# Dynamic Hashing

*NDBI007: Practical class 4*

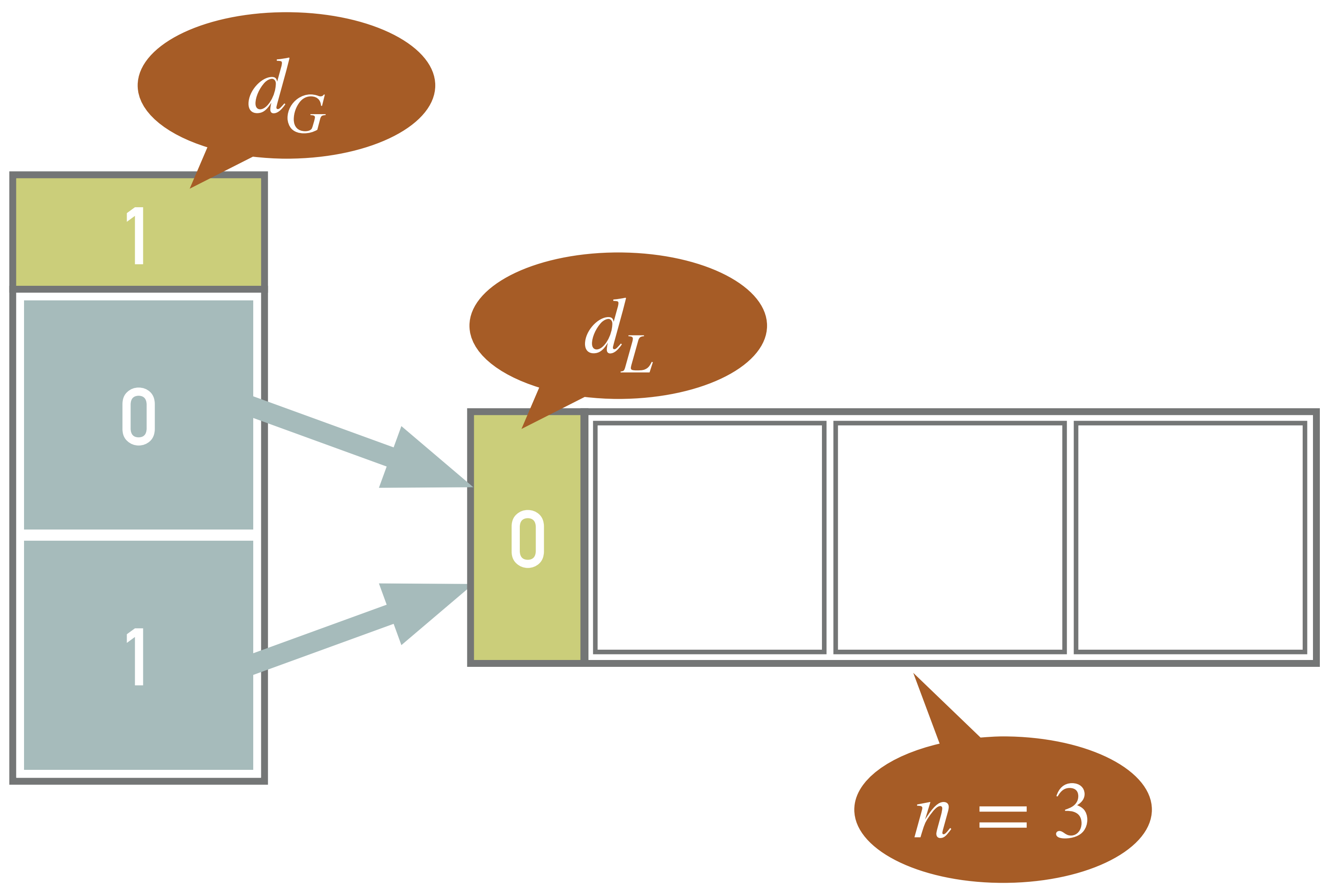
# Dynamic Hashing

- Static forms of hashing lose its good performance as the table utilization comes to its maximum
- Conversely, dynamic hashing algorithms allow to increase the size of the table with increasing number of stored records

- *Fagin*
- *Litwin*
- *LHPE-RL*

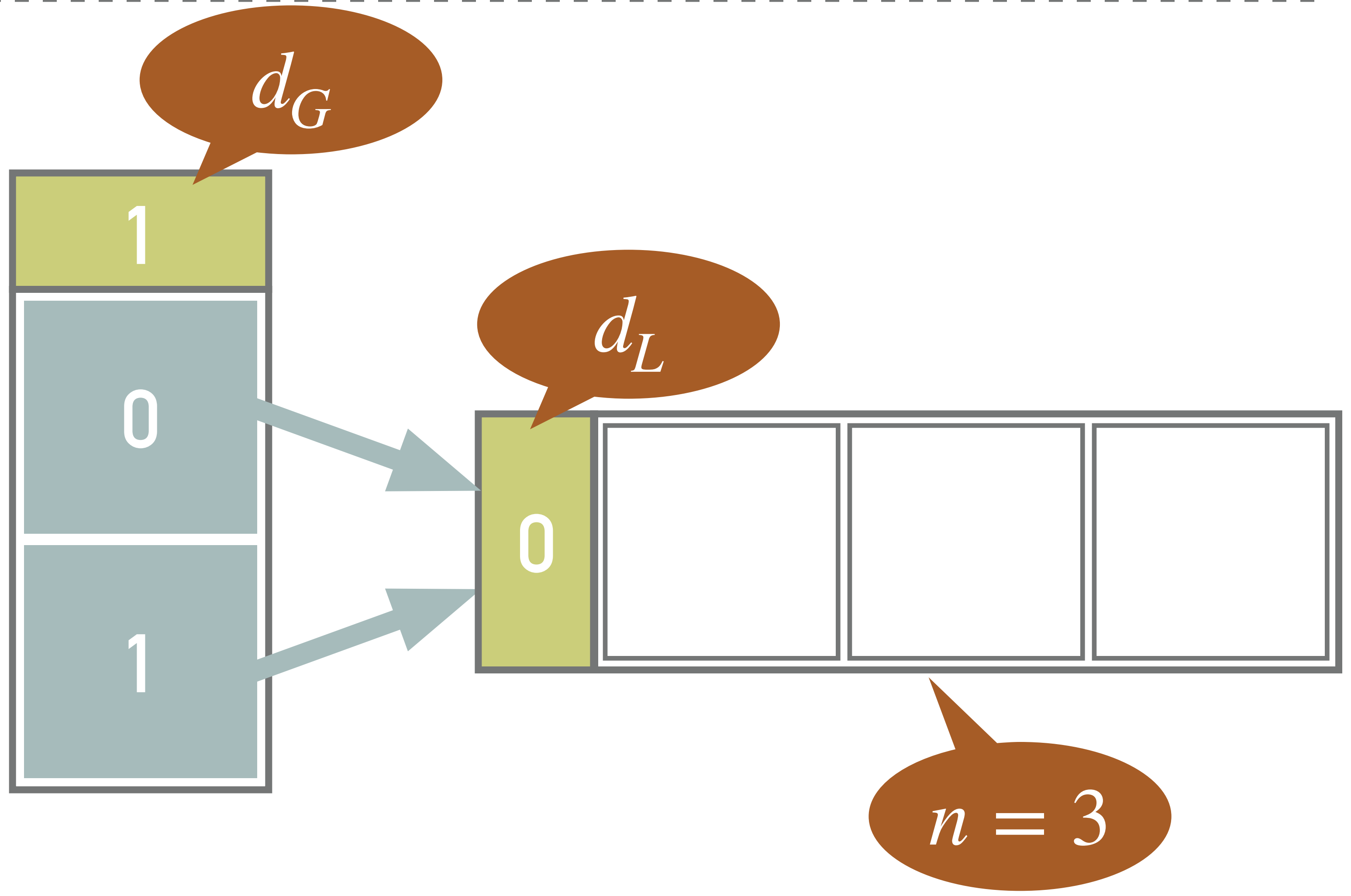
# Fagin

- Directory
    - *List of entries* in the *main memory* that points to the pages in the primary file
    - *Global depth* $d_G$ - number of least significant bits of the hash $h(k)$ needed to *address an entry* in the directory
- Primary file
    - Distributed *collection of pages* stored in the *secondary memory*, i.e., continuous space is not required
    - Each page has a constant size $n$
    - Each page remembers *local depth* $d_L$ - number of least significant bits of the hash $h(k)$ *common to all records*
    - $2^{d_G - d_L}$ tells how many directory entries points to the particular page in the primary file

