Multiverse Recommendation: N-dimensional Tensor Factorization for Context-aware Collaborative Filtering

Alexandros Karatzoglou[1] Xavier Amatriain [1] Linas Baltrunas [2]
Nuria Oliver [2]

[1]Telefonica Research
Barcelona, Spain
[2]Free University of Bolzano
Bolzano, Italy

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Context in Recommender Systems

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Current State of the Art in Context- aware Recommendation

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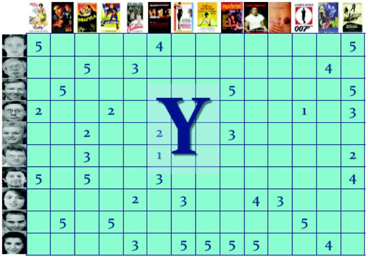
The approach presented here fits in the Contextual Modeling category

Collaborative Filtering problem setting

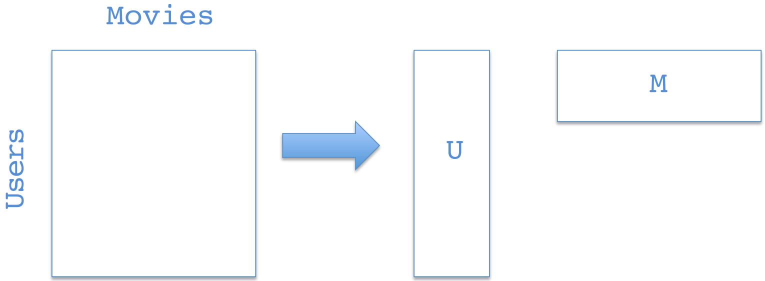


Typically data sizes e.g. Netlix data $n = 5 \times 10^5$, $m = 17 \times 10^3$

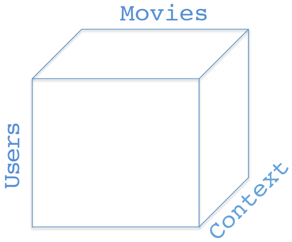
Standard Matrix Factorization

Find $U \in R^{n \times d}$ and $M \in R^{d \times m}$ so that $F = UM$

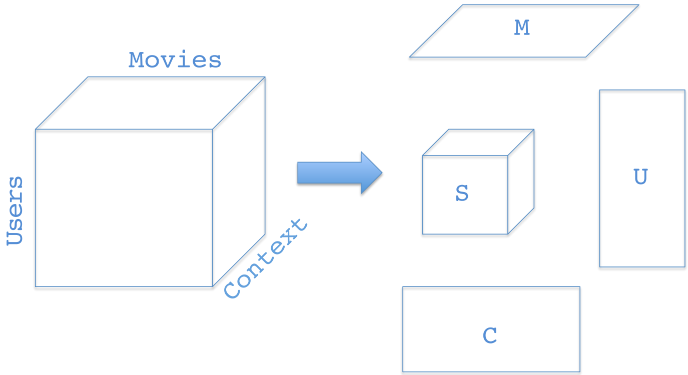
$$\text{minimize}_{U,M} L(F, Y) + \lambda \Omega(U, M)$$



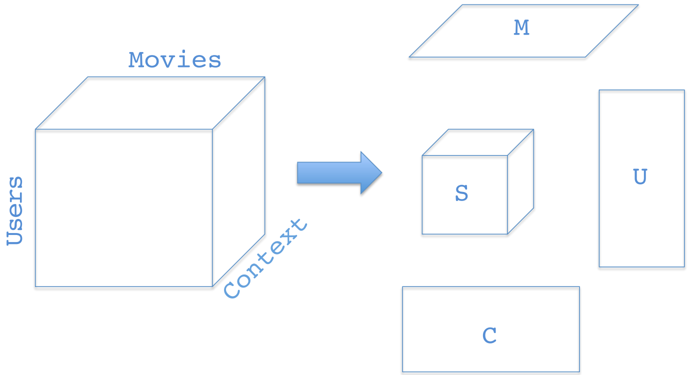
Multiverse Recommendation: Tensors for Context Aware Collaborative Filtering



