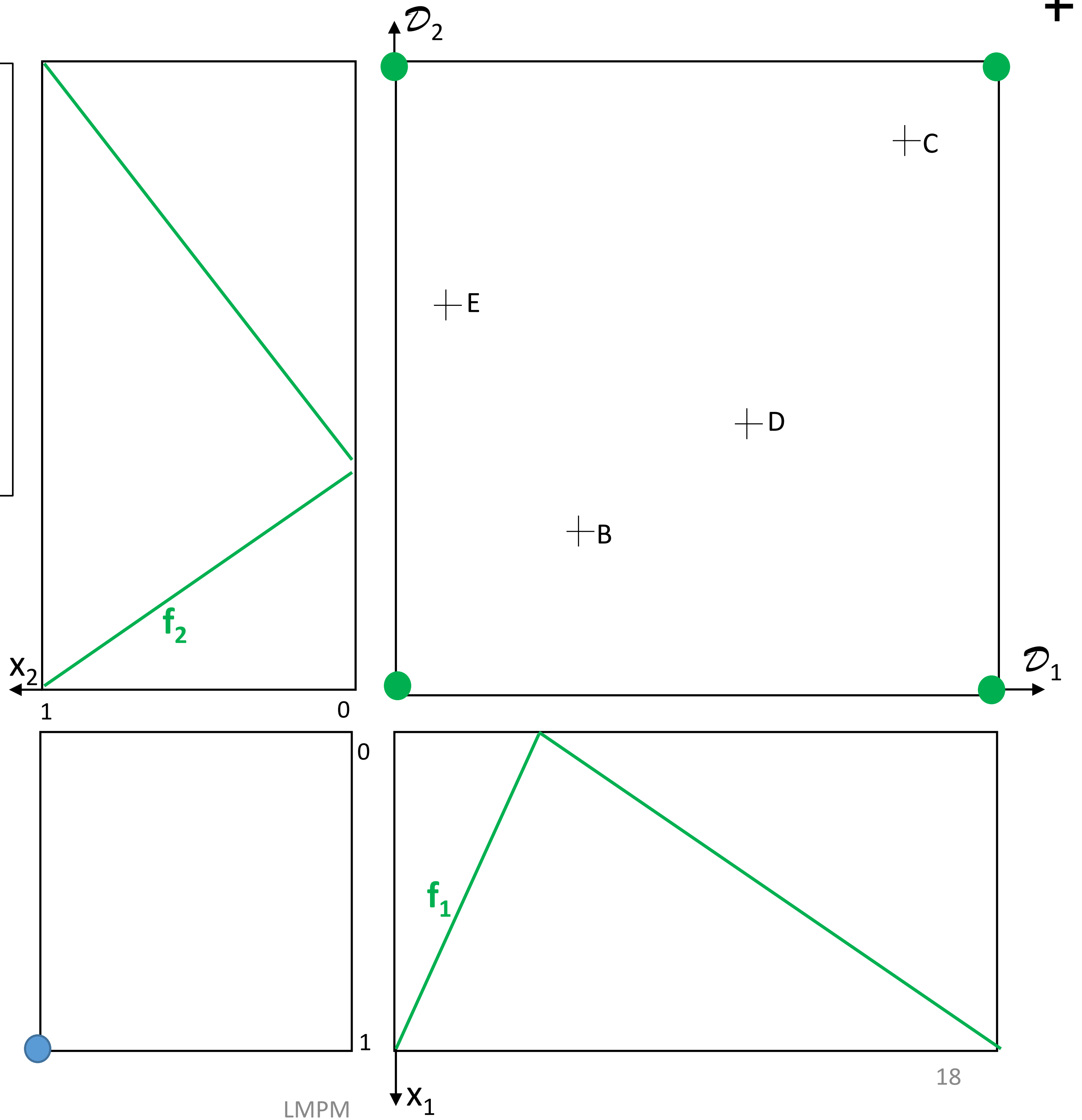
trapezoidal degree of preference of $\mathcal{A}_j$, a value from $\mathcal{D}_j$ (local preference) is given by an ideal interval $[i_j^l, i_j^r]$ and analogically defined functions f_j



degree of preference of $\mathcal{A}_j$, a value from $\mathcal{D}_j$ (local preference) can have different shapes depending on position of ideal points / intervals



degree of preference of $\mathcal{A}_j$, a value from $\mathcal{D}_j$ (local preference) can have different shapes depending on position of ideal points / intervals (and all possible combinations)



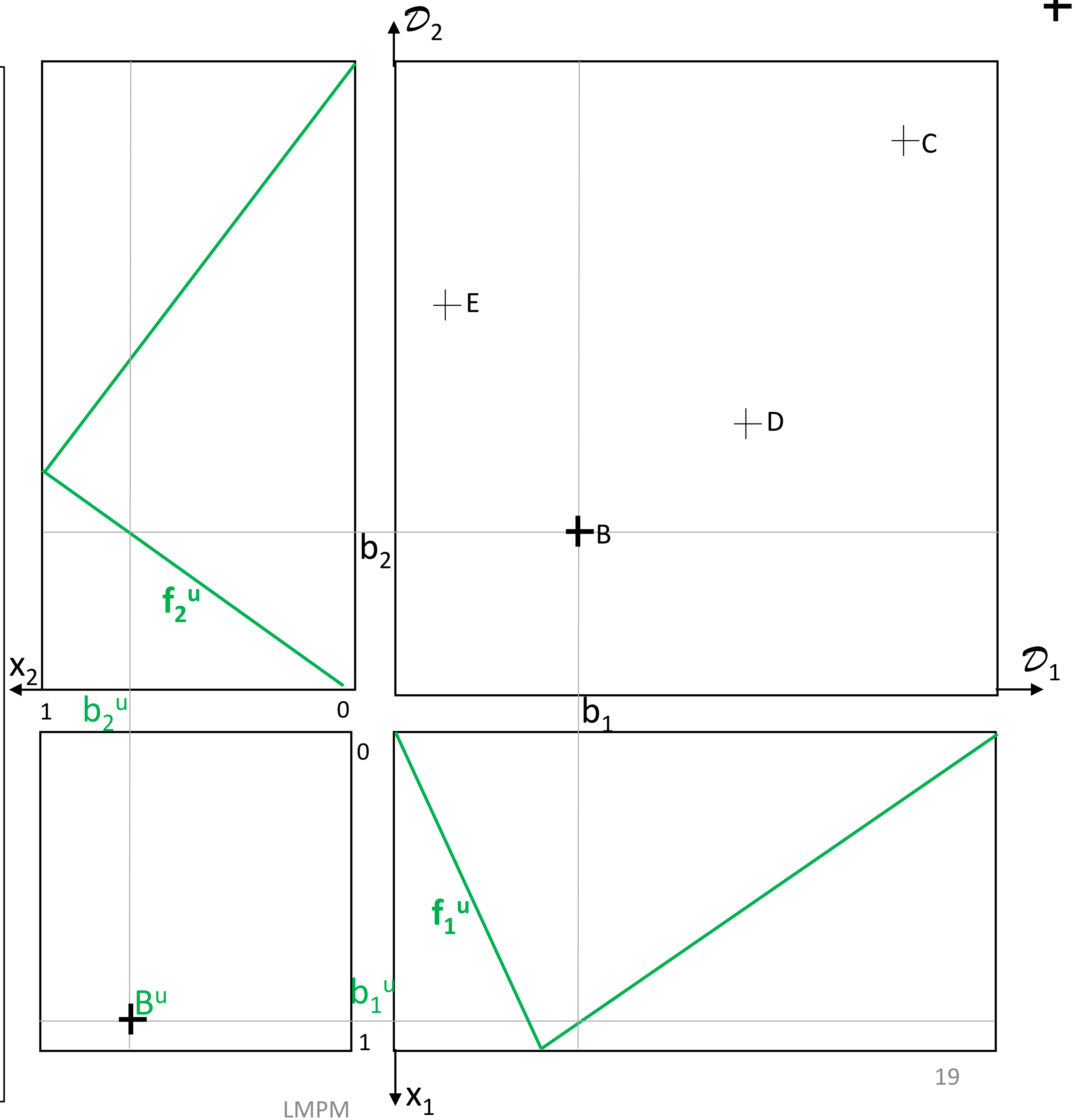
Let us describe steps leading to calculation of preference degree of item B (for user u), we first describe mapping DC-data cube to PC-preference cube

Assume degree of preference $f_i^u: \mathcal{D}_i \rightarrow [0, 1]$ (for an user $u \in U$),

Object with objectID = B has attribute values $B.\mathcal{A}_1 = b_1$ and $B.\mathcal{A}_2 = b_2$, sometimes we write $B=(b_1, b_2)$.

Attribute preference degrees $f_j^u(B.\mathcal{A}_j) = b_j^u$ and corresponding point in preference cube is $B^u = (b_1^u, b_2^u)$

other points analogically



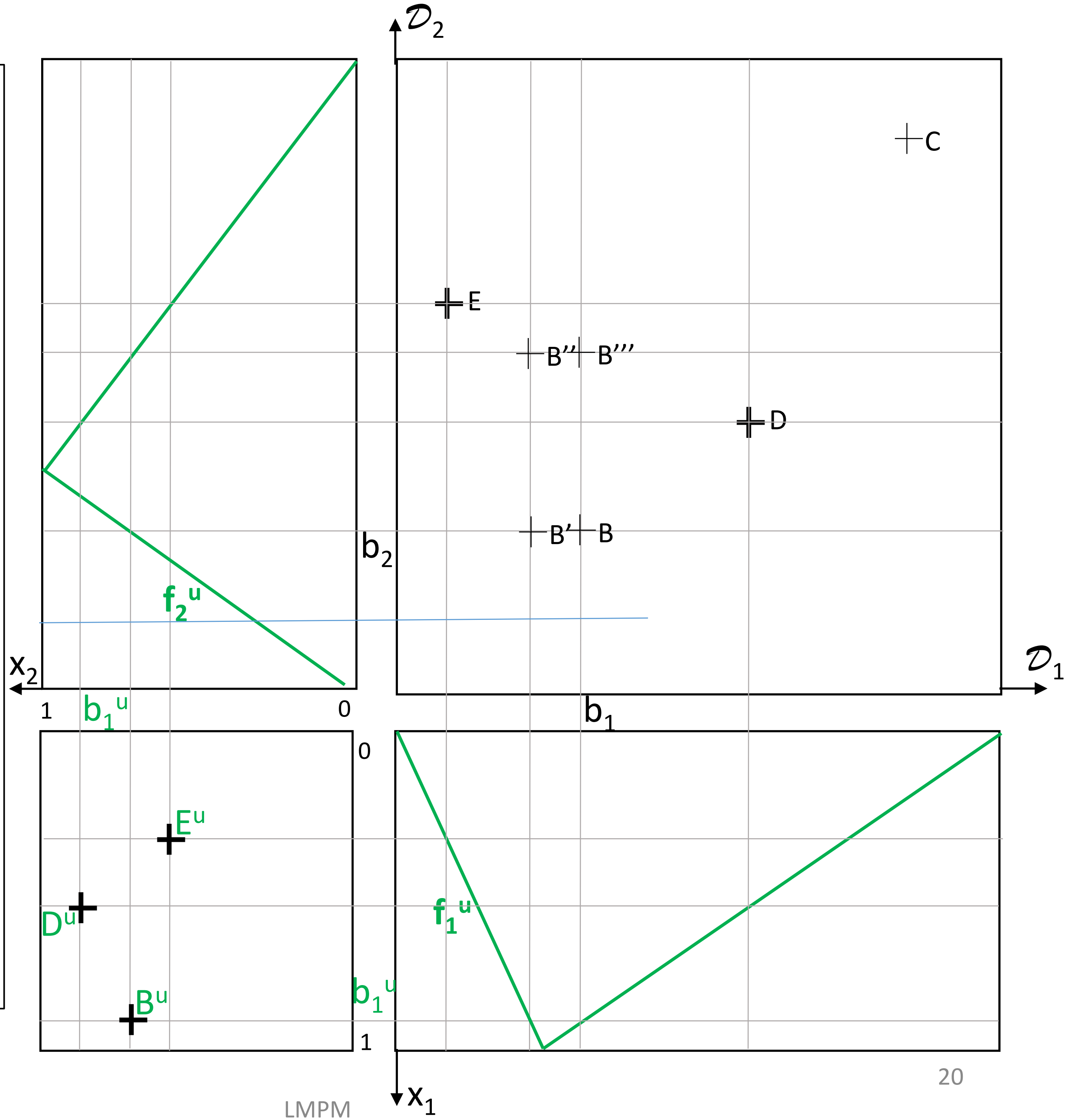
Note, that point B^u has 4 coimages B, B', B'', B'''

Degree of preference for user $u \in U$ are given by f_1^u and f_2^u .

Object with objectID = B has attribute values $B.\mathcal{A}_1 = b_1$ and $B.\mathcal{A}_2 = b_2$, sometimes we write $B=(b_1, b_2)$.

For B, D, E attribute preference degrees the corresponding points in preference cube are B^u, D^u, E^u,

Note that B^u and D^u are incomparable in Pareto order and E^u is dominated by both B^u and D^u,



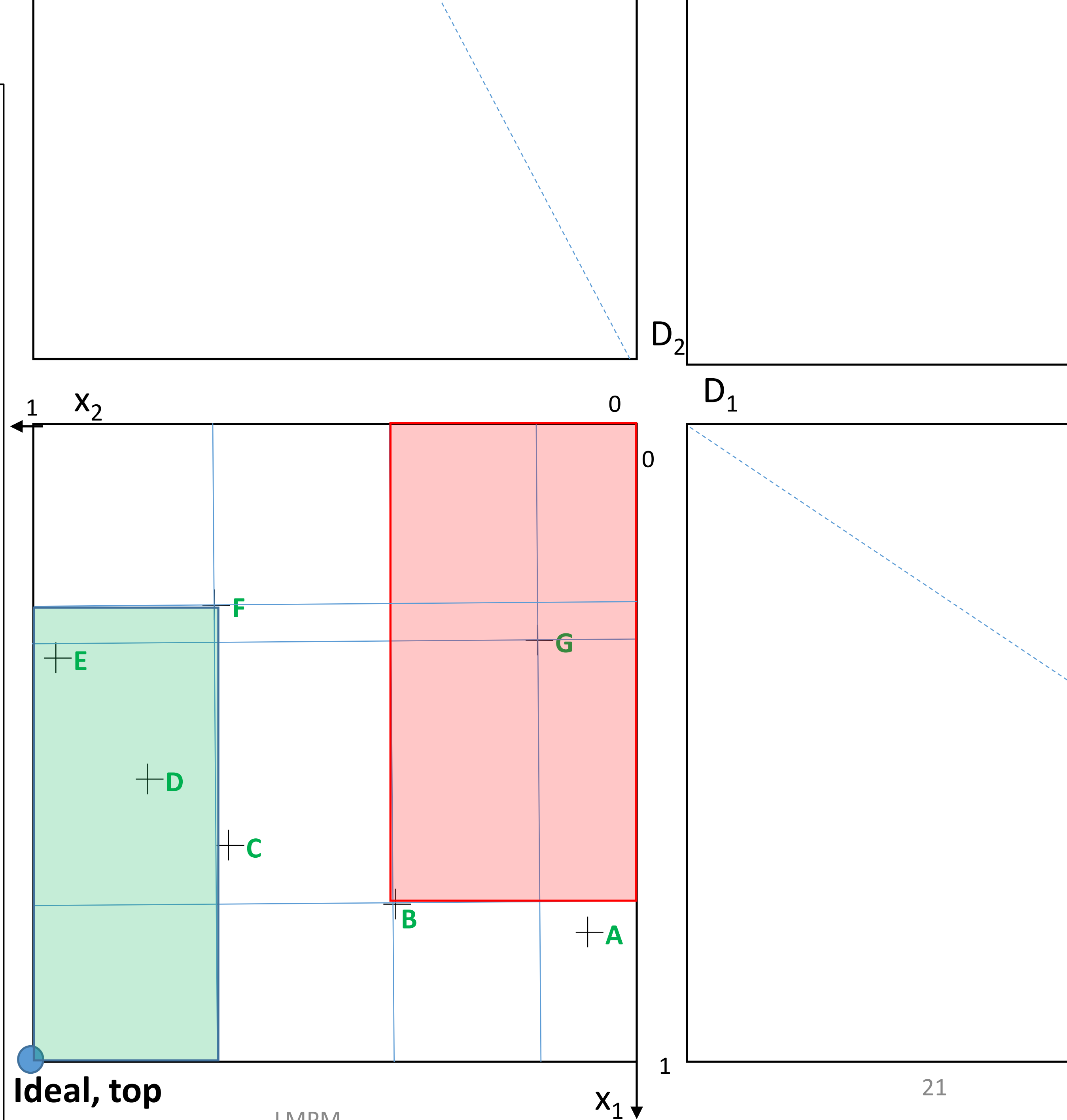
Pareto ordering of pref. cube ($\underline{x}$) $<_{Pareto}$ ($\underline{y}$) iff (for each i) $x_i \leq y_i$ & ($\exists$i) $x_i < y_i$