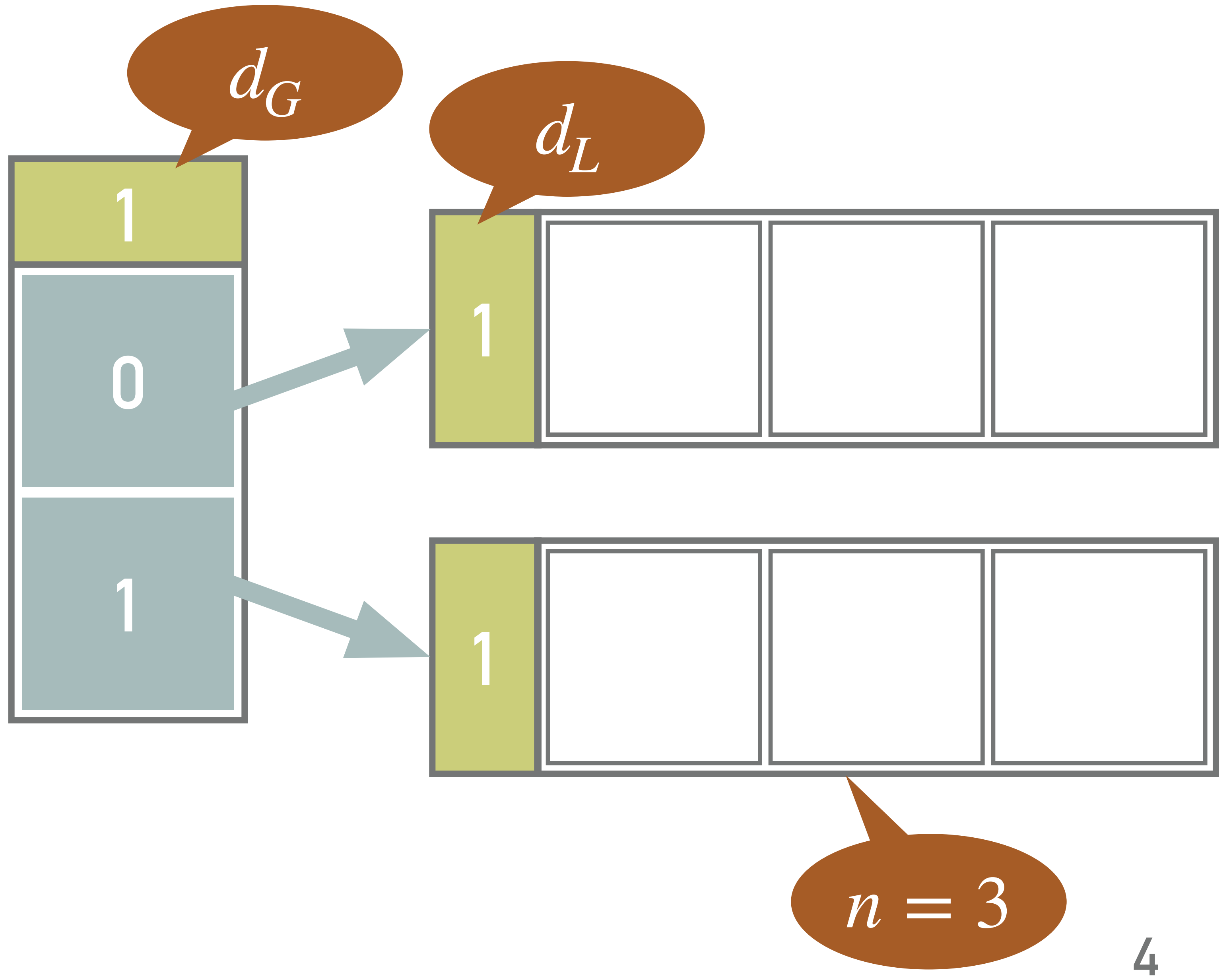


# Fagin

- *Overflowing* causes a *change in the structure* of the directory and primary file
  - $d_L < d_G$ - the particular *page can be split*, i.e., the page is split and $d_L$ incremented
  - $d_L = d_G$ - the *directory* must be *expanded*, i.e., the directory is doubled and $d_G$ incremented

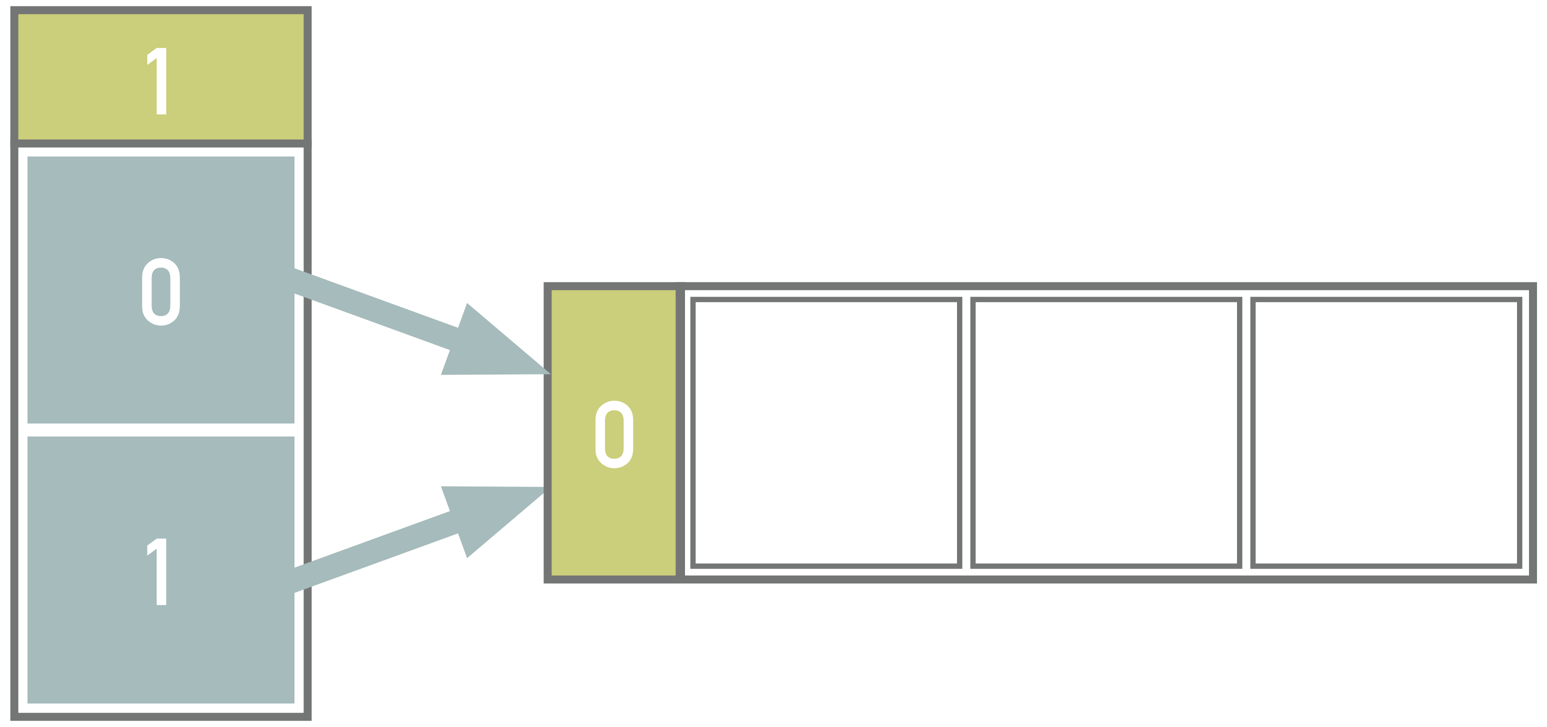



- Inserting or searching for a record with key $k$
  - Compute $k' = h(k)$
  - Convert $k'$ into directory entry $k''$ by leaving the $d_G$ least significant bits
  - The pointer in the corresponding entry points to the page where the record should be inserted / searched