Tensors for Context Aware Collaborative Filtering



Tensors for Context Aware Collaborative Filtering



$$F_{ijk} = S \times_U U_{i*} \times_M M_{j*} \times_C C_{k*}$$

$$R[U, M, C, S] := L(F, Y) + \Omega[U, M, C] + \Omega[S]$$

Regularization

$$\Omega[F] = \lambda_M \|M\|_F^2 + \lambda_U \|U\|_F^2 + \lambda_C \|C\|_F^2$$

$$\Omega[S] := \lambda_S \, \|S\|_{\mathrm{F}}^2 \tag{1}$$

Squared Error Loss Function

Many implementations of MF used a simple squared error regression loss function

$$l(f, y) = \frac{1}{2}(f - y)^2$$

thus the loss over all users and items is:

$$L(F, Y) = \sum_{i}^{n} \sum_{j}^{m} l(f_{ij}, y_{ij})$$

Note that this loss provides an estimate of the conditional mean

Absolute Error Loss Function

Alternatively one can use the absolute error loss function

$$l(f, y) = |f - y|$$

thus the loss over all users and items is:

$$L(F, Y) = \sum_{i}^{n} \sum_{j}^{m} l(f_{ij}, y_{ij})$$

which provides an estimate of the conditional median

Optimization - Stochastic Gradient Descent for TF

The partial gradients with respect to U, M, C and S can then be written as:

$$\partial_{U_{i*}} l(F_{ijk}, Y_{ijk}) = \partial_{F_{ijk}} l(F_{ijk}, Y_{ijk}) S \times_M M_{j*} \times_C C_{k*}$$
$$\partial_{M_{j*}} l(F_{ijk}, Y_{ijk}) = \partial_{F_{ijk}} l(F_{ijk}, Y_{ijk}) S \times_U U_{i*} \times_C C_{k*}$$
$$\partial_{C_{k*}} l(F_{ijk}, Y_{ijk}) = \partial_{F_{ijk}} l(F_{ijk}, Y_{ijk}) S \times_U U_{i*} \times_M M_{j*}$$
$$\partial_S l(F_{ijk}, Y_{ijk}) = \partial_{F_{ijk}} l(F_{ijk}, Y_{ijk}) U_{i*} \otimes M_{j*} \otimes C_{k*}$$

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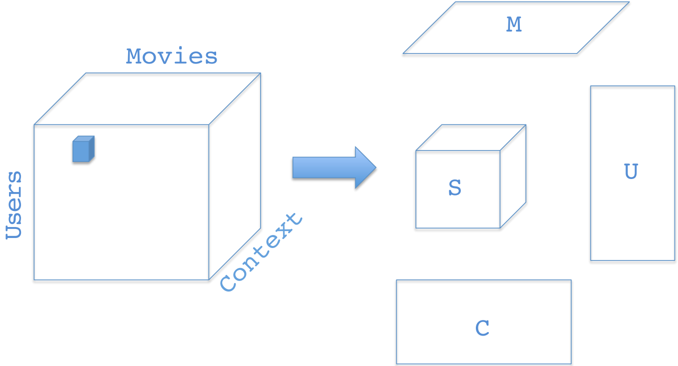
We then iteratively update the parameter matrices and tensors using the following update rules:

$$U_{i*}^{t+1} = U_{i*}^{t} - \eta \partial_U L - \eta \lambda_U U_{i*}$$
$$M_{j*}^{t+1} = M_{j*}^{t} - \eta \partial_M L - \eta \lambda_M M_{j*}$$
$$C_{k*}^{t+1} = C_{k*}^{t} - \eta \partial_C L - \eta \lambda_C C_{k*}$$
$$S^{t+1} = S^{t} - \eta \partial_S l(F_{ijk}, Y_{ijk}) - \eta \lambda_S S$$

where η is the learning rate.

Optimization - Stochastic Gradient Descent for TF



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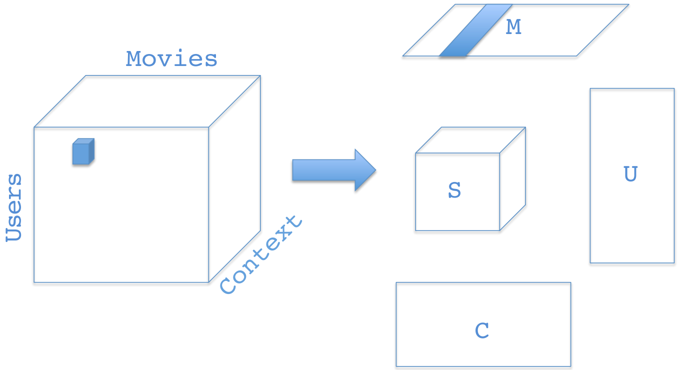