Assume **A, B, C, D, E, F, G** are images of respective items under some attribute preference

We say that item **B** dominates item **G** in $<_{Pareto}$ (**G** is dominated by **B**), in fact **B** dominates whole red area

F is dominated by whole green area

$<_{Pareto}$ is not linear, e.g. **B** and **C** are not comparable



LMPM

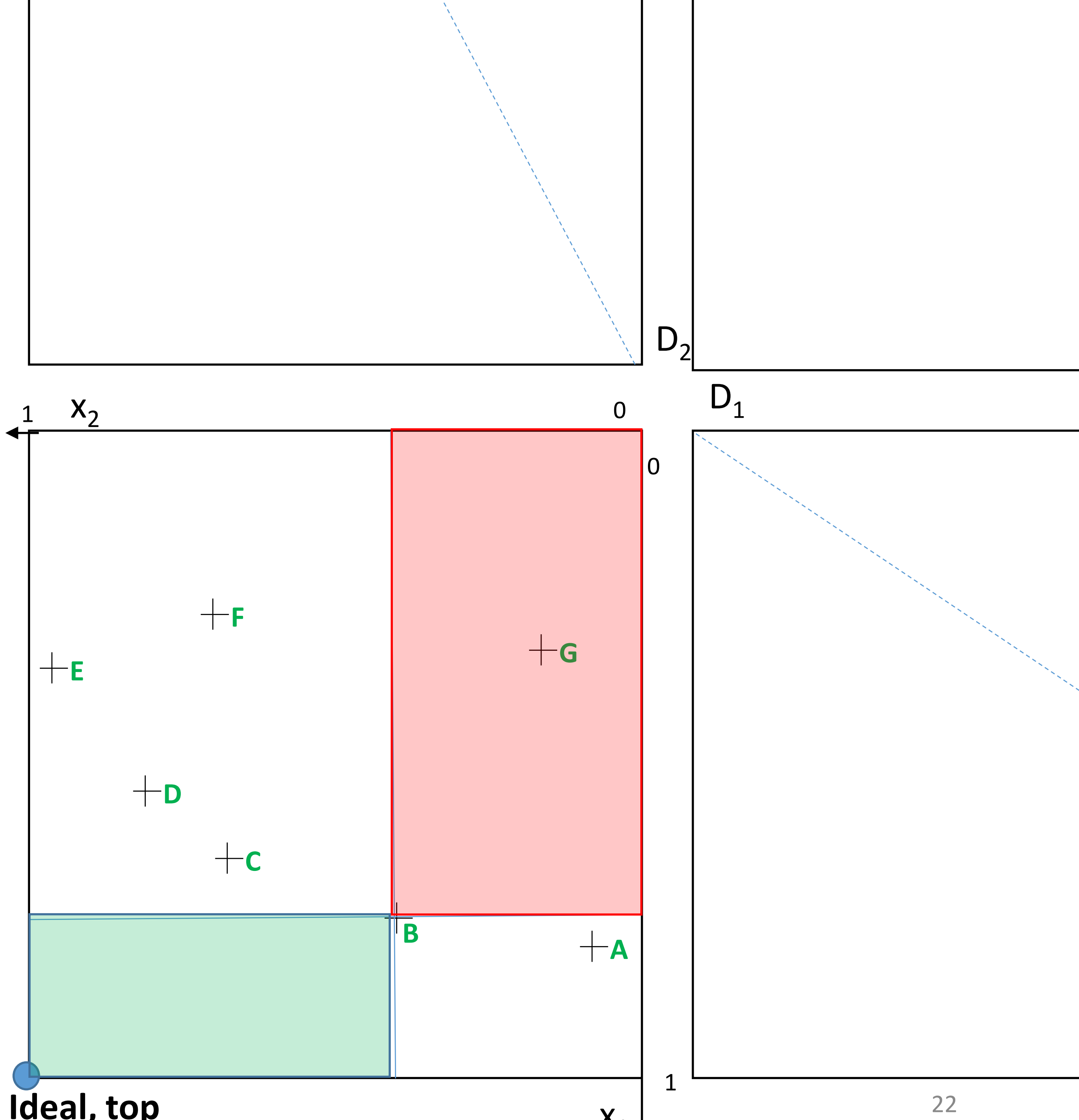
Pareto ordering of pref. cube ($\underline{x}$) $<_{Pareto}$ ($\underline{y}$) iff (for each i) $x_i \leq y_i$ & ($\exists$i) $x_i < y_i$

Assume **A, B, C, D, E, F, G** are images of respective items under some attribute preference

Repeat, that item **B** dominates whole red area and is dominated by whole green area

$<_{Pareto}$ is not linear, e.g. **B** and **C** are not comparable

All item images in white areas are incomparable with **B**



To get global preference degree of items we need aggregation functions. It is a function $[0,1]^2 \rightarrow [0,1]$

$t(x_1, x_2) = w_1 * x_1 + w_2 * x_2$,

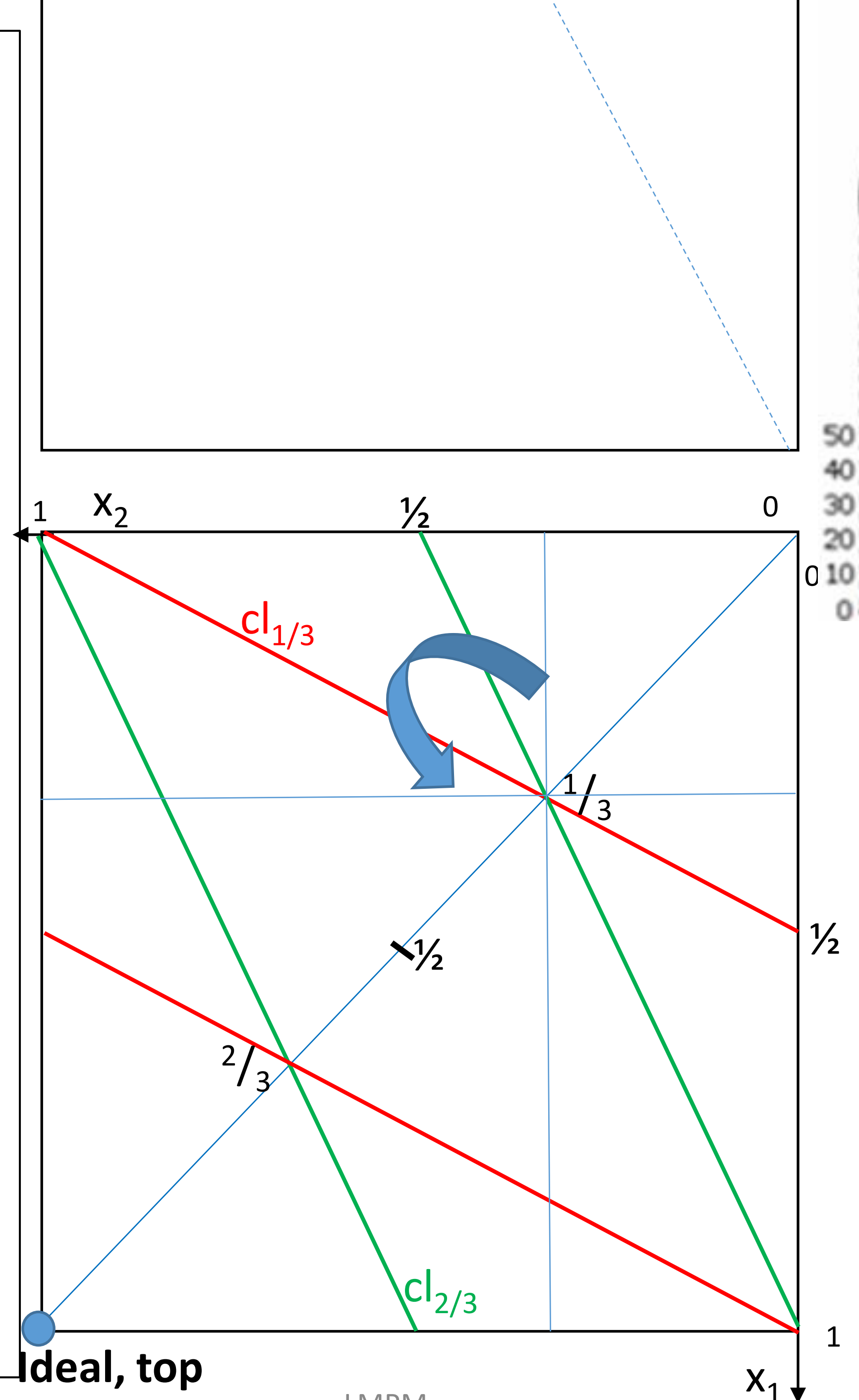
where $w_1, w_2 \geq 0$ are attribute weights with $w_1 + w_2 = 1$

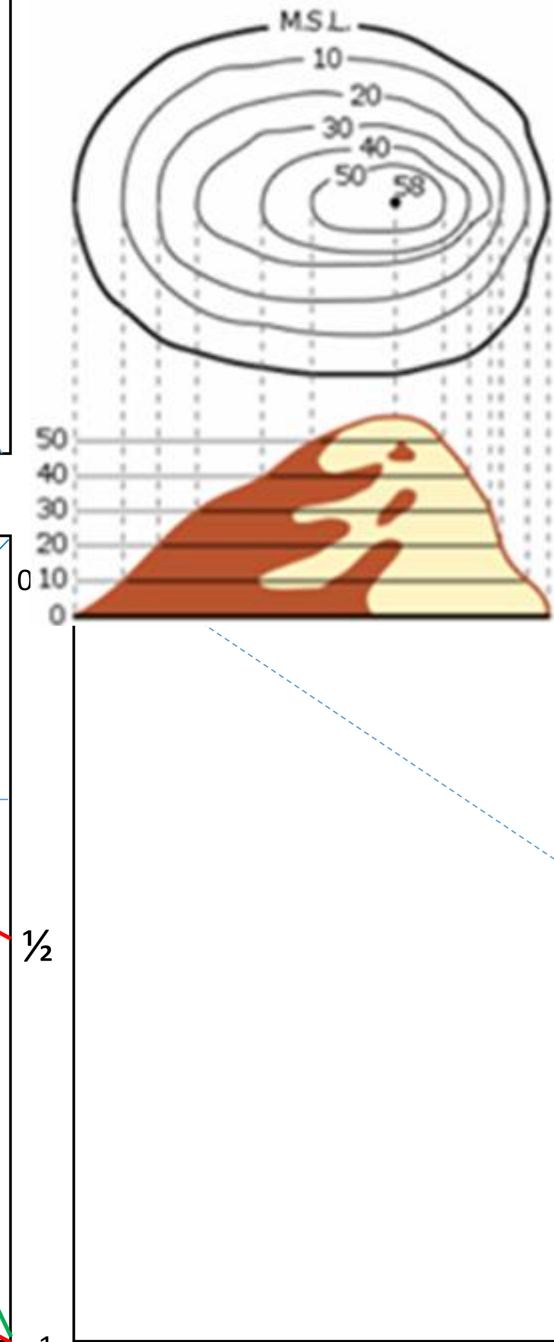
Graph of t is a 3D object. Intuition behind display of aggregation function are contour lines (e.g. $cl_{1/3}$, $cl_{2/3}$) for users **u**, resp. **u**

Note, that on the preference cube diagonal corresponding contour line of preference degree $y \in [0,1]$ intersect the diagonal at point (y, y) , because

$w_1 * x + w_2 * x = y$ gives

$x * (w_1 + w_2) = y$, i.e. $x = y$





Assume, we have users **u**, and **u**. Red are item image using **u**'s preference, green that of **u**.

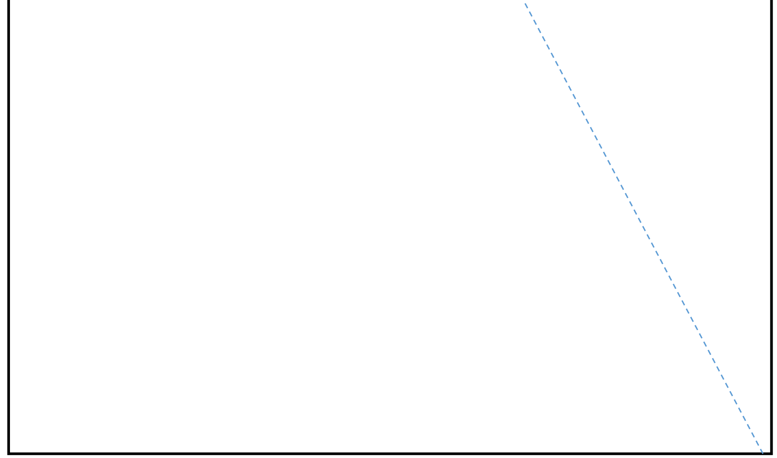
Notice e.g. $t^u(A) = 0.54$

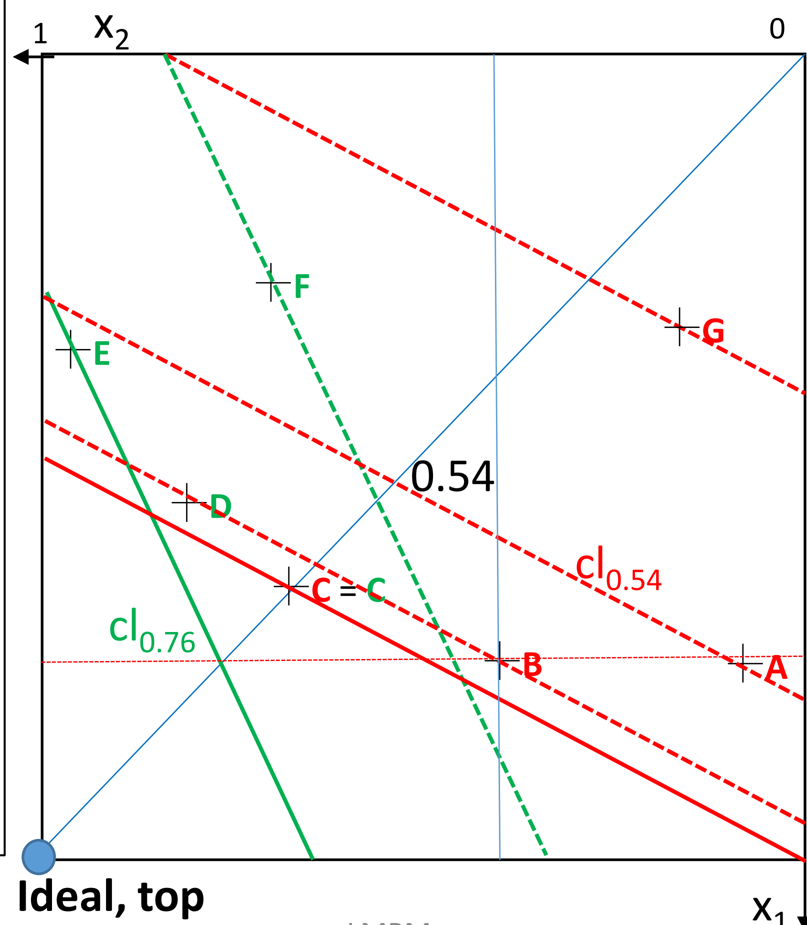
B and **C** are not $<_{Pareto}$ comparable, aggregation makes **C** more preferable for user **u** than **B** (w_1 is sufficiently bigger than w_2)

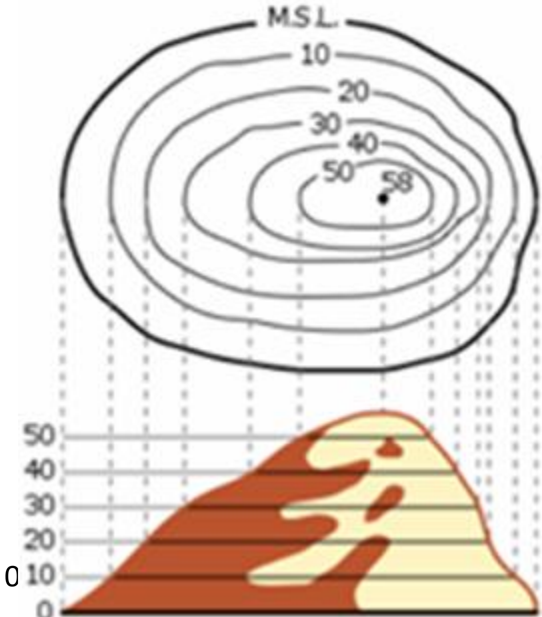
E is best for **u**

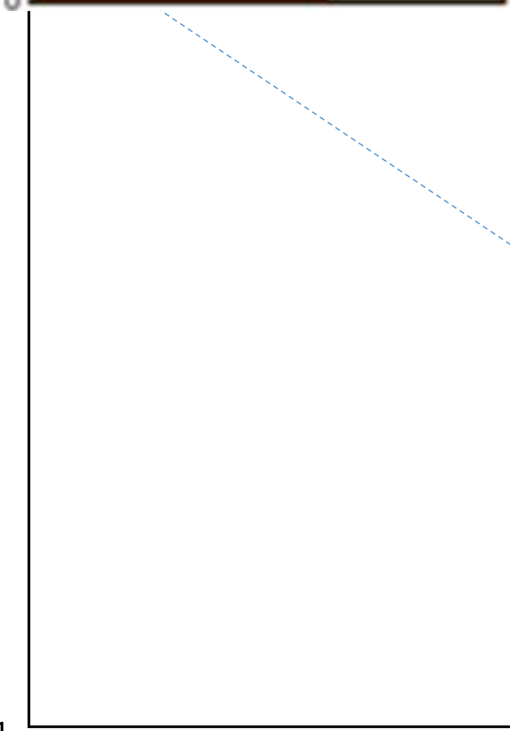
Can **C** be better than **E**?
Can **B** be better than **C**?