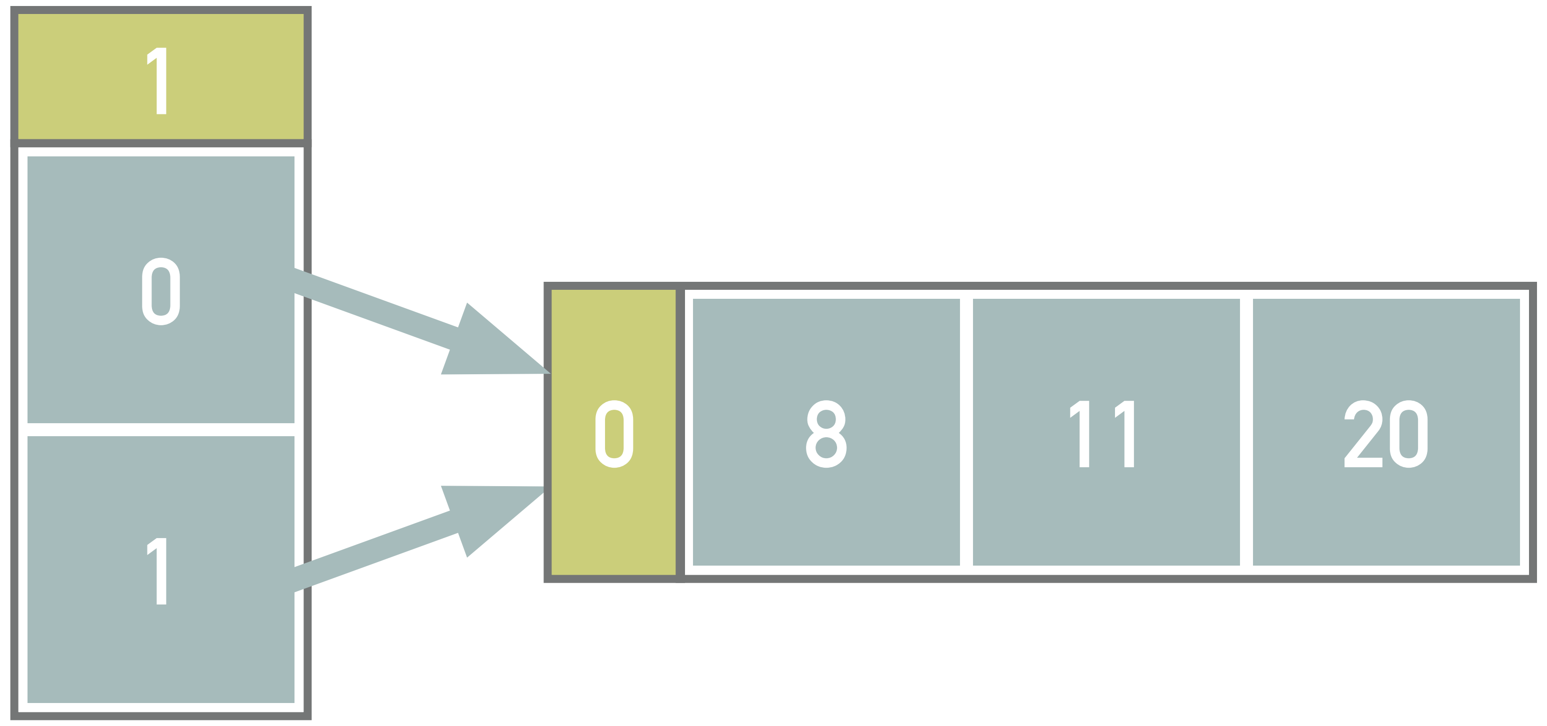
# Example 4.1

- Insert records with keys 20, 11, and 8




- $h(20_{10}) = 10100_2$
  - Using the least significant bit of key 20, that is 0, the corresponding record is inserted into the page using entry 0

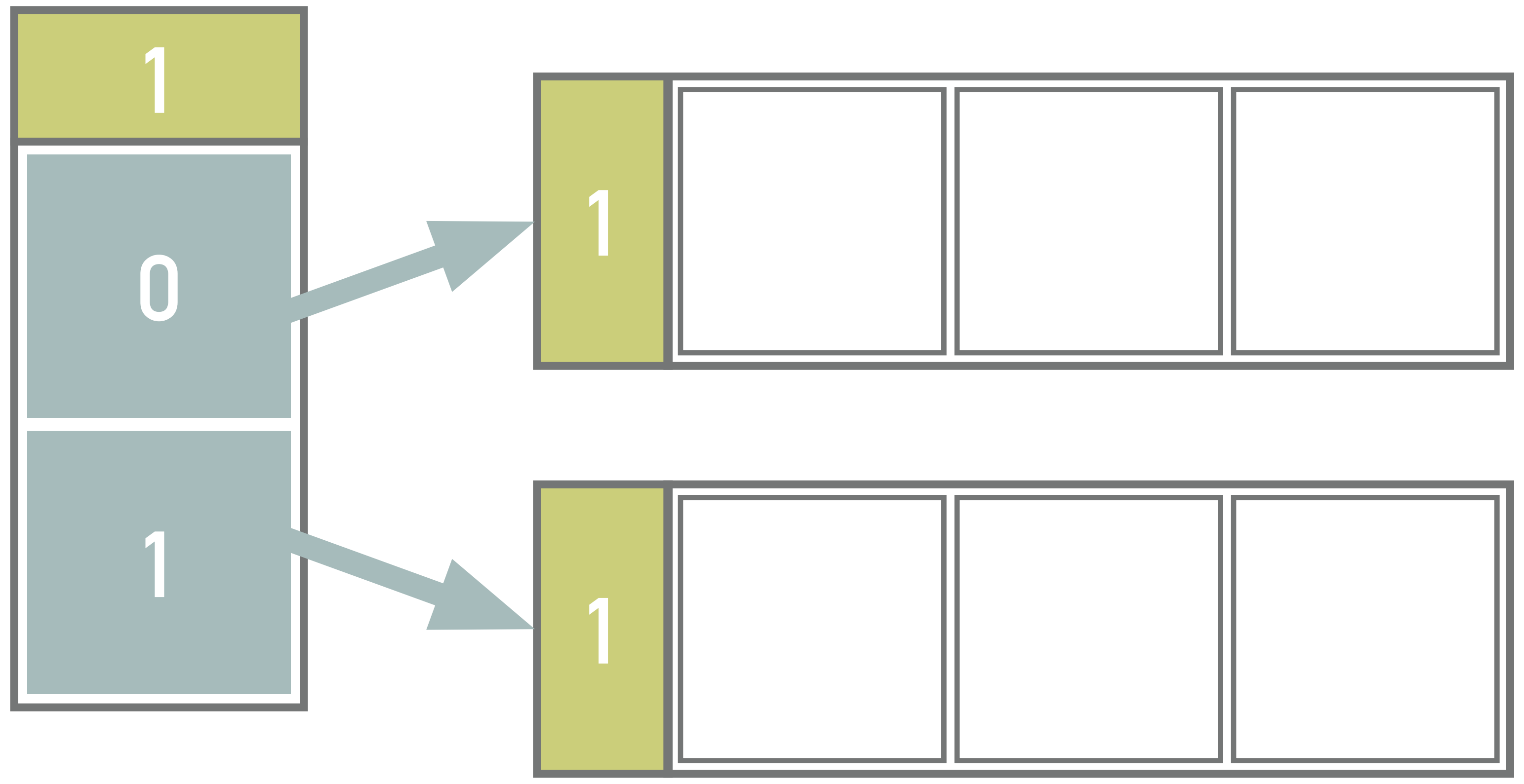
- $h(11_{10}) = 1011_2$
  - Record with key 11 is stored to the same page using entry 1