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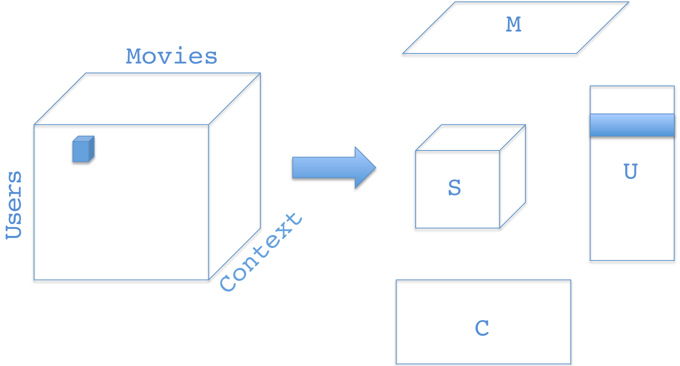

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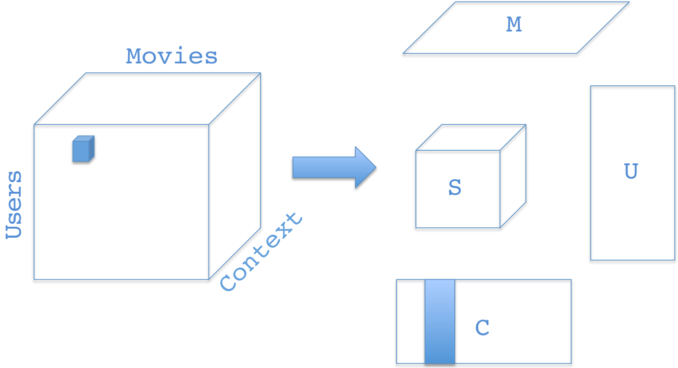

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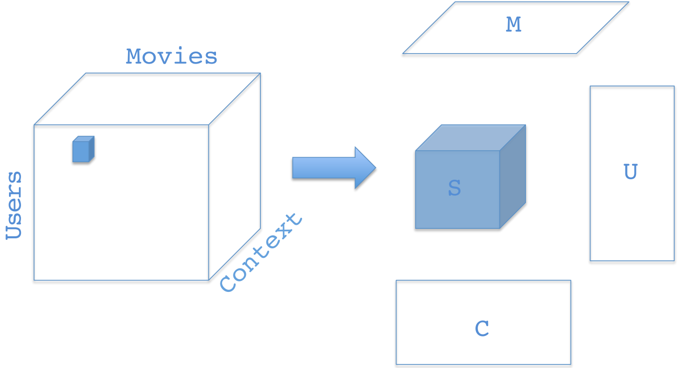


Experimental evaluation

We evaluate our model on contextual rating data and computing the *Mean Absolute Error (MAE)*,using 5-fold cross validation defined as follows:

$$MAE = \frac{1}{K}\sum_{ijk}^{n,m,c} D_{ijk}|Y_{ijk} - F_{ijk}|$$

Data

Data set	Users	Movies	Context Dim.	Ratings	Scale
Yahoo!	7642	11915	2	221K	1-5
Adom.	84	192	5	1464	1-13
Food	212	20	2	6360	1-5

Table: Data set statistics

Context Aware Methods

- Pre-filtering based approach, (*G. Adomavicius et.al*), computes recommendations using *only* the ratings made in the same context as the target one
- Item splitting method (*L. Baltrunas, F. Ricci*) which identifies items which have significant differences in their rating under different context situations.

Results: Context vs. No Context

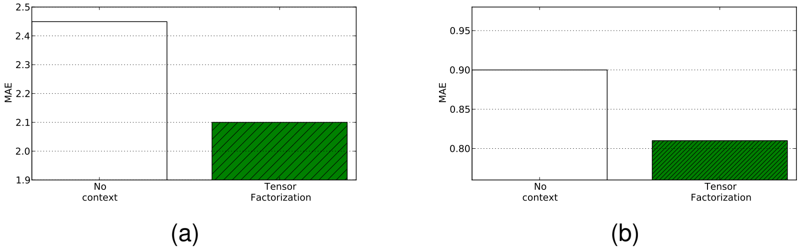


Figure: Comparison of matrix (no context) and tensor (context) factorization on the Adom and Food data.