If two PC cube points are Pareto incomparable, then any ordering of these is possible – prove or disprove!









How does it work together?

Vector of attribute preferences **f**=[f_1, …, f_m] and aggregation **t** define a user $\mathbf{u}_{\mathbf{f,t}}$ = **u**

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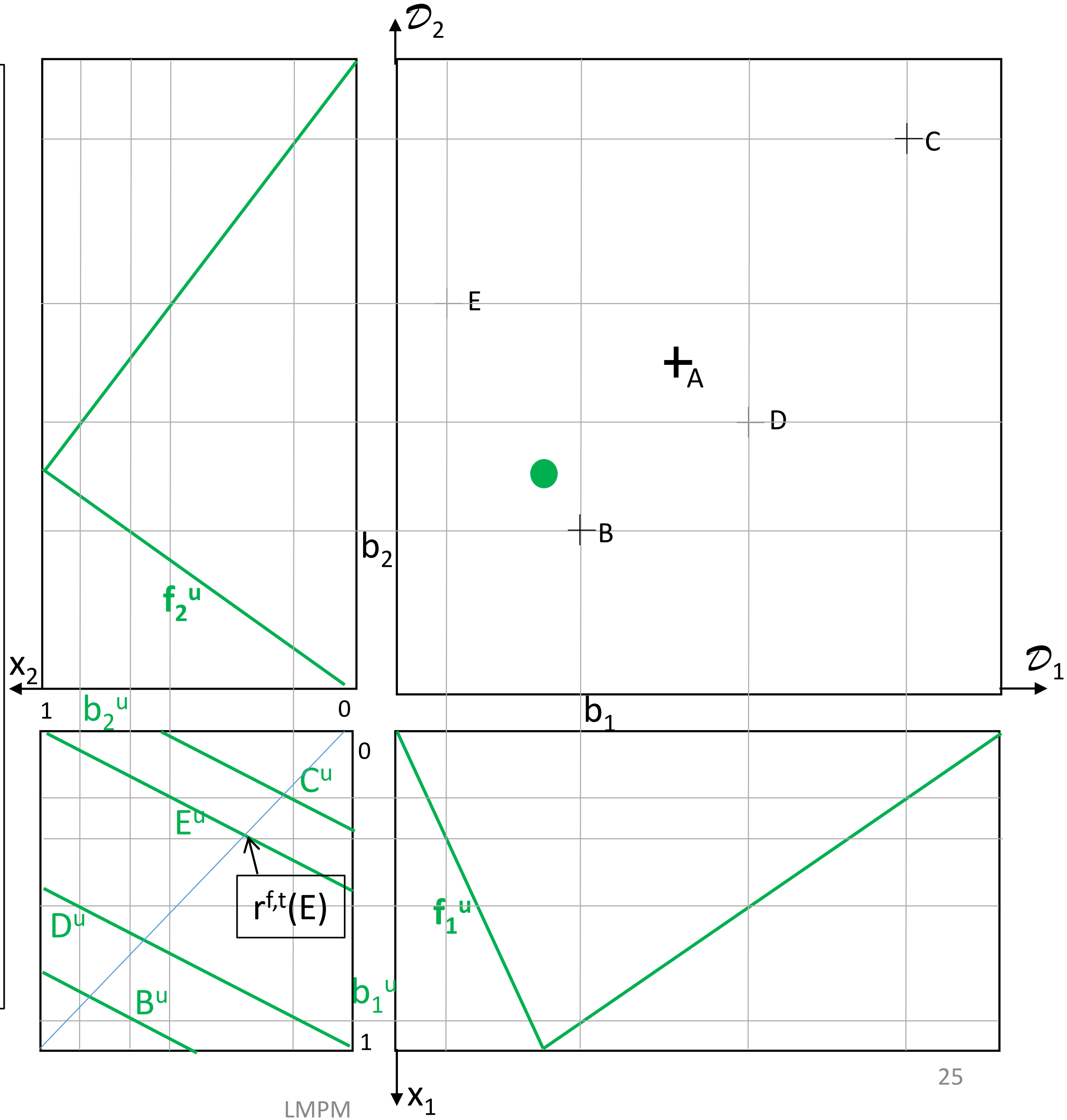
Depict contour line (i.e. items of same preference degree) in DC-data cube is a little bit trickier (depending on position of ideal points and/or intervals)



Dynamical model – three sessions – moving ideal points (aggregations remain same)

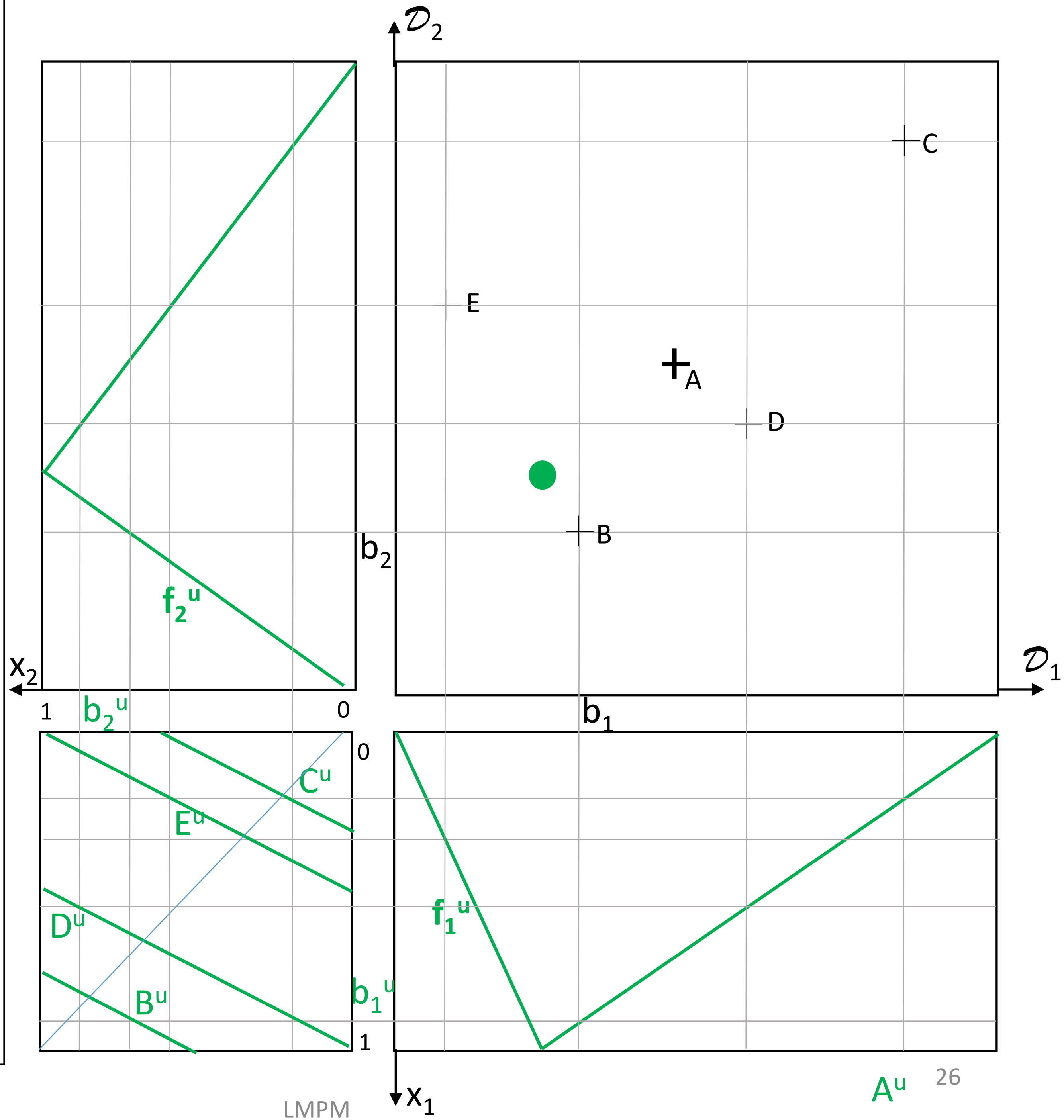
Simulation of development in time

Starting vector of attribute preferences $\mathbf{f^0}$ and aggregation $\mathbf{t^0}$ define an user $\mathbf{u^0_{f,t}} = \mathbf{u^0}$ in time 0. Depict contour line in DC-data cube.

Assume user clicks on third item. In time 1, $\mathbf{t^0} = \mathbf{t^1}$, ideal is clicked item (triangular max-min shape remains).

In time 1 user clicks on second item – this becomes ideal in time 2.

Describe order in time 2. Use copy of DC, PC in pptx.



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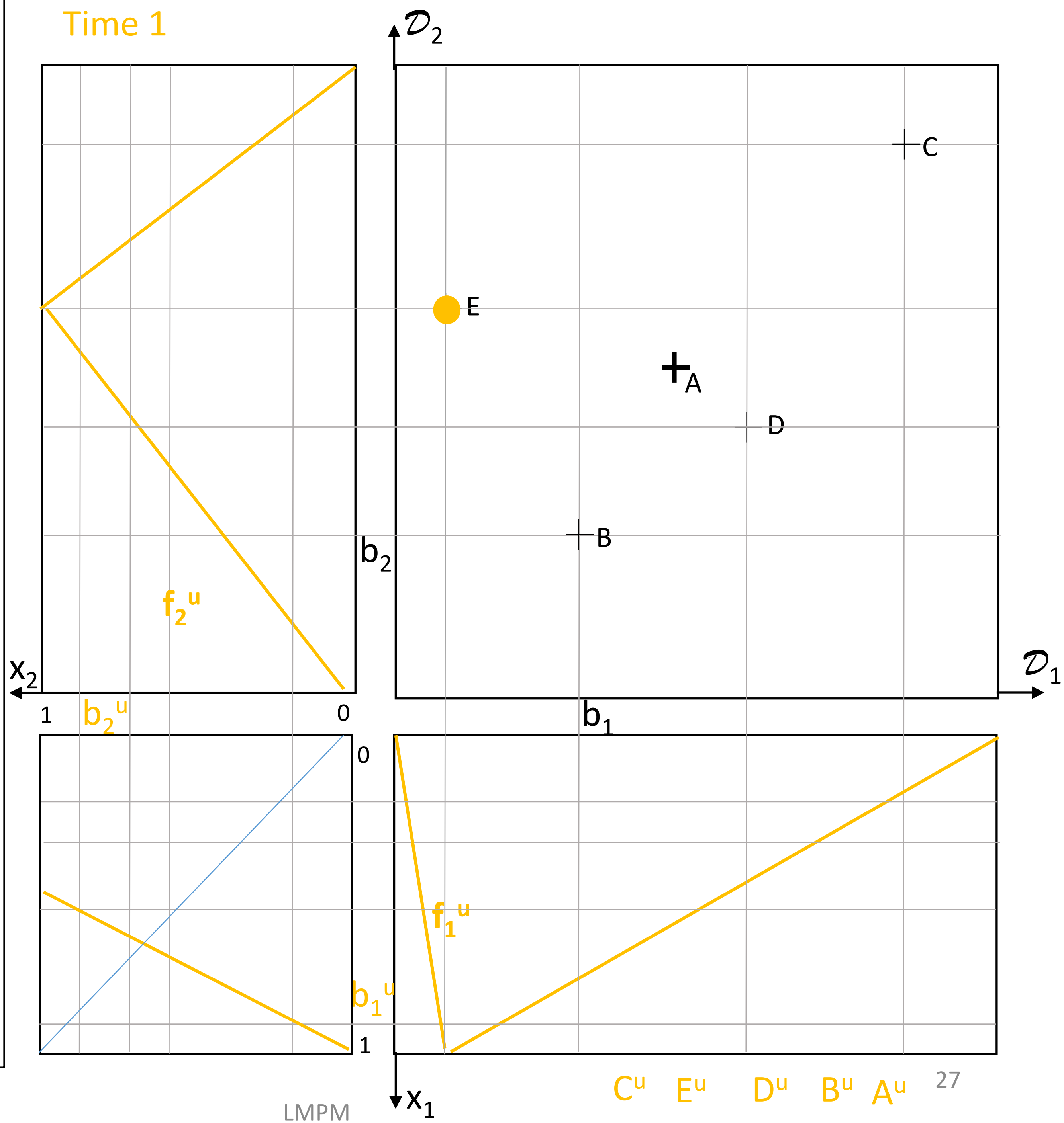

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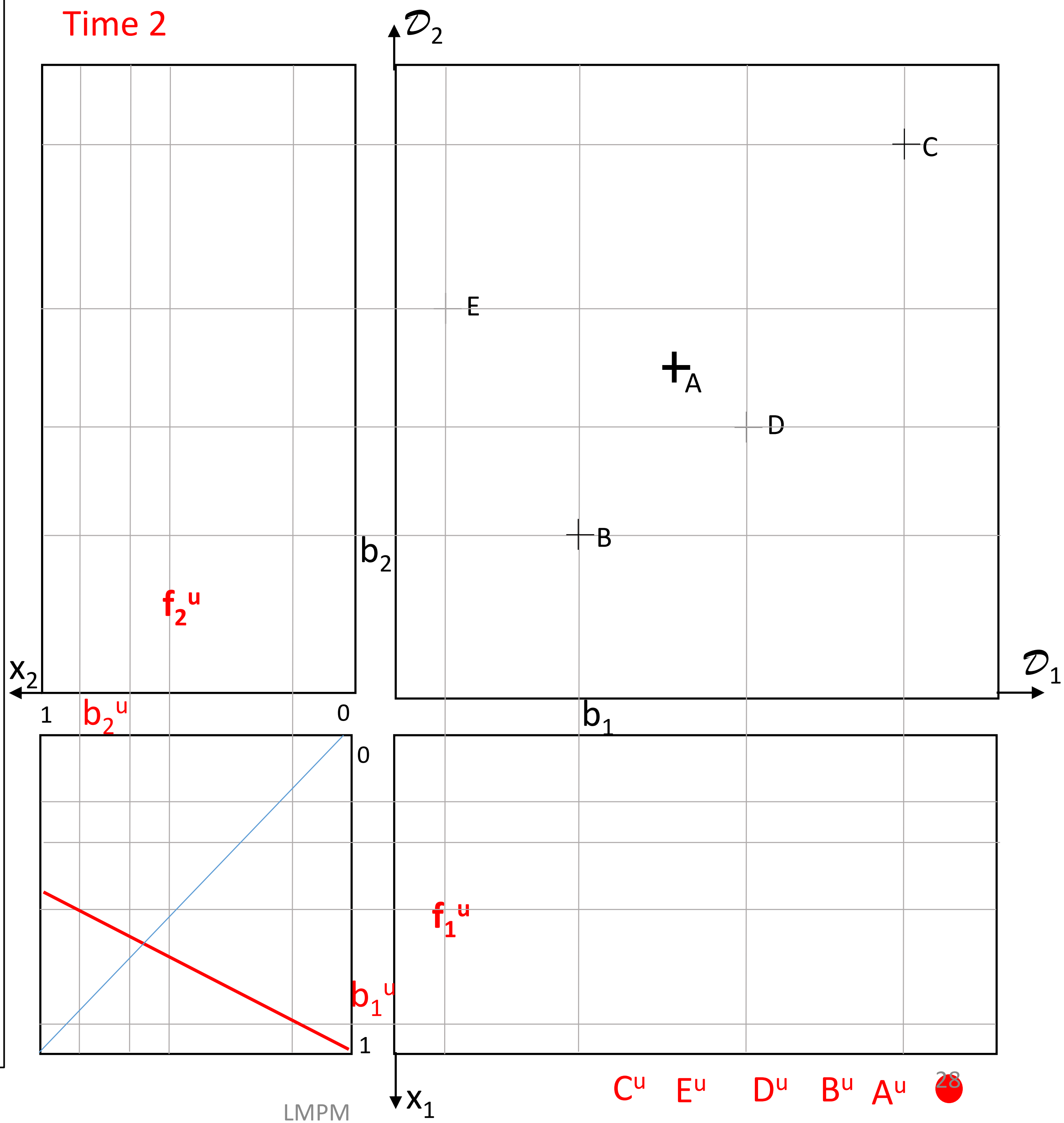

Dynamical model – three sessions – moving ideal points and **moving aggregation**

Simulation of development in time

Starting vector of attribute preferences $\mathbf{f^0}$ and aggregation $\mathbf{t^0}$ define an user $\mathbf{u^0_{f,t}} = \mathbf{u^0}$ in time 0. Depict contour line in DC-data cube.