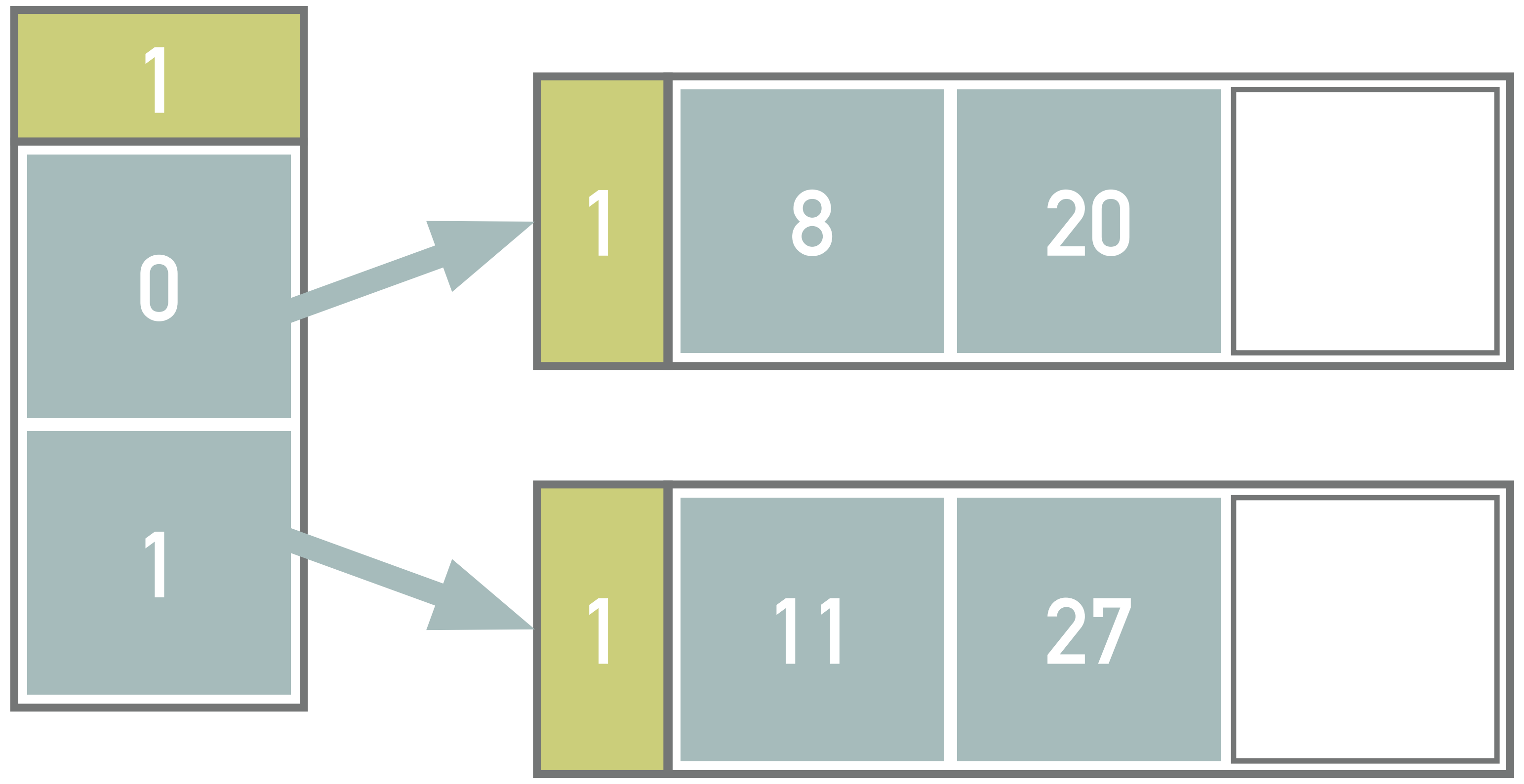

- $h(8_{10}) = 1000_2$
  - Record with key 8 is inserted into the same page using entry 0

# Example 4.2: Splitting a Page

- Insert record with key 27 (into the structure from example 4.1)




- $h(27_{10}) = 11011_2$
- Page is overflown
  - The local value $d_L$ of the page is less than the global value $d_G$ of the directory
  - Therefore we can split the page into two new pages and increment $d_L$ values of both the pages




- Finally, we reinsert the records previously allocated into the page being split
  - After the reinsert, the even keys are stored in the page referenced from the zero-th directory entry while the off records are referenced from the first entry