Yahoo Artificial Data

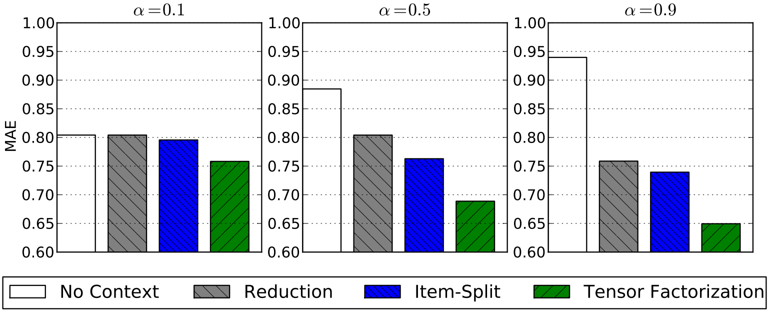
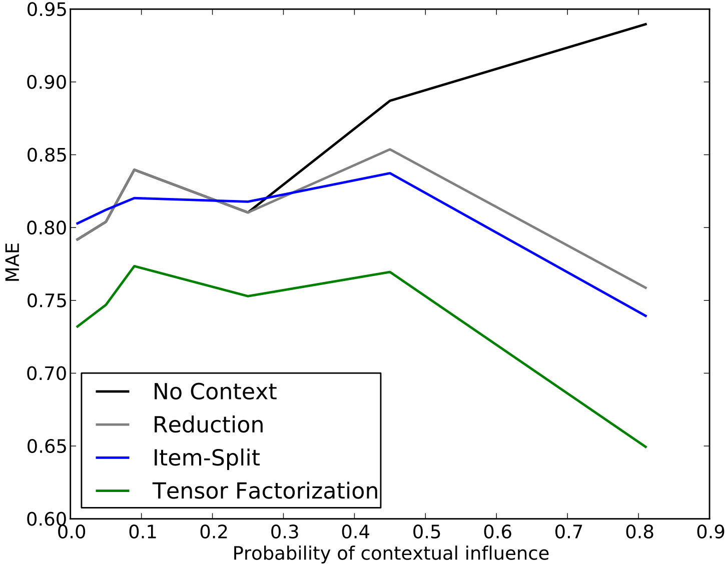


Figure: Comparison of context-aware methods on the Yahoo! artificial data

Yahoo Artificial Data



Tensor Factorization

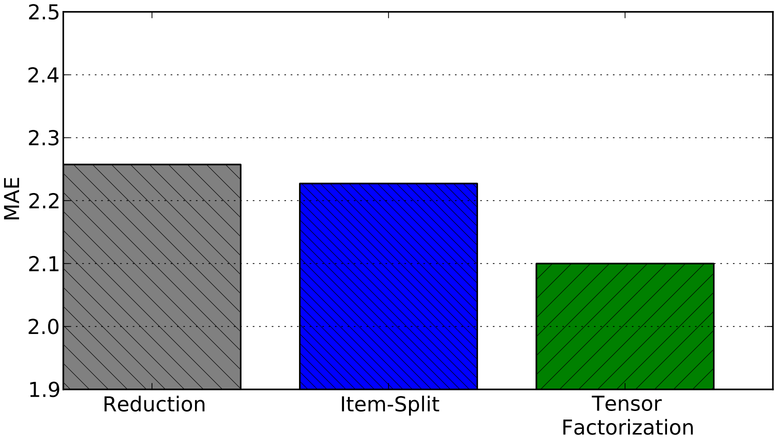


Figure: Comparison of context-aware methods on the Adom data.

Tensor Factorization

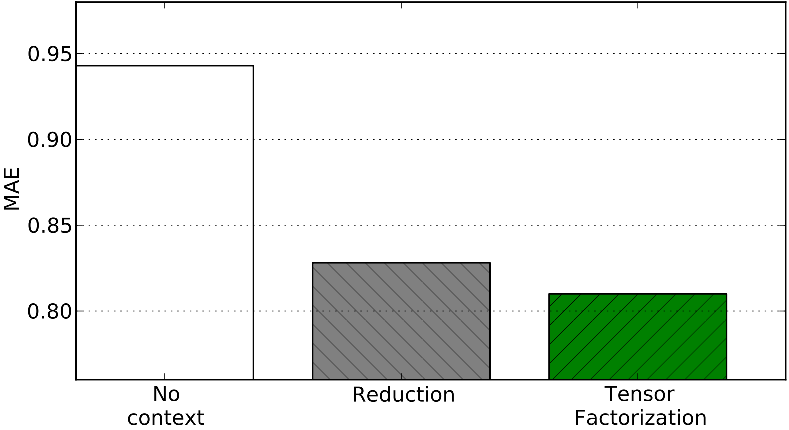


Figure: Comparison of context-aware methods on the Food data.

Conclusions

- Tensor Factorization methods seem to be promising for CARS
- Many different TF methods exist
- Future work: extend to implicit taste data
- Tensor representation of context data seems promising

Thank You !