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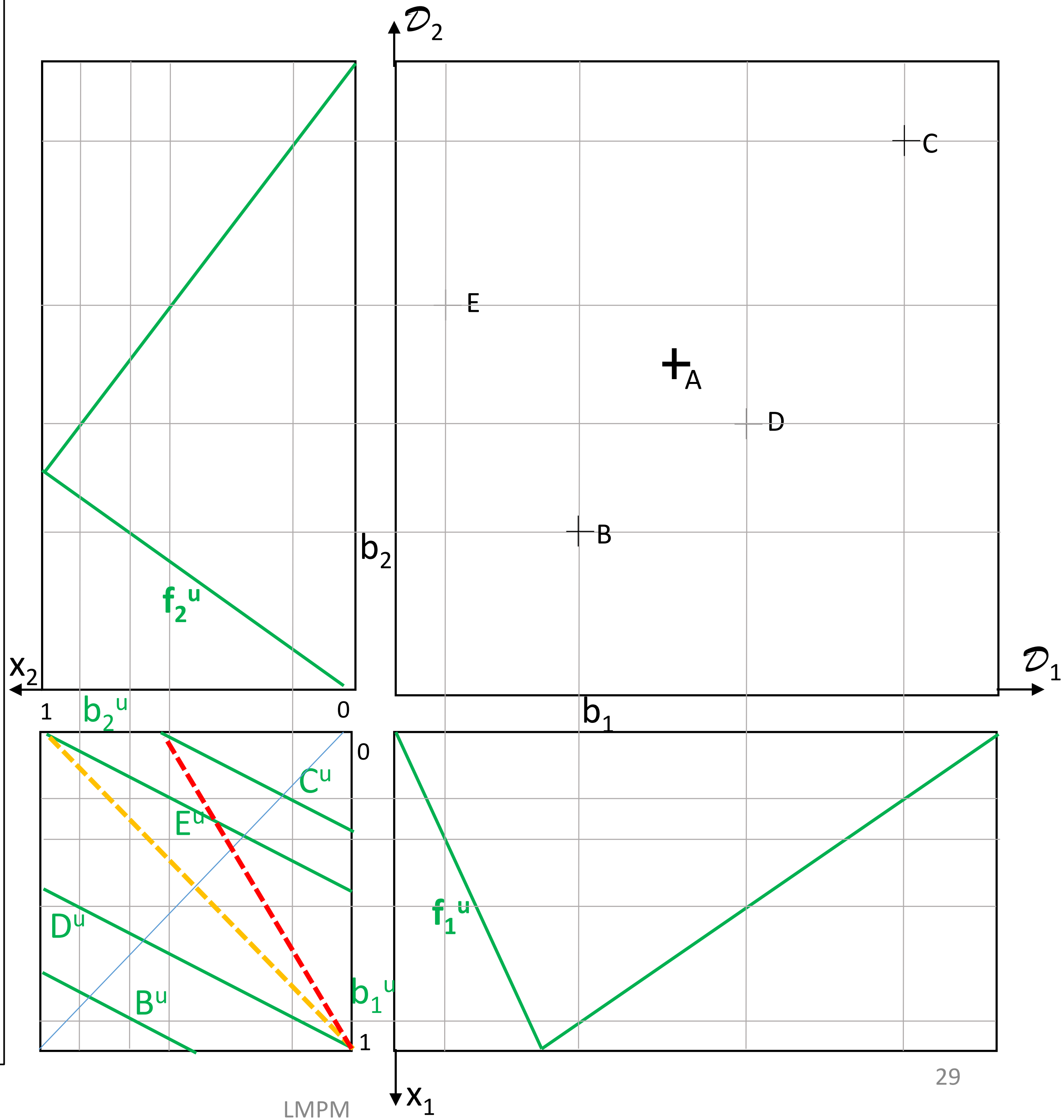


Preference model of user $u_{f,t}$ on data cube

Function $R^{f,t}: \Pi D_i \rightarrow [0,1]$

$R^{f,t}(a_1, \ldots, a_m) = t\,([f_i\,(a_i) : i = 1, \ldots, m])$

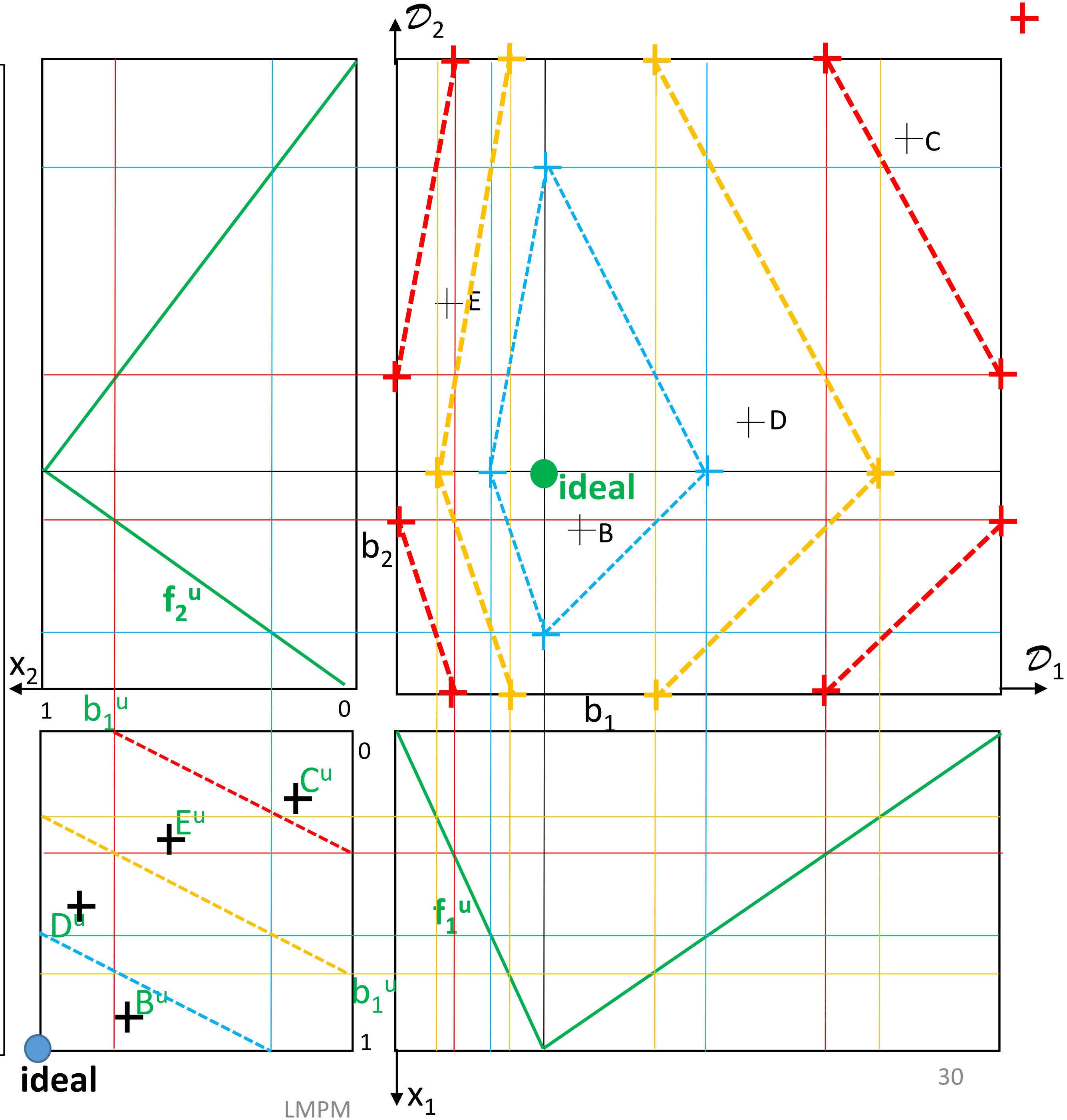
Ordering on data cube$(a_1, \ldots, a_m) \geq^{f,t} (b_1, \ldots, b_m)$ iff $R^{f,t}(a_1, \ldots, a_m) \geq R^{f,t}(b_1, \ldots, b_m)$ Odering can be vizualized as contour lines on ΠD_i

For better understanding are different contour lines (of same t) in colors

User $\mathbf{u_{f,t}}$, preference of user $u_{f,t}$, $R^{f,t}: \Pi D_i \rightarrow [0,1]$

$R^{f,t}(a_1, \ldots, a_m) = t\,([f_i\,(a_i) : i = 1, \ldots, m])$

(a) $\geq^{f,t}$ ($\underline{b}$) iff $R^{f,t}(\underline{a}) \geq R^{f,t}(\underline{b})$



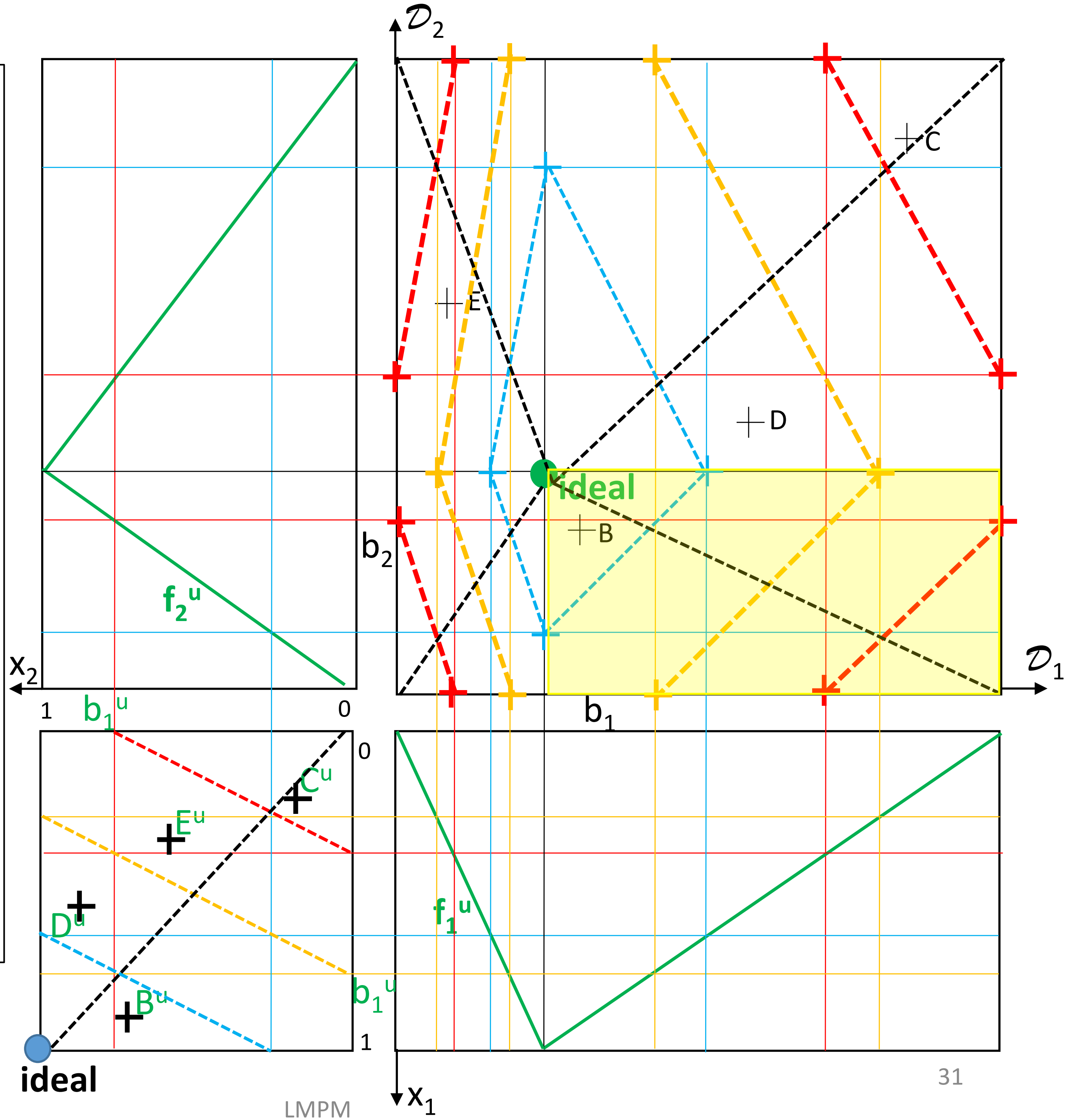
Let us insert into previous slide the preference cube diagonal and depict it in DC. Does it preserve preference degree?

We have now mappings from DC→PC and from PC→DC – what are properties?

DC→PC is injective, PC→DC need not

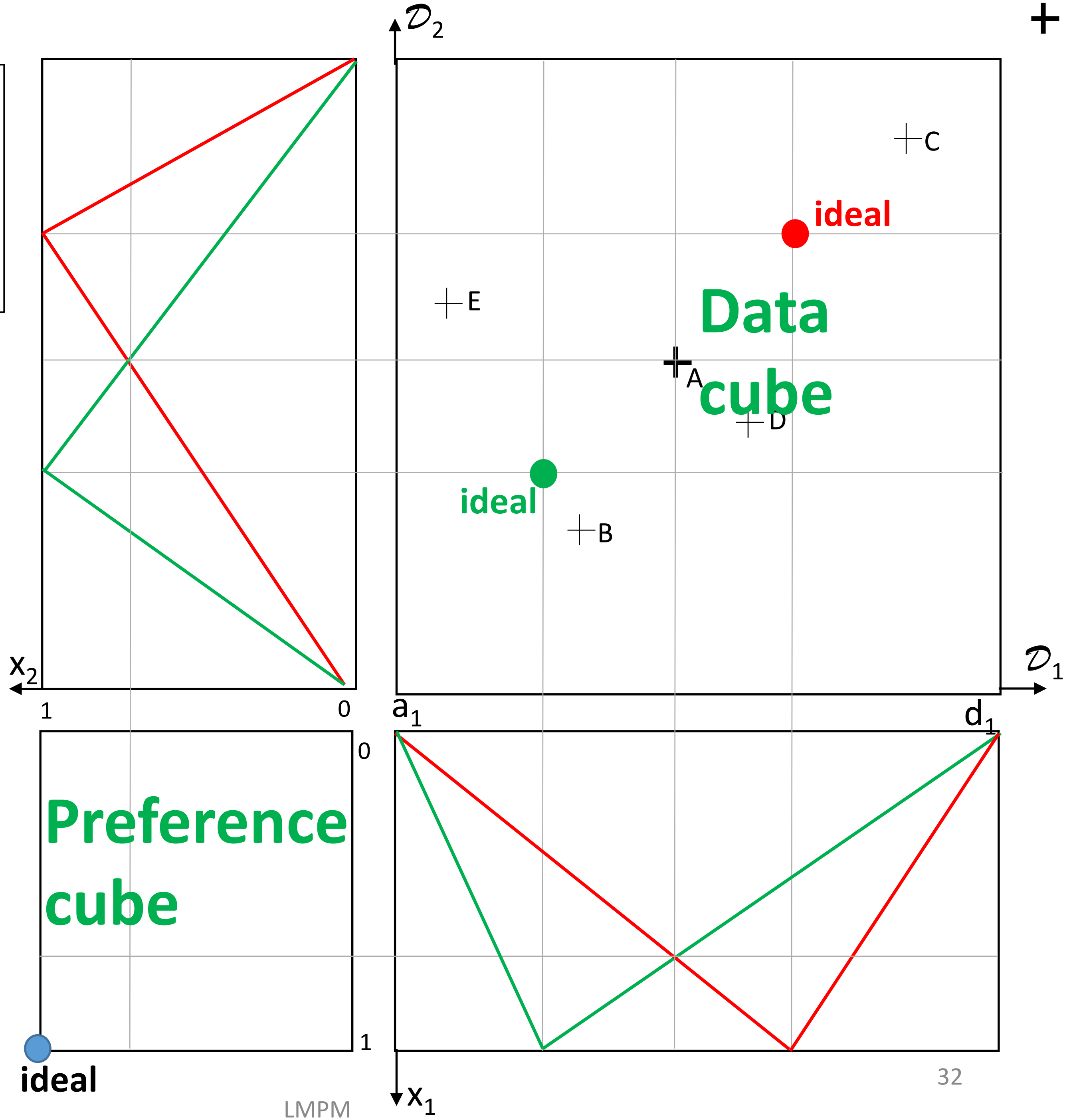
Both mappings preserve line segments (maybe in quadrants , see quadrant in yellow)

Mapping of areas, e.g. quadrilaterals can be more complicated



Two users **u** and **u**

Preference scale L = [0, 1]



Data model: attributes $\mathcal{A}_1, \mathcal{A}_2$; domains $\mathcal{D}_1, \mathcal{D}_2$;