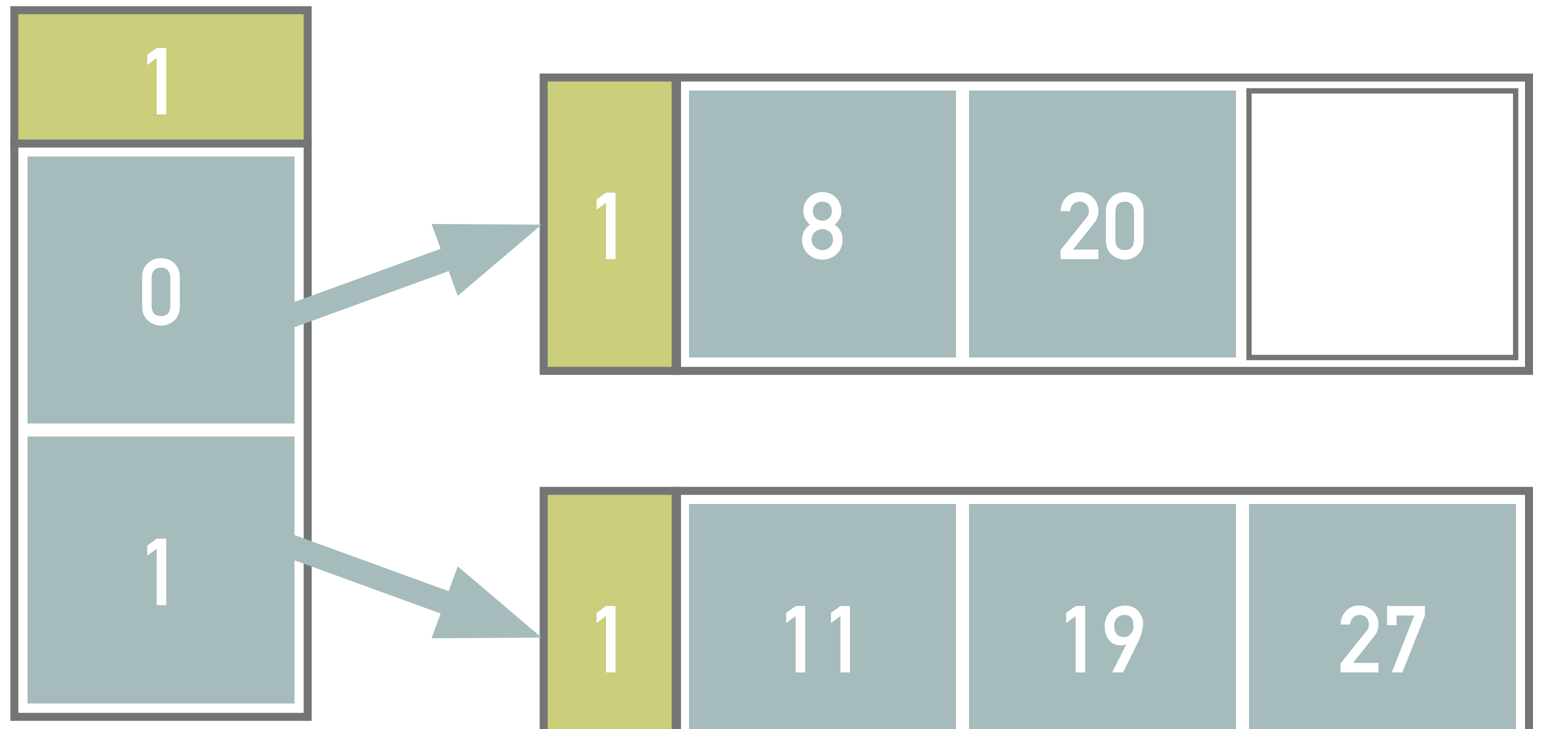
# Example 4.3: Expanding the Directory

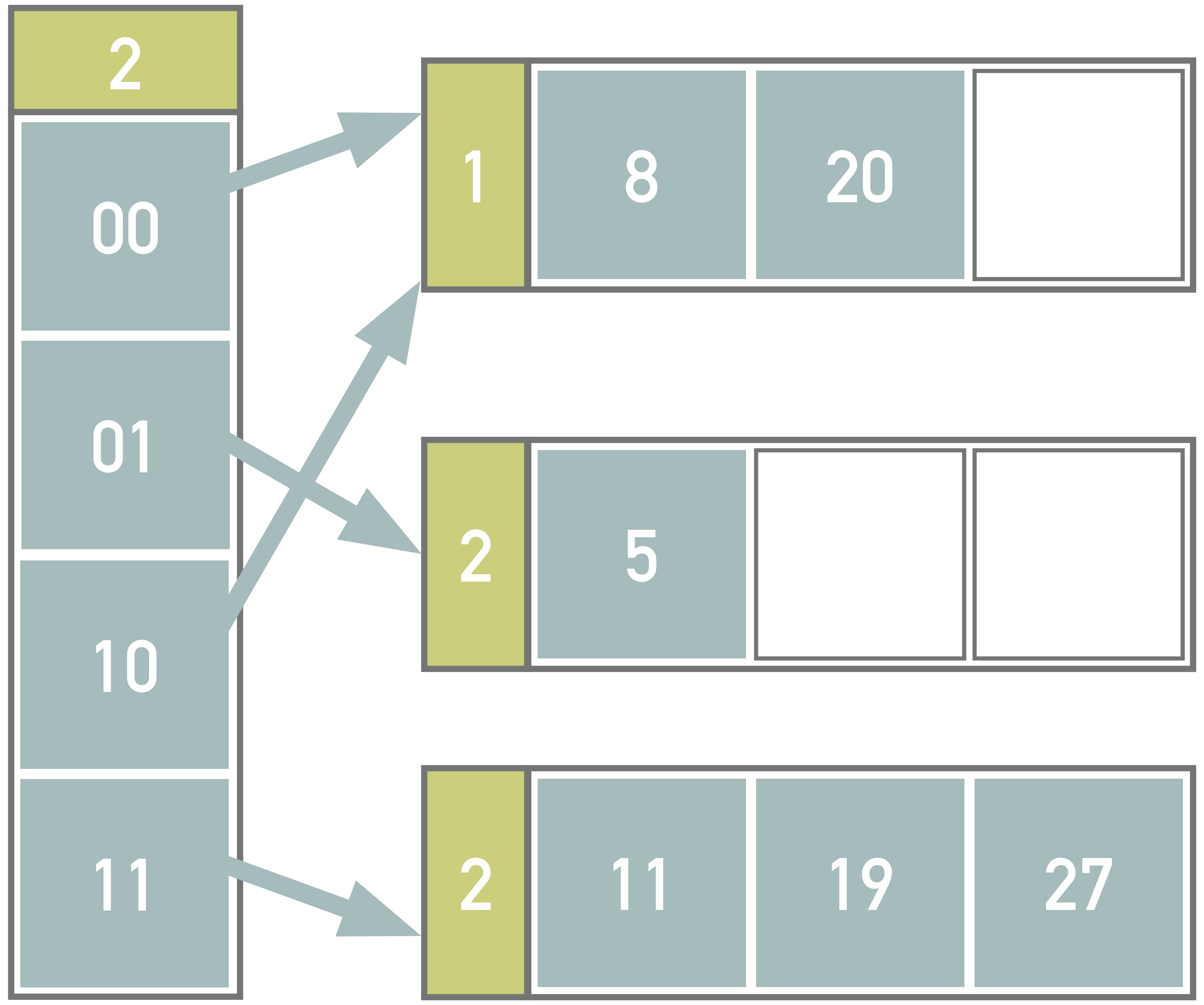
- Insert records with keys 19 and 5 into the structure from example 4.2

- $h(19_{10}) = 10011_2$
  - After inserting record with key 19, a page is filled

- $h(5_{10}) = 101_2$
  - The insert of the record having key 5 causes:
    - Expanding the directory, i.e., $d_L = d_G$
    - Splitting of the second page, i.e., $d_L = 2$
    - Reinserting of records with keys 5, 11, 19, and 27