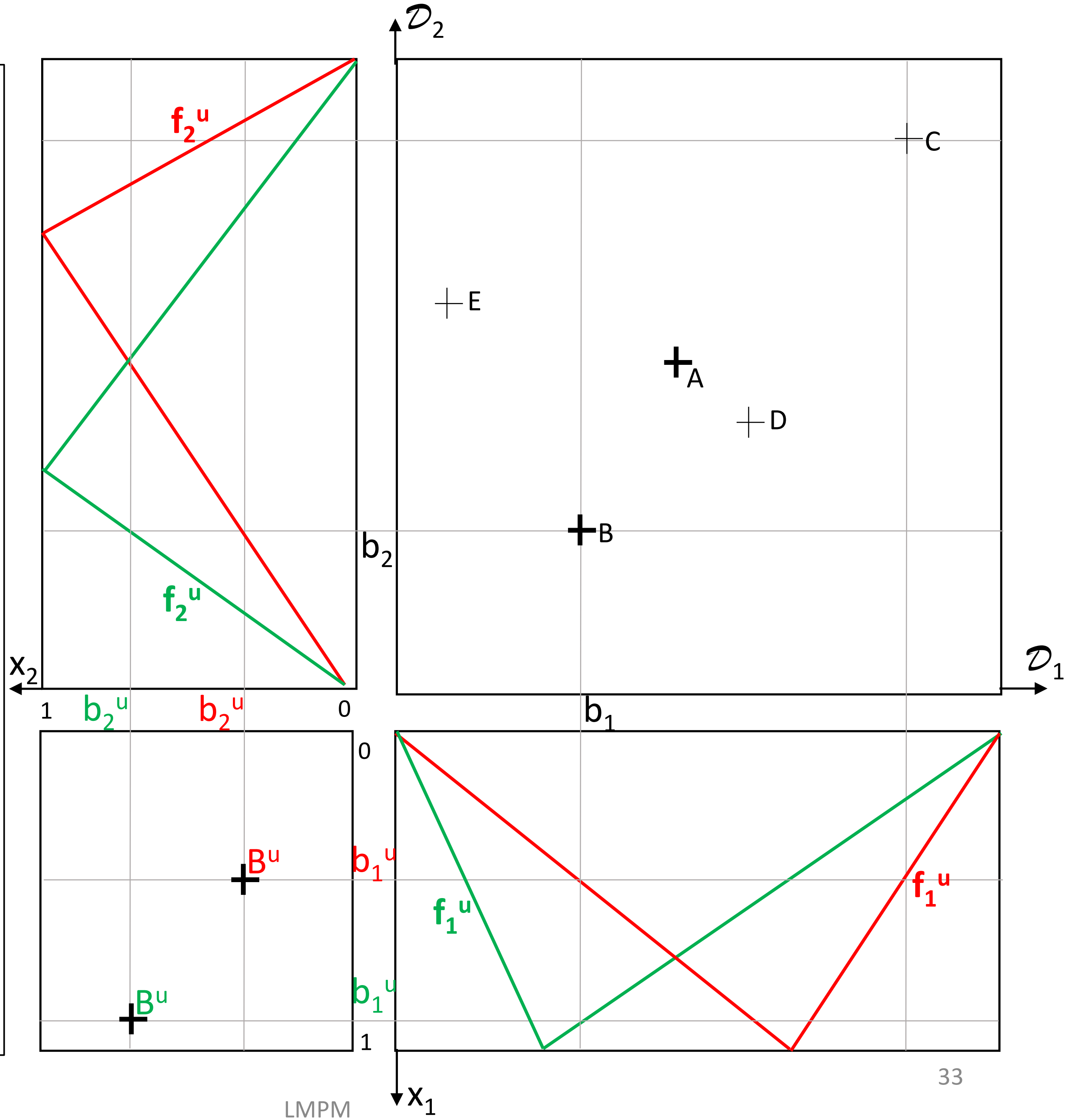
Ideal points can be for each user different, we consider users **u** and **u**.

Degree of preference $f_i^u: \mathcal{D}_i \rightarrow [0, 1]$ (for an user $u \in U$), so we have $\mathbf{f_i^u}$ and $\mathbf{f_i^u}$.

Object with objectID = B has attribute values $B.\mathcal{A}_1 = b_1$ and $B.\mathcal{A}_2 = b_2$, sometimes we write $B=(b_1, b_2)$.

Attribute preference degrees $f_i^u(B.\mathcal{A}_i) = b_i^u$ and corresponsing point in preference cube is $B^u = (b_1^u, b_2^u)$

Find both images of C, …



Data model: attributes $\mathcal{A}_1, \mathcal{A}_2$; domains $\mathcal{D}_1, \mathcal{D}_2$;

Ideal points can be for each user different, we consider users **u** and **u**. Both have same aggregation average AVG

As before we have $f_i^u: \mathcal{D}_i \rightarrow [0, 1]$ (for an user $u \in U$), so we have $\mathbf{f_i^u}$ and $\mathbf{f_i^u}$.

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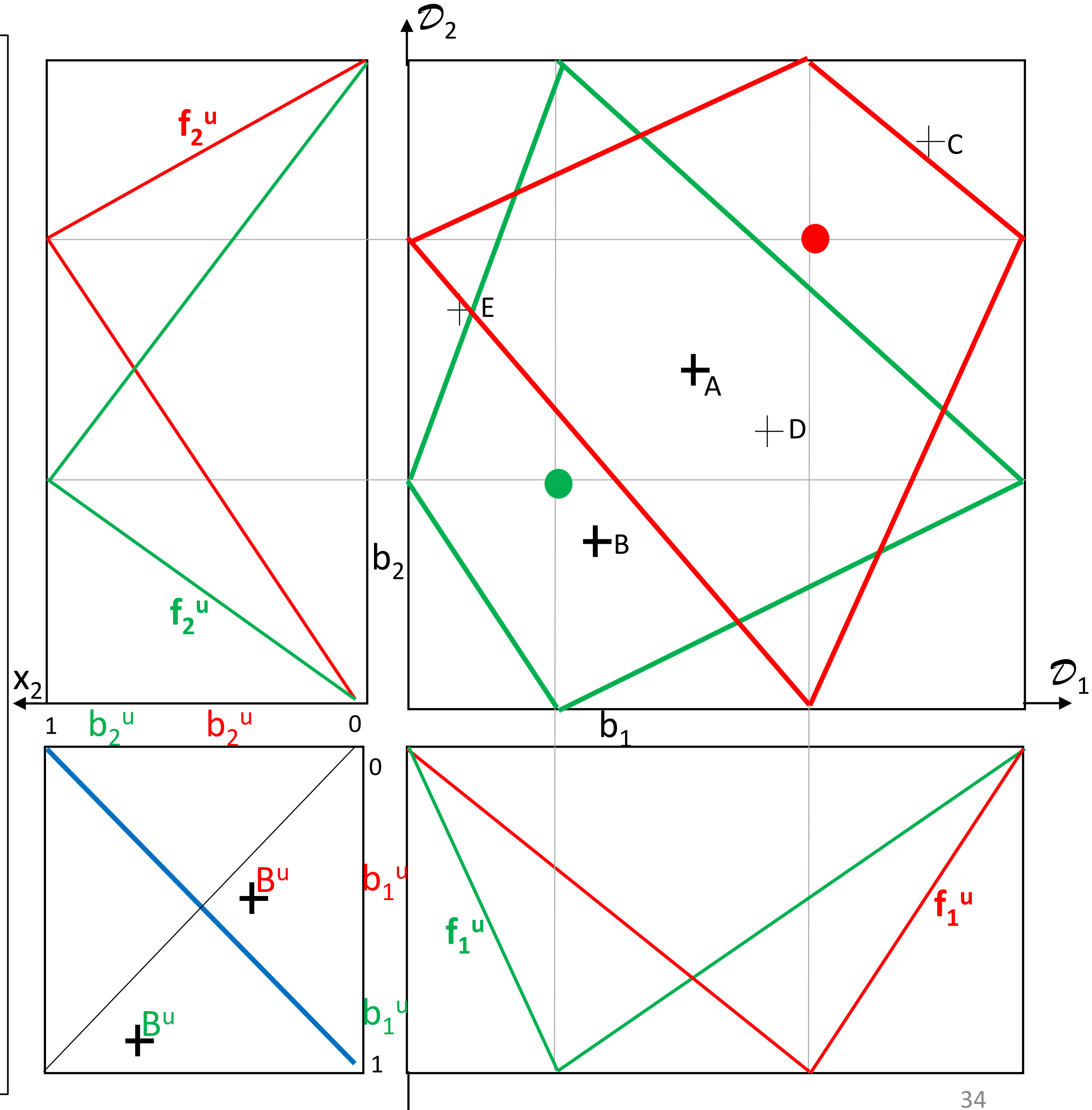
Let us depict ½ contour line in DC, interpret result, discuss intuitiveness



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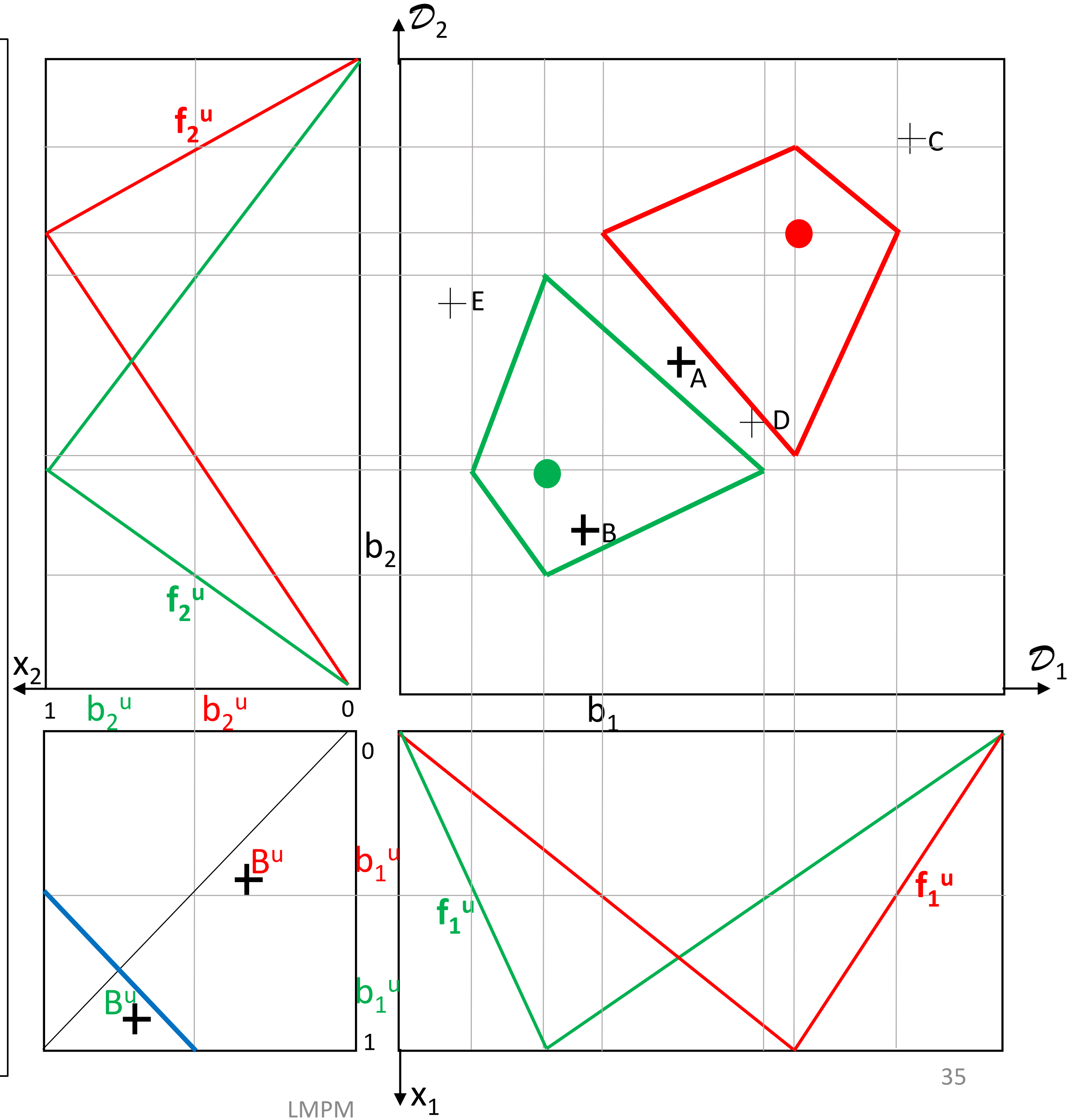
Let us depict ¾ contour line in DC, interpret result, discuss intuitiveness



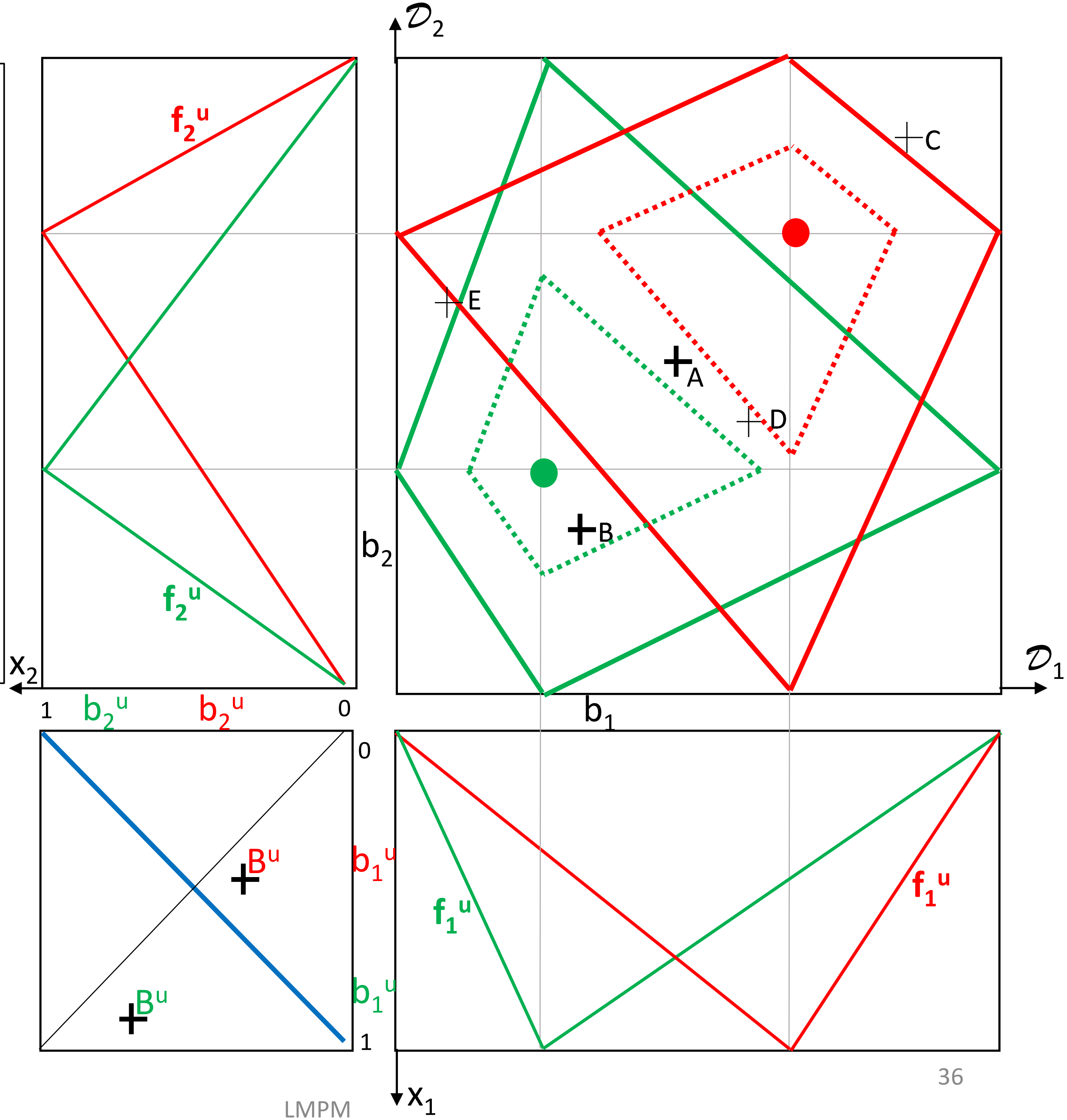
Previous two slides in one.

Observe ½ and ¾ contour lines in DC.

It seems that there is some parallelism.

Formulate statement, prove or disprove.

Interpret result, discuss intuitiveness



Discuss all possible combinations for two users:

Different attribute preferences same aggregation

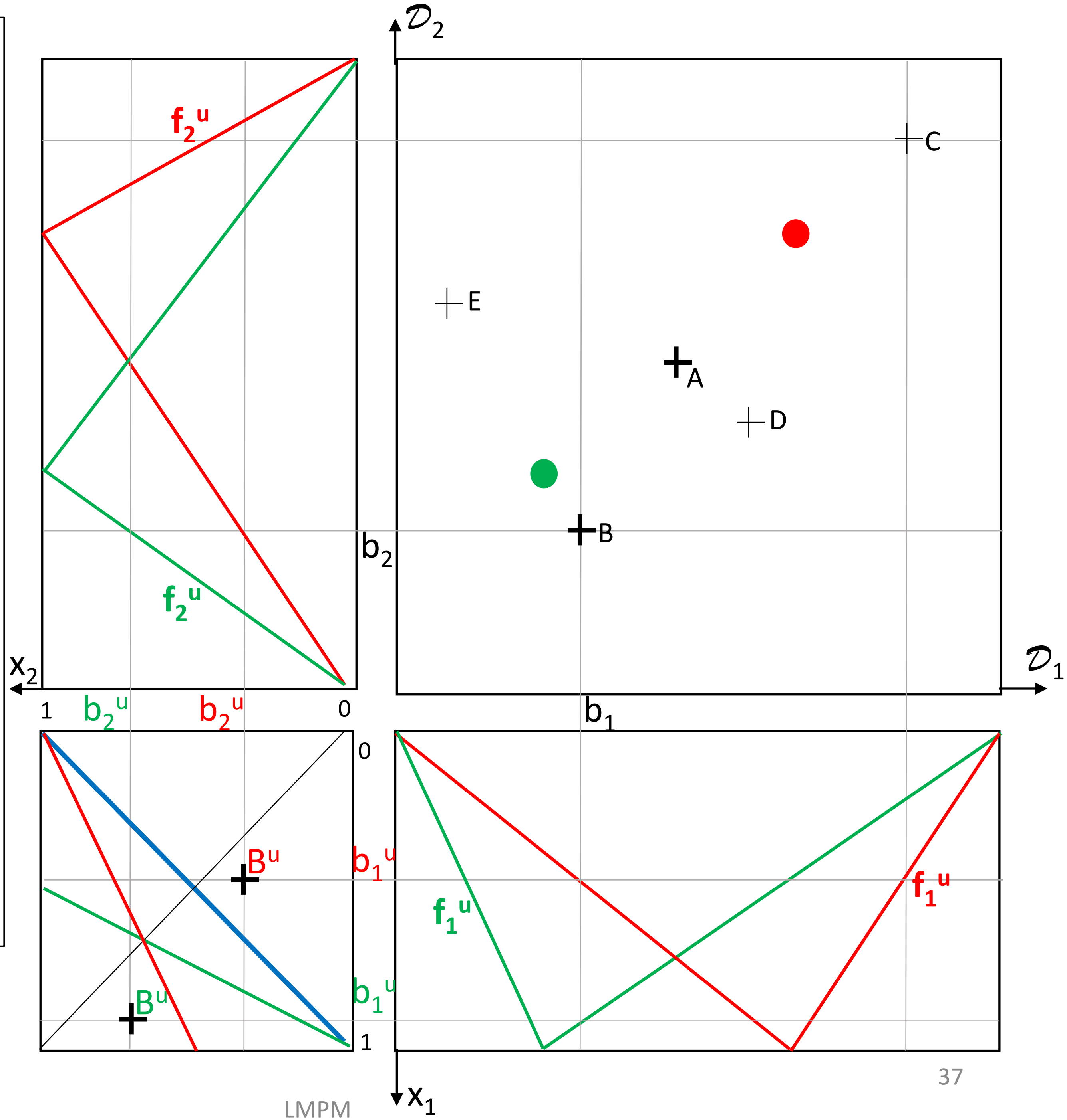
Same attribute preferences and different aggregations

All is different, both attribute preferences and aggregations

?All possible shapes of f_j^u

Interpret result, discuss intuitiveness

Some comments on market segmentation

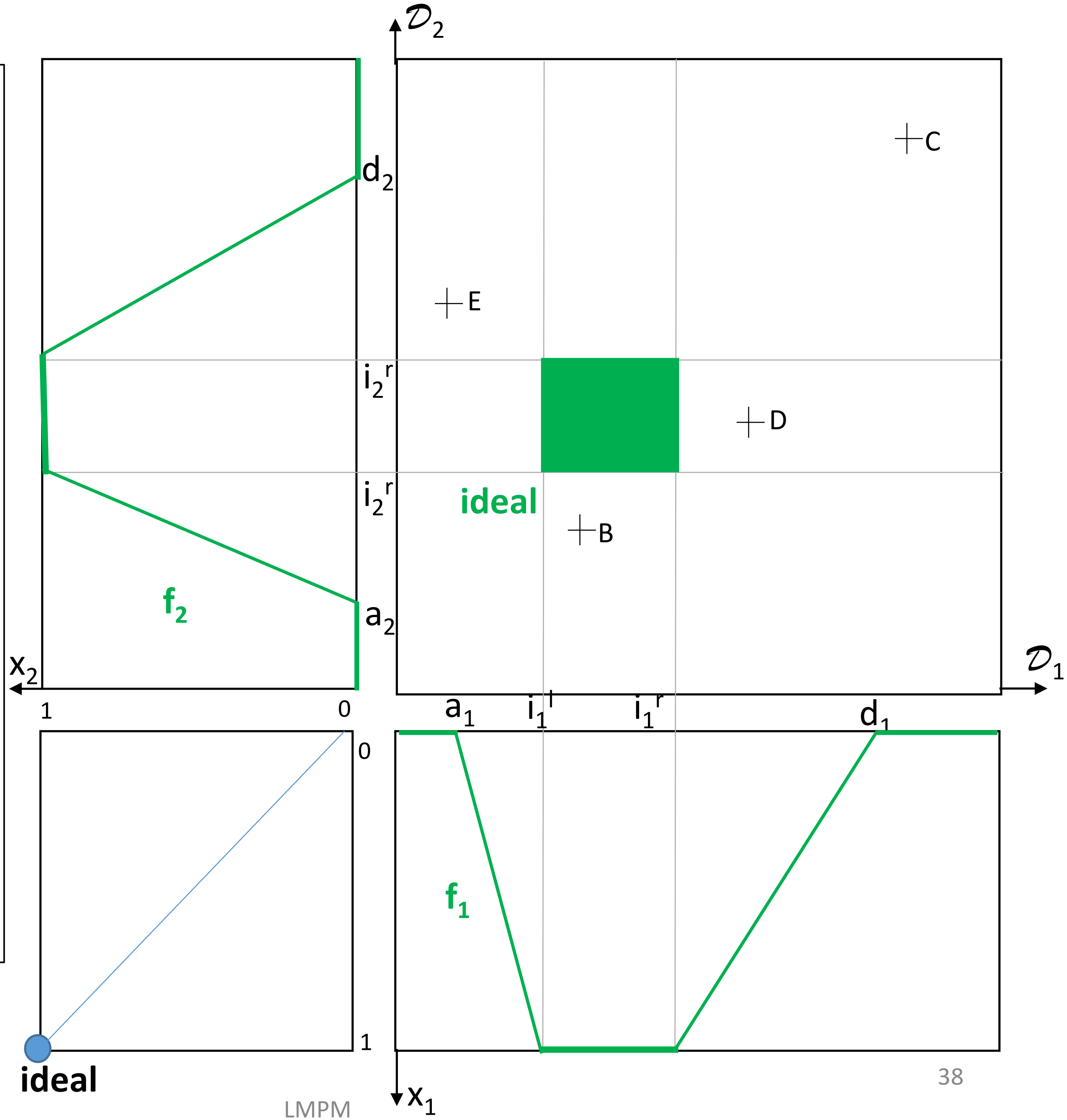


Similarly

trapezoidal degree of preference of $\mathcal{A}_j$, a value from $\mathcal{D}_j$ (local preference) is given by an ideal interval $[i_j^l, i_j^r]$ and analogically defined functions f_j

Instead of minimum/max of domains $\mathcal{D}_j$ we can/have to consider the possibility that trapezoid is based on some interval $[a_j, d_j]$

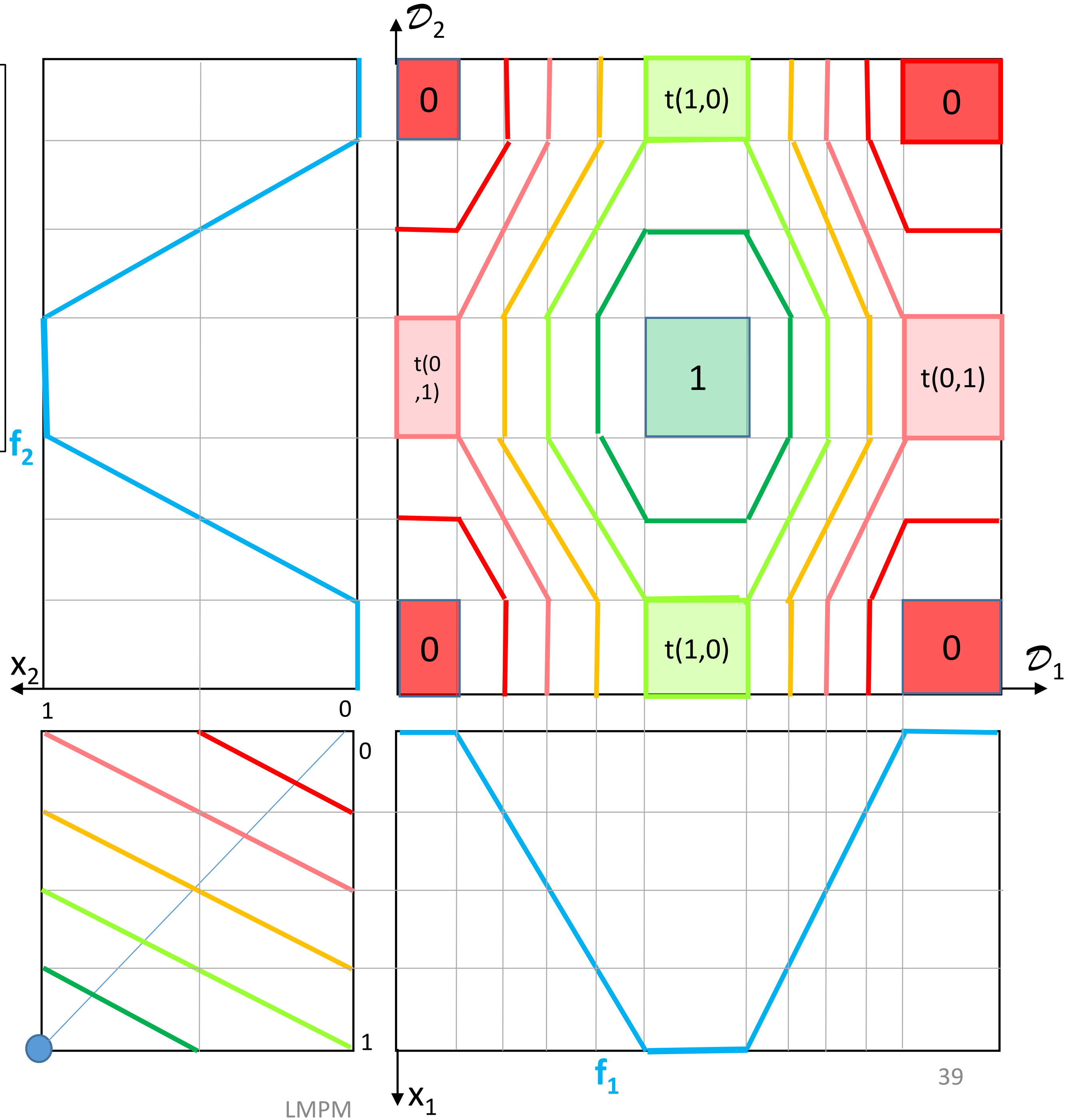
Depicting contour lines continues on blackboard