- $h(11_{10}) = 1011_2$
- $h(19_{10}) = 10011_2$
- $h(27_{10}) = 11011_2$

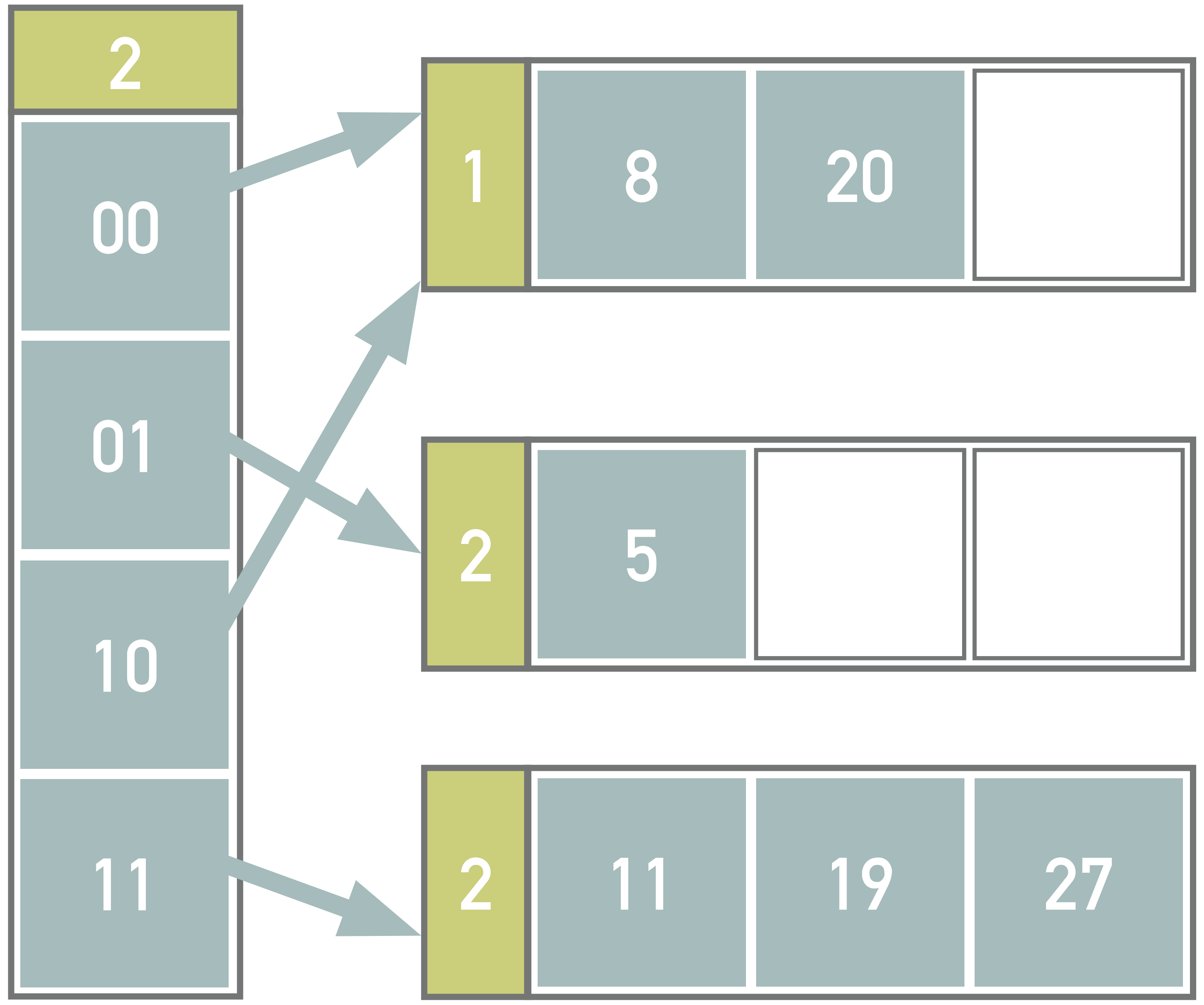


# Exercise 4.4

- Insert records with keys 24 and 32 into the following structure

- Note all the computations and illustrate the solution

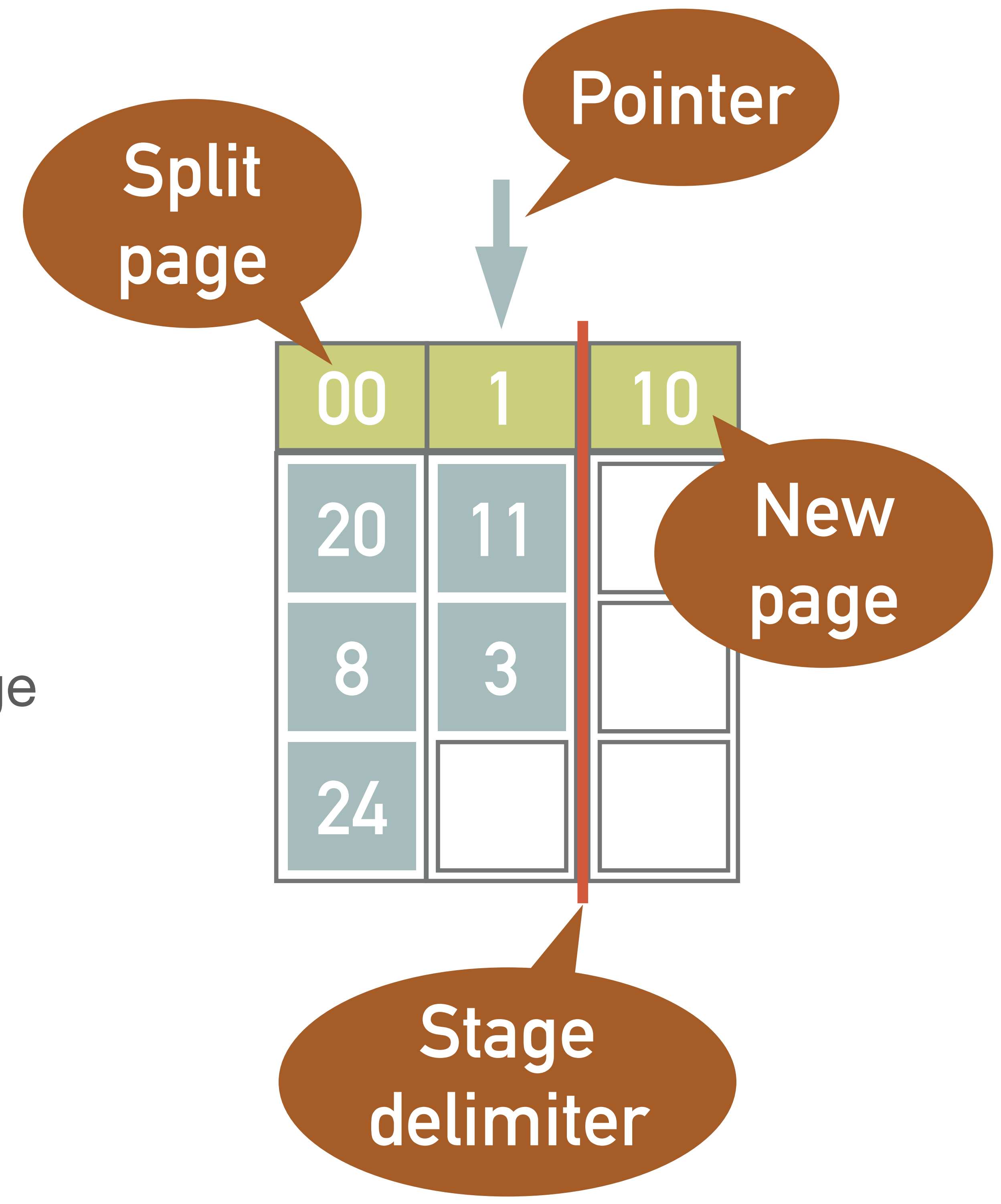


# Litwin

- Directory-less schema that avoids *exponentially increasing* size of the directory, but we need a continuous space in the secondary memory
  - Addition of a single page after pre-defined condition

- The primary file is *linearly expanded* with time (stages), i.e., adding one page after another
  - *Stage* $d$ starts with $s = 2^d$ pages and ends when the number reaches $s = 2^{d+1}$ (i.e., stage $d + 1$ begins)
- During the stage, a *split pointer* $p \in \{0,\ldots,2^d - 1\}$ identifies the next page to be split
  - At the beginning of stage $d$, the pointer points to page 0
  - After every split operation, the *pointer is incremented* by 1, or moved to the start when we enter a new page
  - Records from page $p$ (and overflow records) will be distributed between *split pages* $p$ and $p + s$ using $h_{d+1}(k)$
  - If a page overflows before its time to split, *overflow page* will be utilized

- At each stage, we have two types of hash functions
  - $h_d(k)$ for pages not yet split, i.e., the least significant $d$ bits of the hashed value $h(k)$ are used
  - $h_{d+1}(k)$ for the already split pages