Contour lines for general trapezoidal case

Please, notice construction

Image ungrouped for further constructions

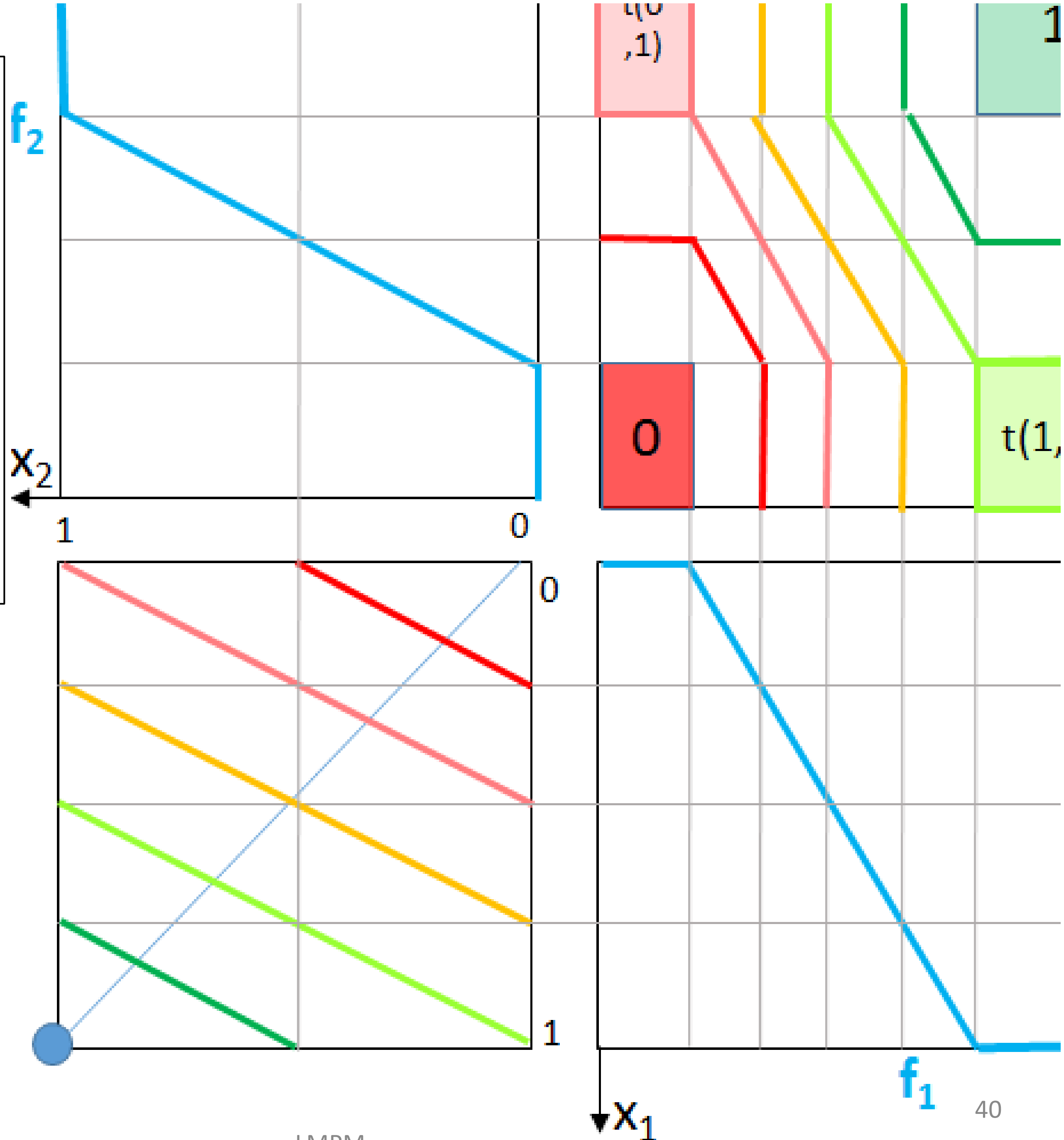


Four corners versus one corner - Makes your solutions faster

Illustration for “one quarter” construction

Saves time

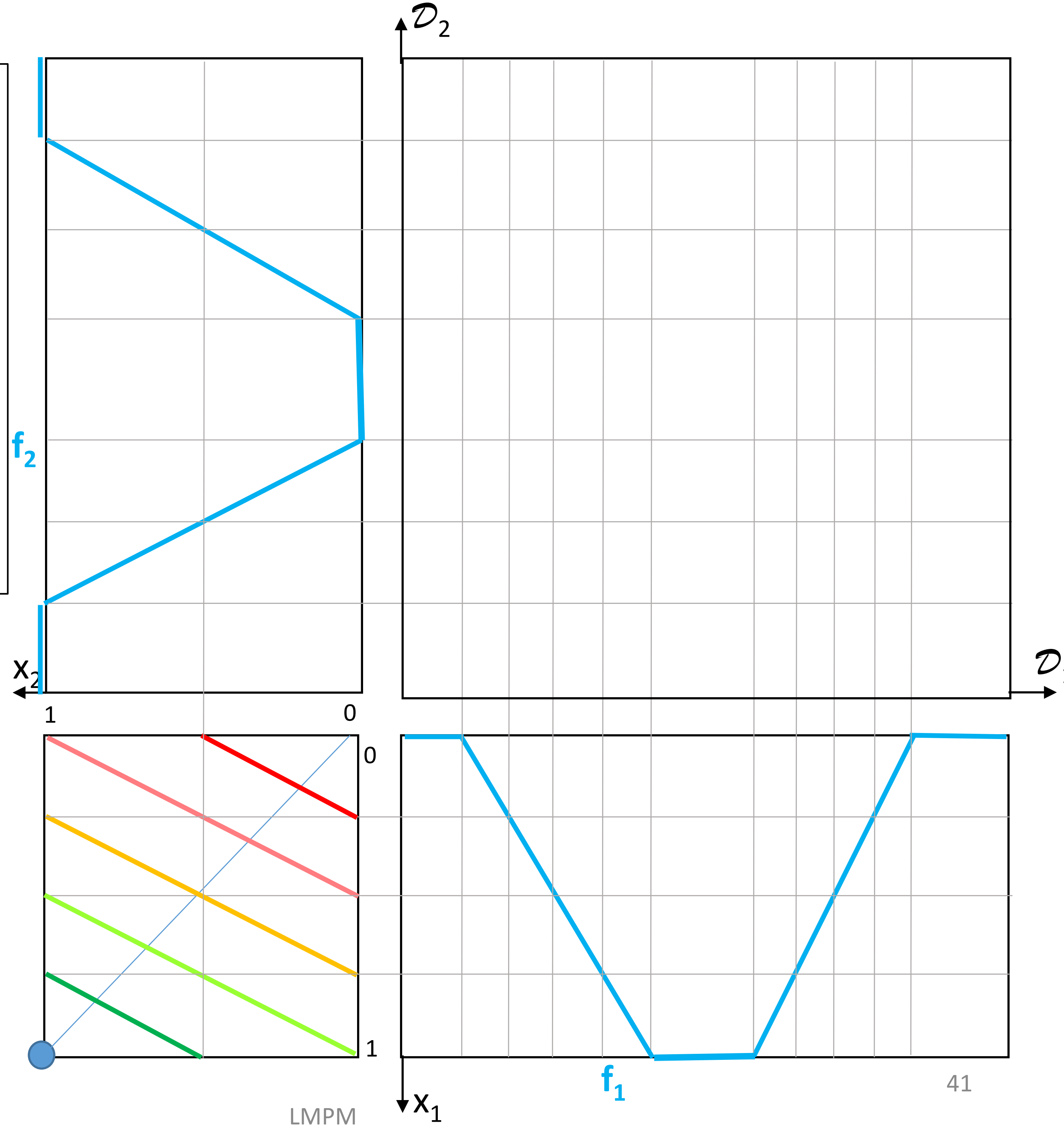
Only illustration, must be constructed



Contour lines for general trapezoidal case

Consider different combination of “hill” “valley” shaped attribute preferences

Illustration for “one quarter” construction …

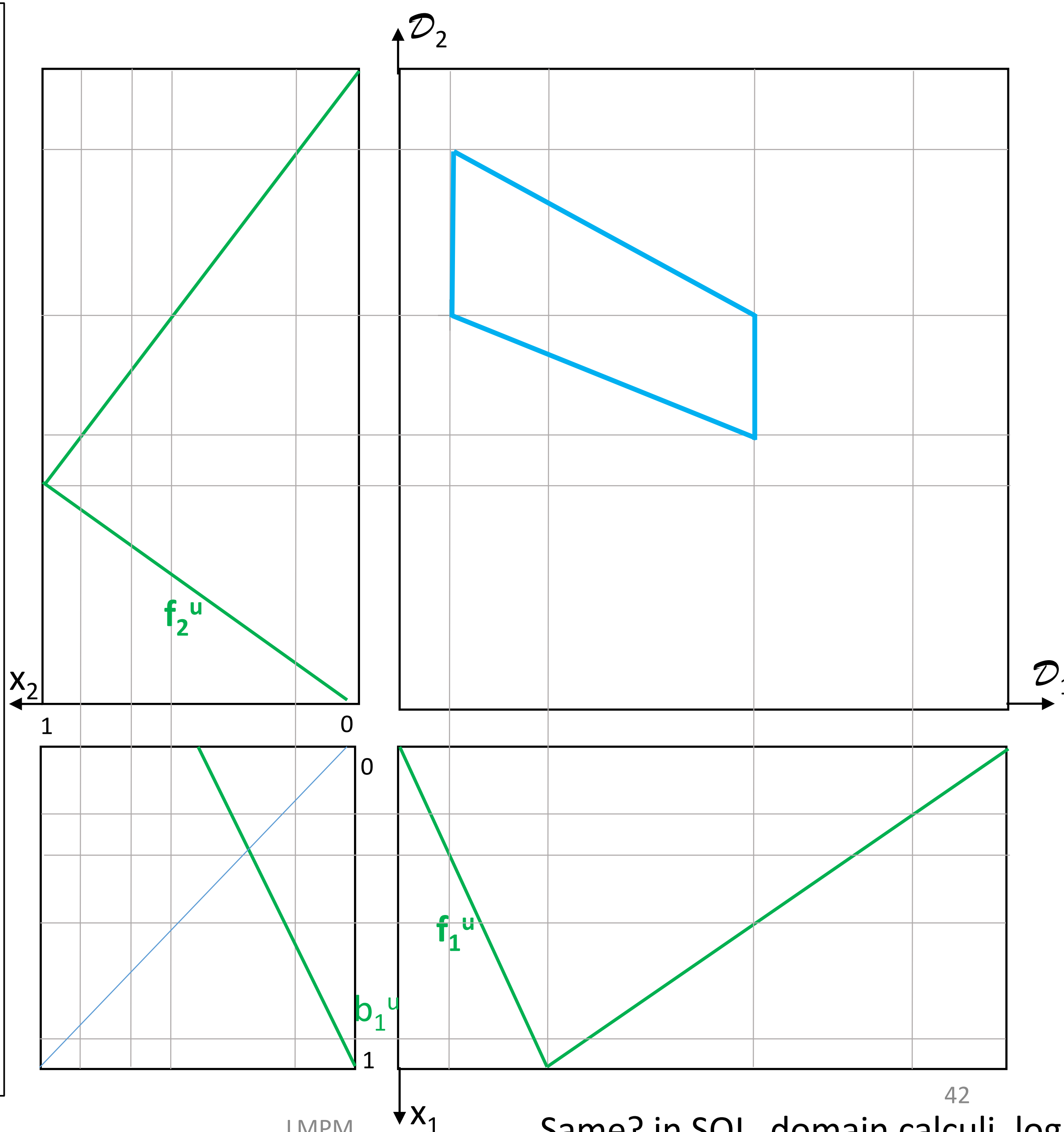


Possible task: assume we know the convex hull of data (as depicted in DC), find the best object, calculate it's preference degree x, find objects with preference degree 0.9*x

Discuss all possible solution strategies, which is/can be most intuitive for an untrained user?

Consider variants of this task, e.g. with trapezoidal attribute preferences; with ideal point in max/min of domains,

Consider $\mathbf{f_i^u}$ and t variable, formulate tasks …



Same? in SQL, domain calculi, logic

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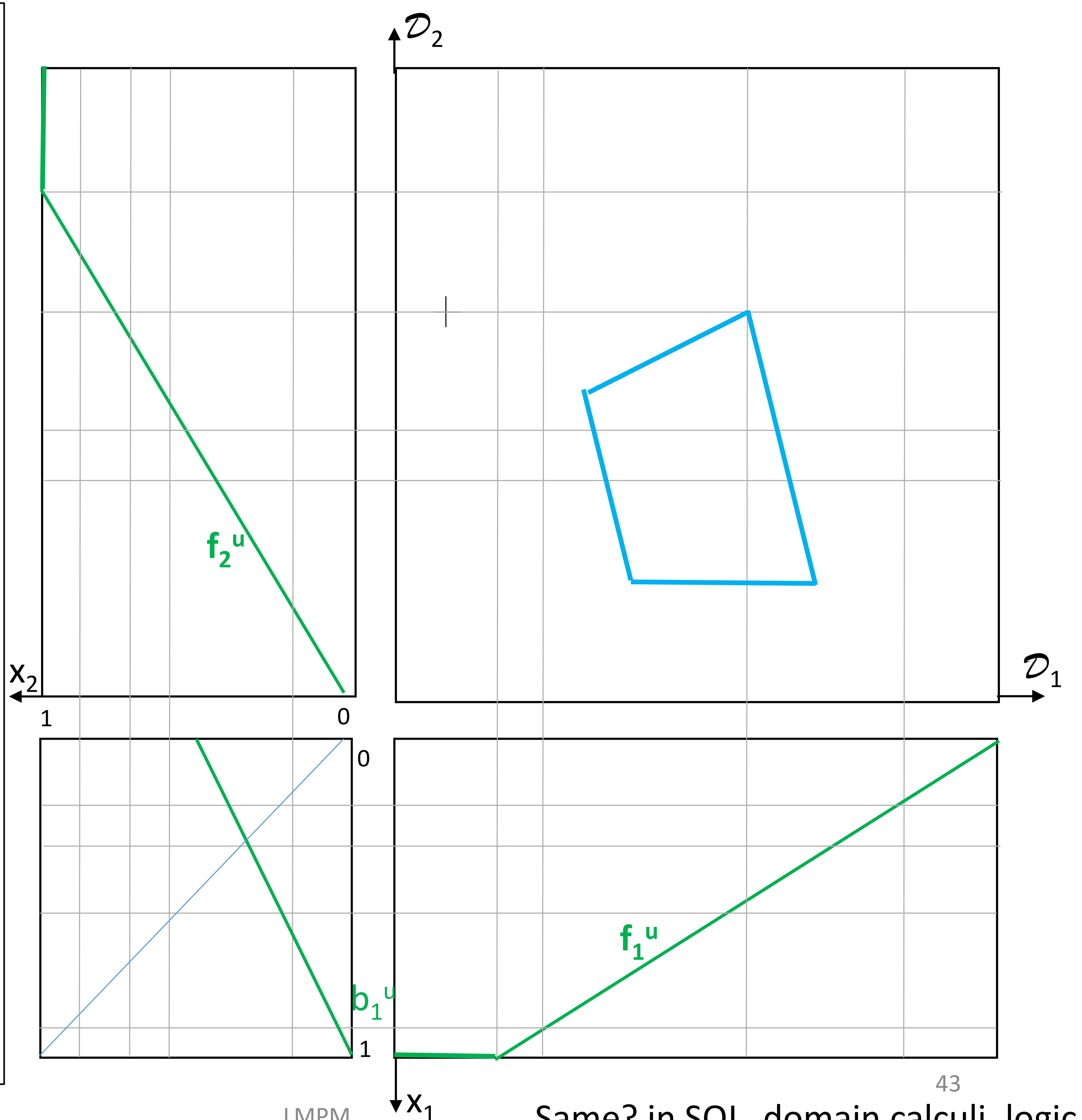

Same? in SQL, domain calculi, logic

Possible task: assume we know the convex hull of data each product group separately (as depicted in DC), find the best object, calculate it's preference degree x, find objects with preference degree 0.9*x

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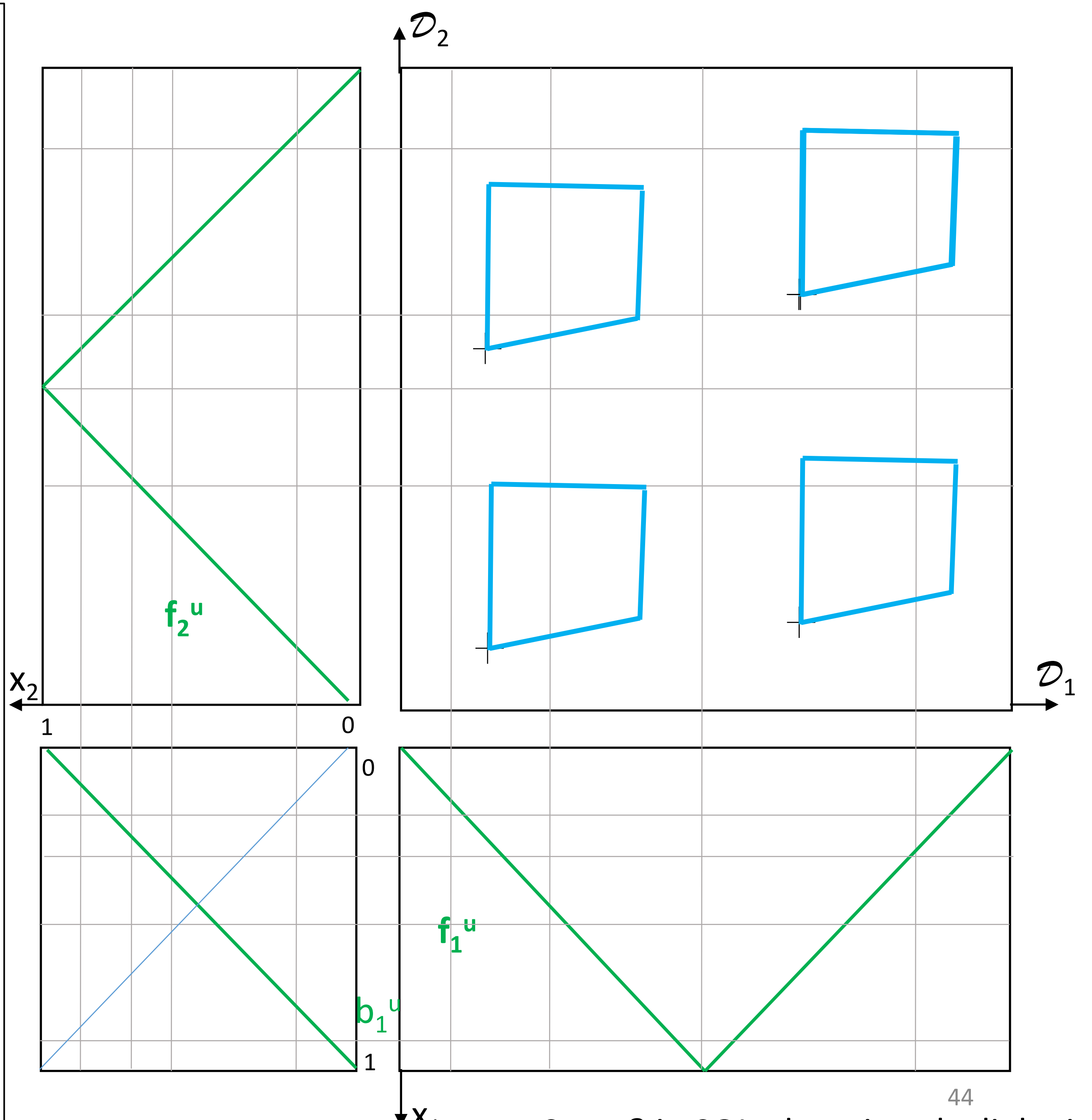

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How does it work together?