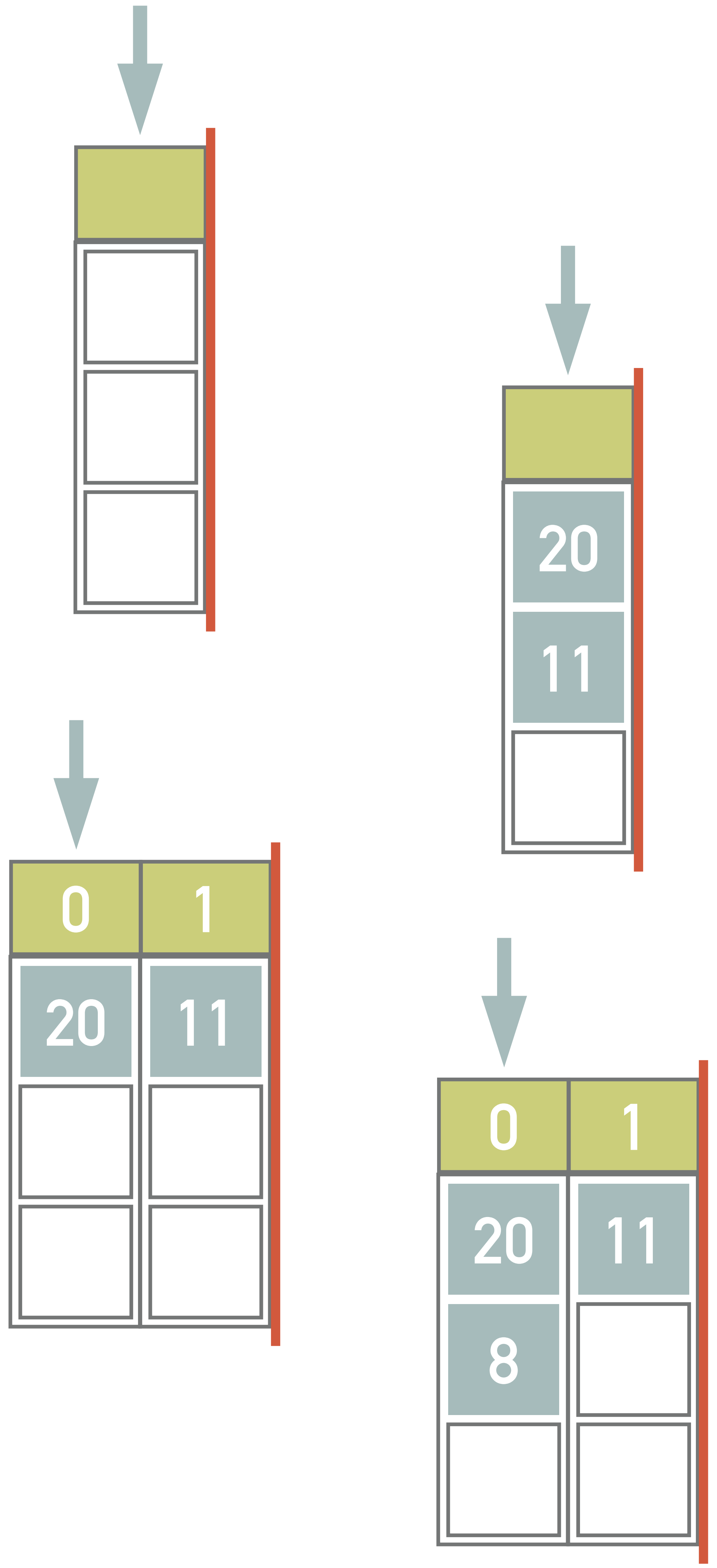
# Example 4.5

- Insert records with keys 20, 11, and 8 into an empty primary file
  - I.e., start the stage $d = 0$ with one page (capacity 3 records), $h(k) = p, p = 0$
  - Pre-defined condition: Splitting occurs after 2 inserts

- The records with key 20 and 11 are inserted into the 0th page disregarding the value of the key
  - $d = 0$ bits of the keys are used at this points

- We have inserted 2 keys, therefore splitting occurs (a new page is created)
  - The records from 0-th page are redistributed using the least significant bit of the hashed key
    - $h(20_{10}) = 10100_2$
    - $h(11_{10}) = 1011_2$
  - Because $p = 2^i$ is reached, the stage changes to $d = 1, p = 0$

- Now, we use $d = 1$ bit for not yet split pages and $d + 1$ bits for split pages
  - The record with key 8 is inserted into the page 0 using the least significant bit
    - $h(8_{10}) = 1000_2$



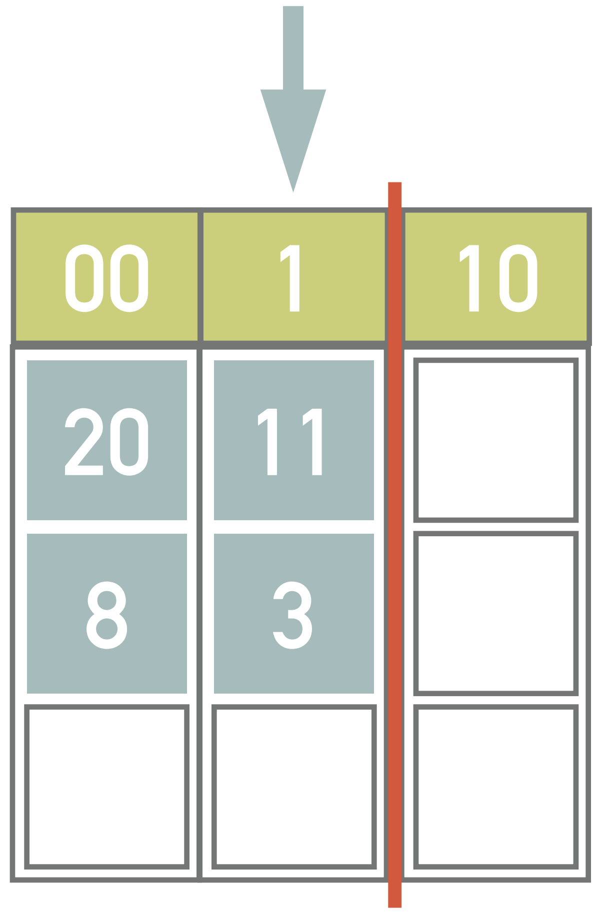
# Example 4.6

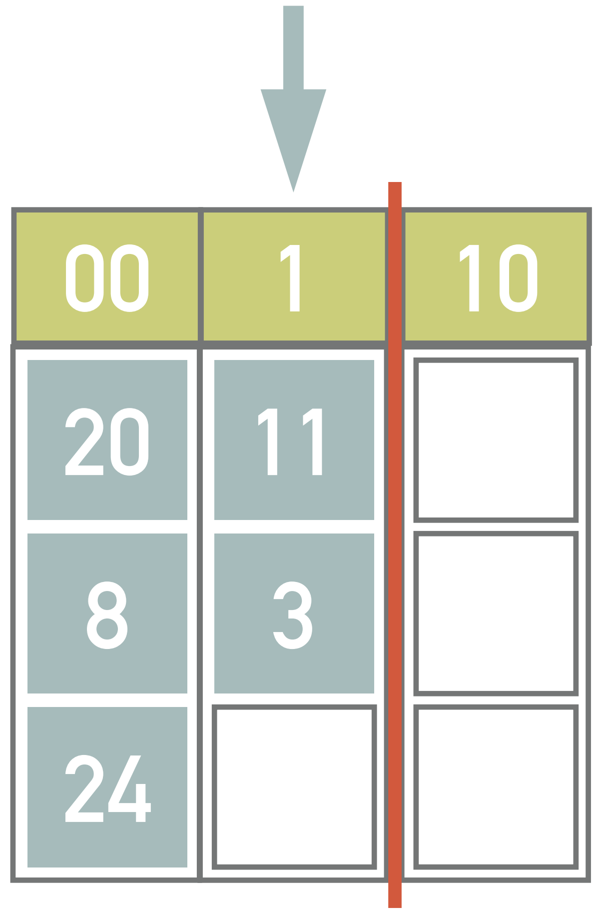
- Insert records with keys 3, 24, and 32 into the structure from example 4.5