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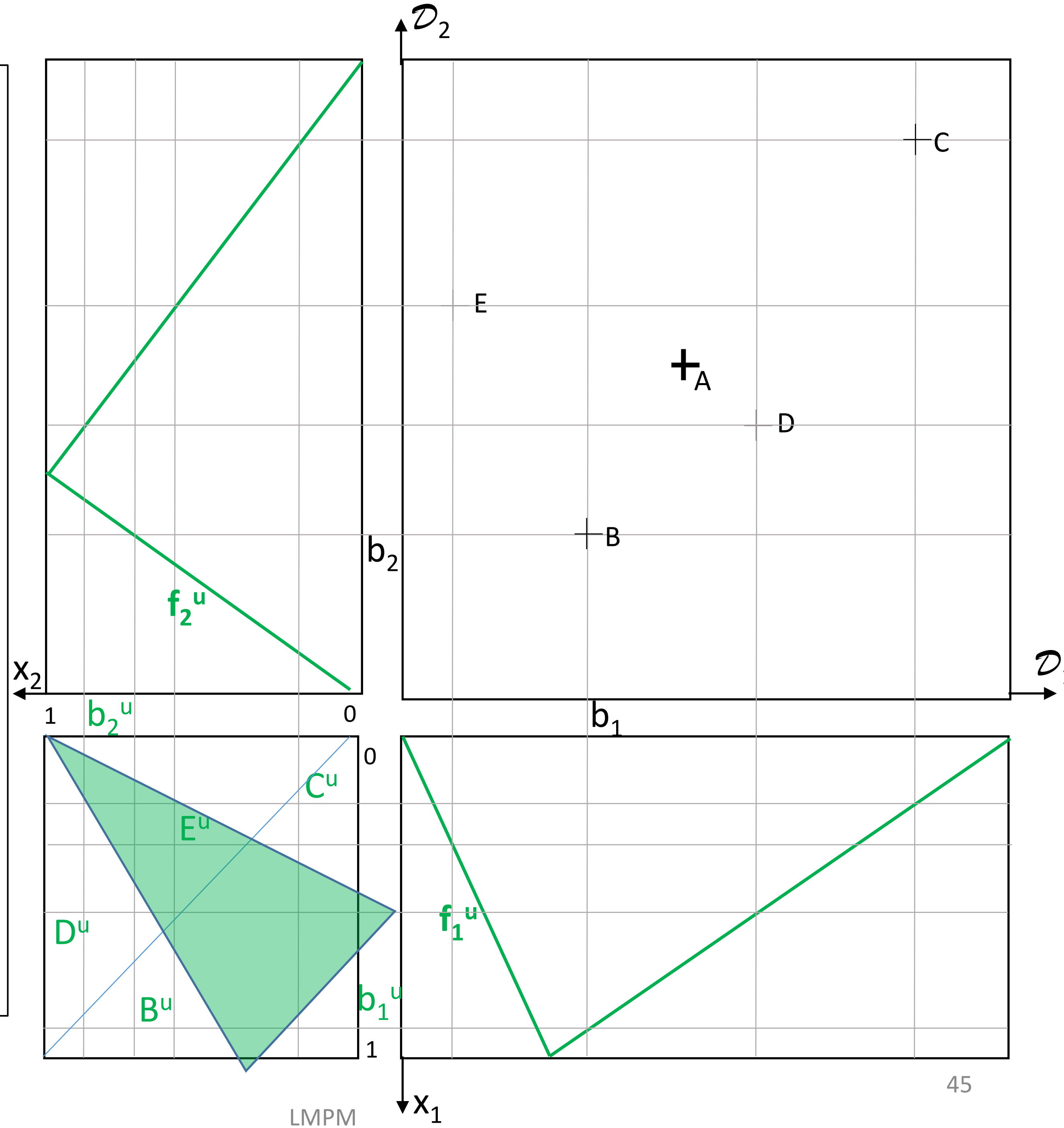

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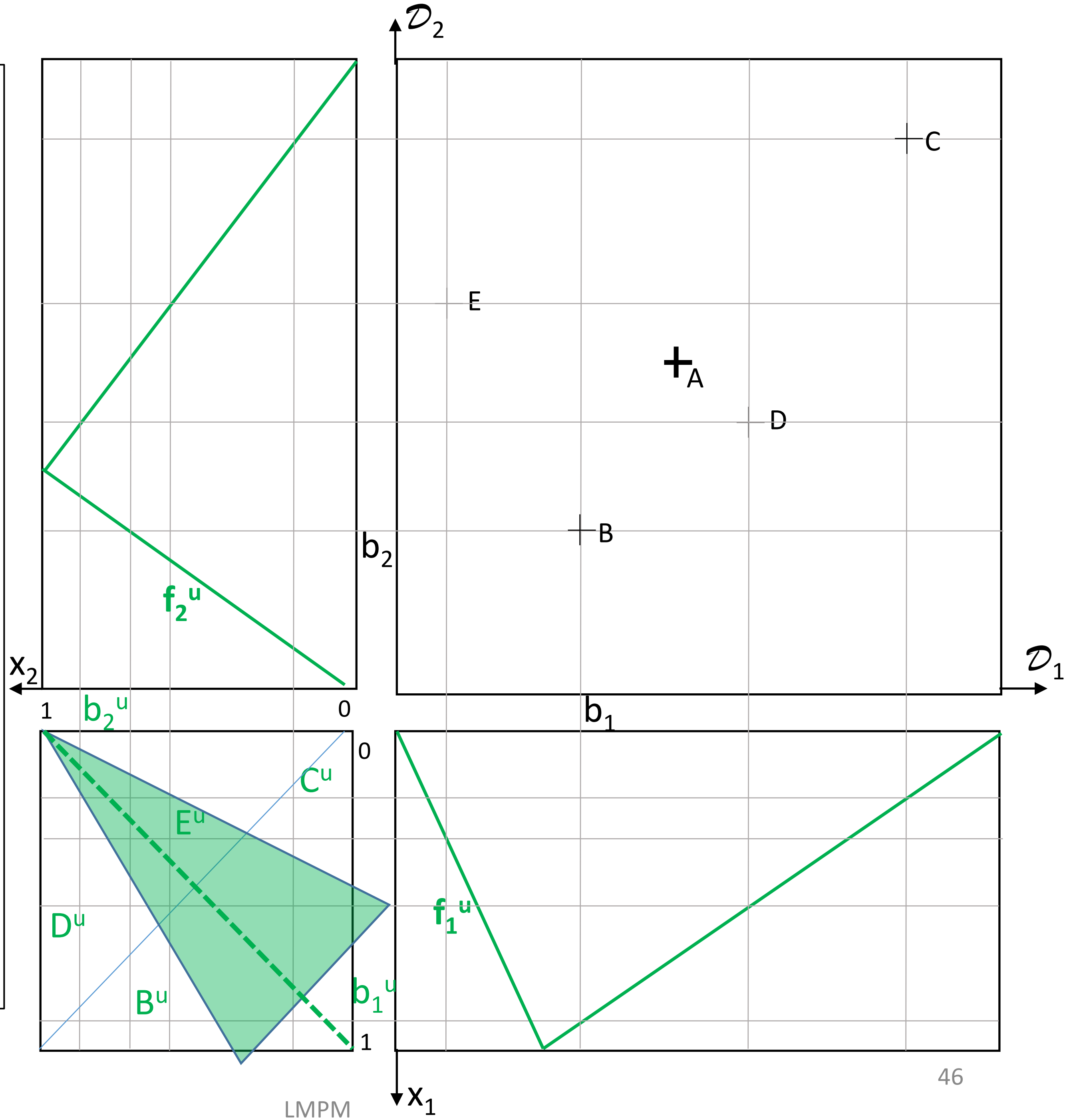

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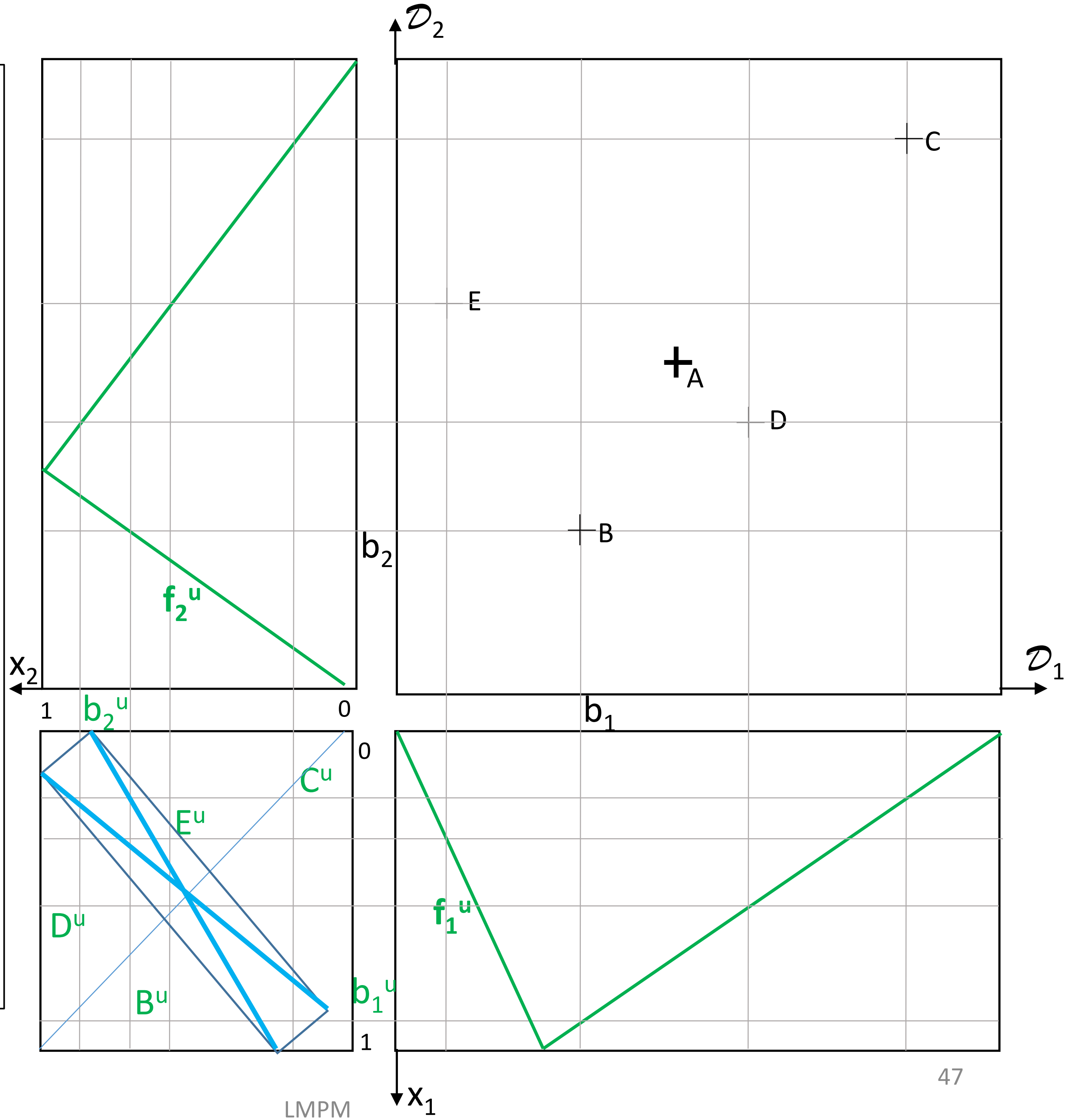

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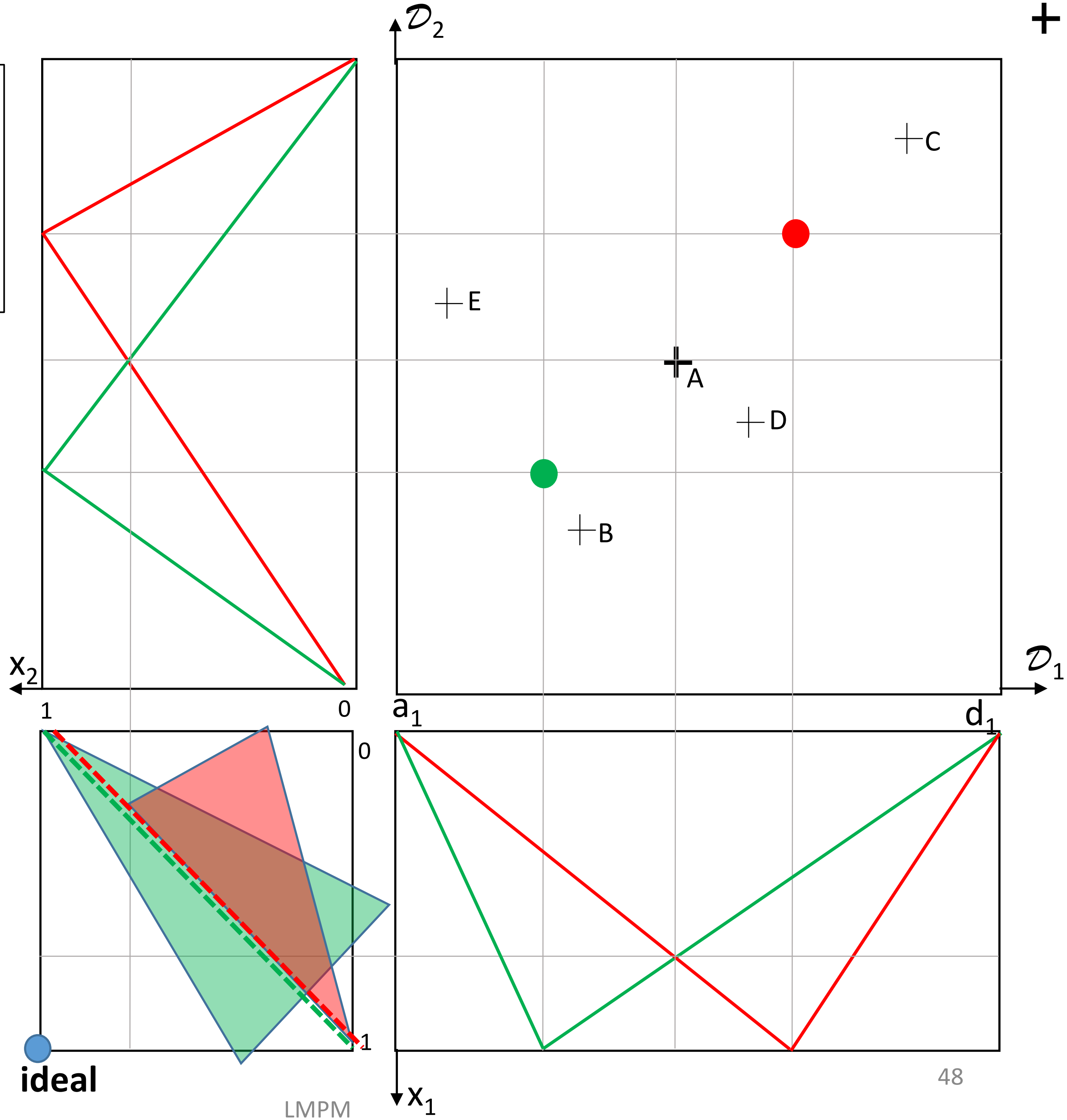


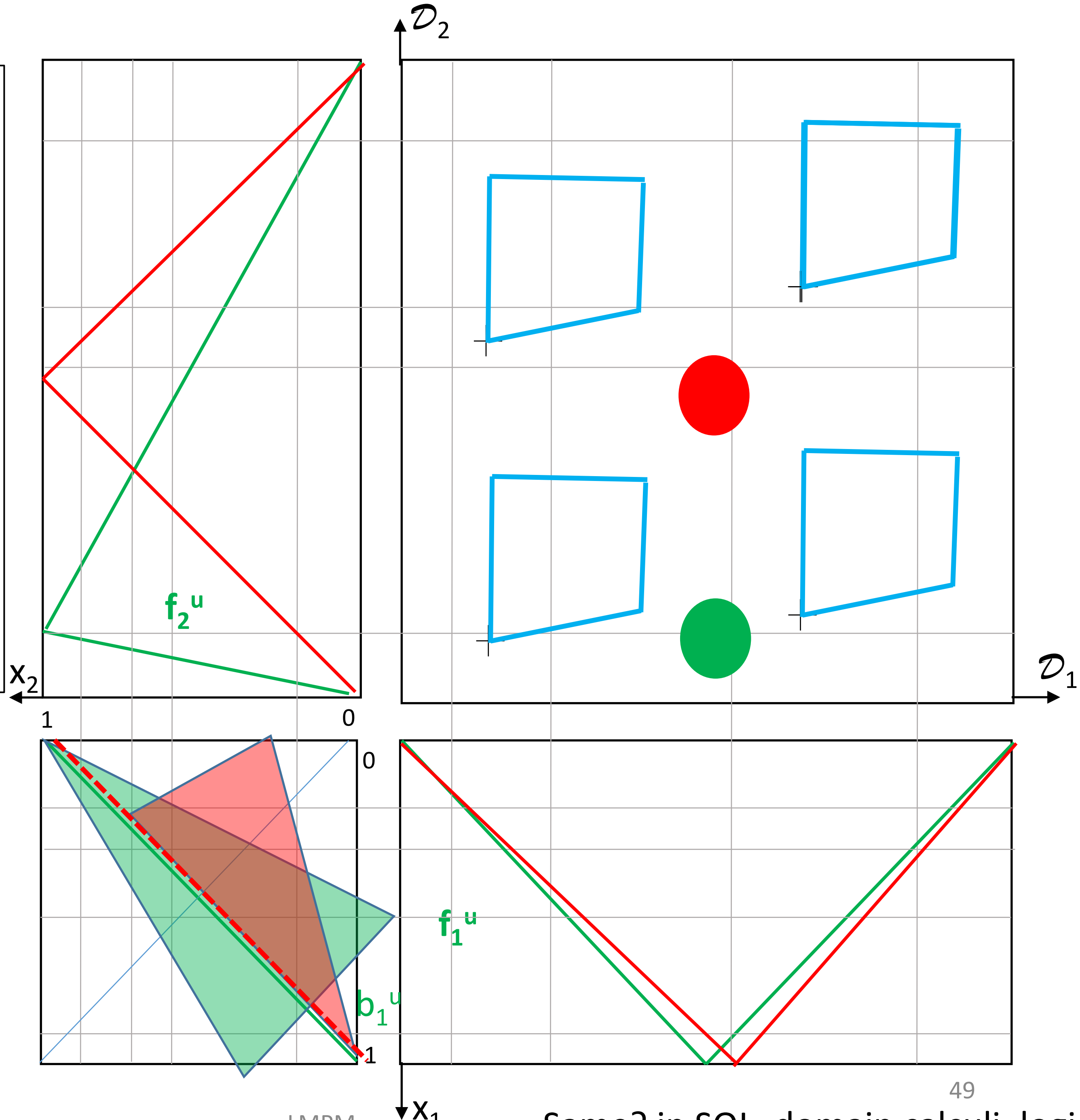
Possible task:

assume a cluster of item segments

Assume a cluster of possible ideal points of users

Assume a cluster of their aggregations

Discuss the situation



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Questions?

Comments?