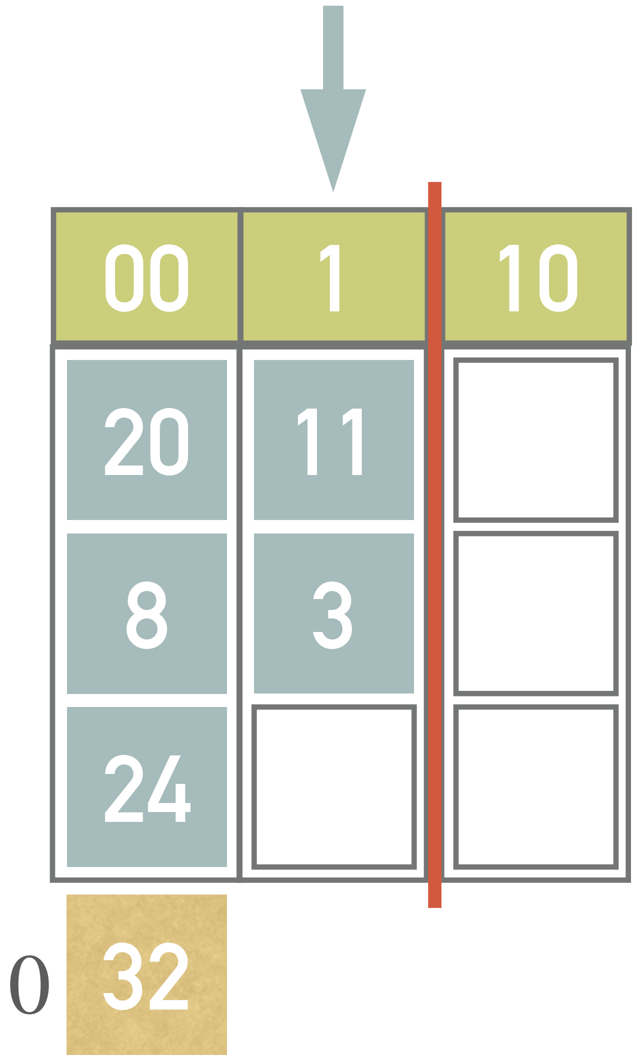
- A record with key 3 will now be inserted into page 1
    - $h(3_{10}) = 11_2$

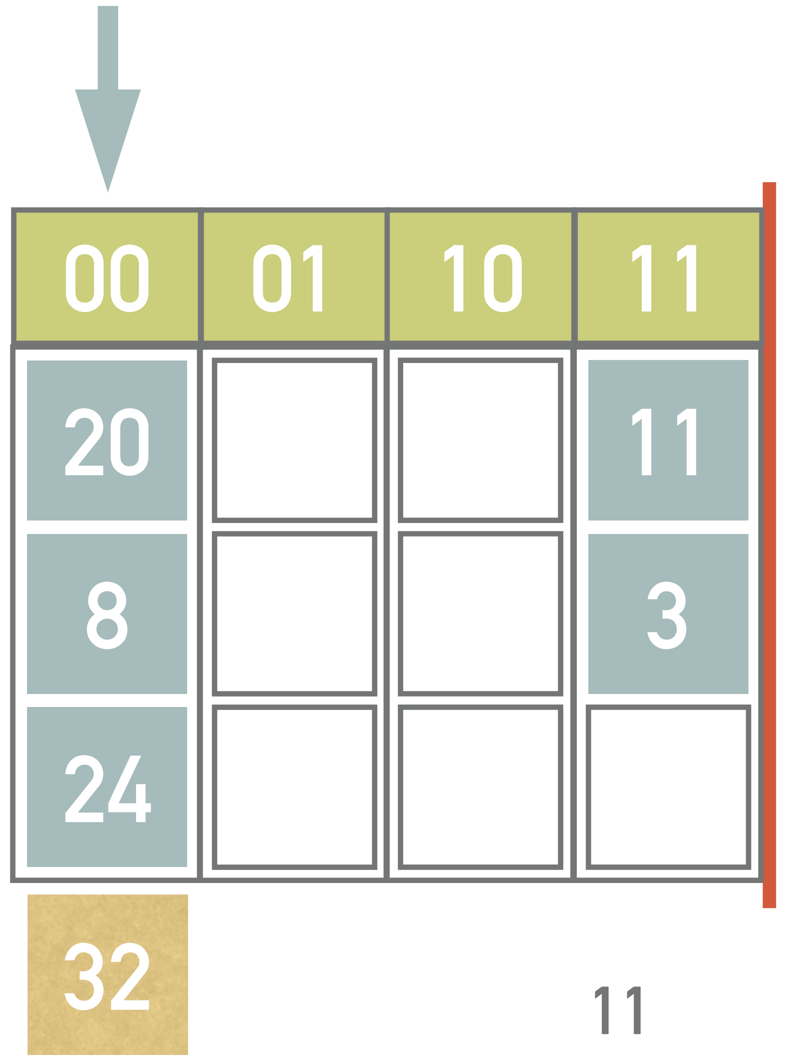
- We have already inserted 2 records in the stage $d = 1$, therefore page $p = 0$ is split into pages $p_0 = 00$, $p_1 = 10$

- Next, we will insert a pair of records with keys 24 and 32
- $h(24_{10}) = 11000_2$
    - Because $h_1(11000_2) = 0$ and $0 < p$, it is necessary to address the keys using 2 least significant bits, i.e., $h_1(100000_2) = 00$, and the key belongs in the page 00
  - $h(32_{10}) = 100000_2$
    - The key 32 belongs to the same page, but that is already filled and thus overflows
  - Finally, the page 1 is split
    - Since the number of pages reaches $s = 2^{1+1} = 4$, the second stage is initiated, i.e., $d = 2$, $p = 0$



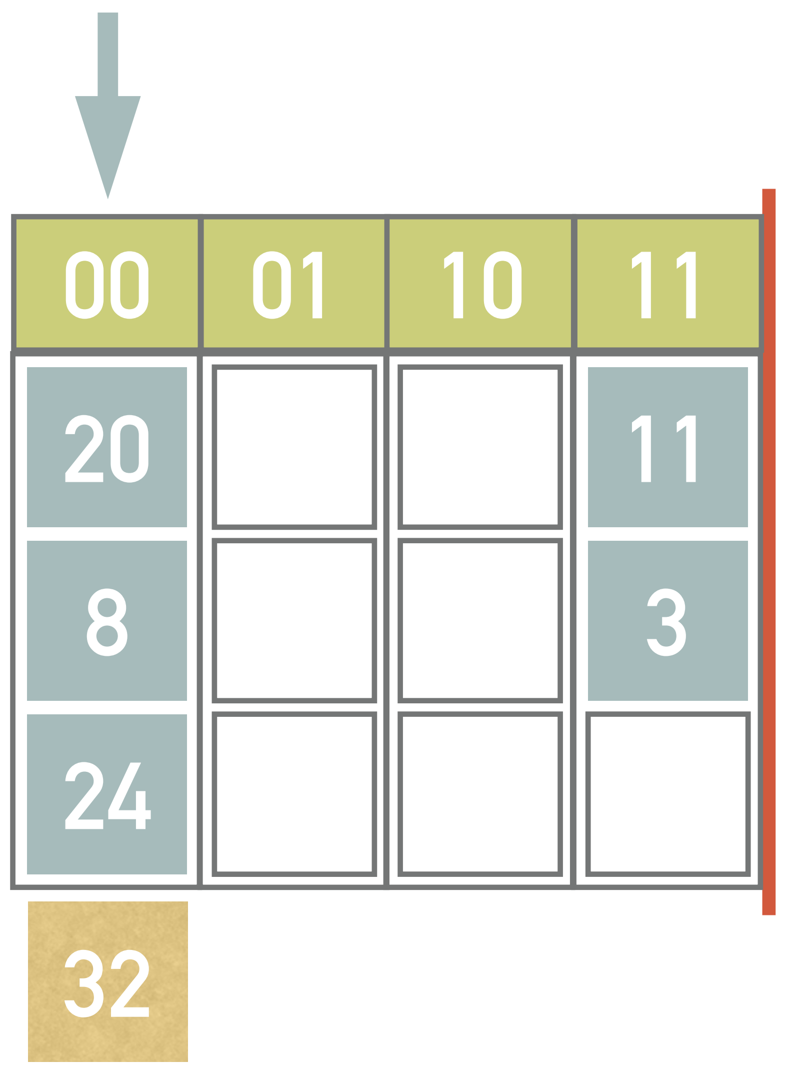



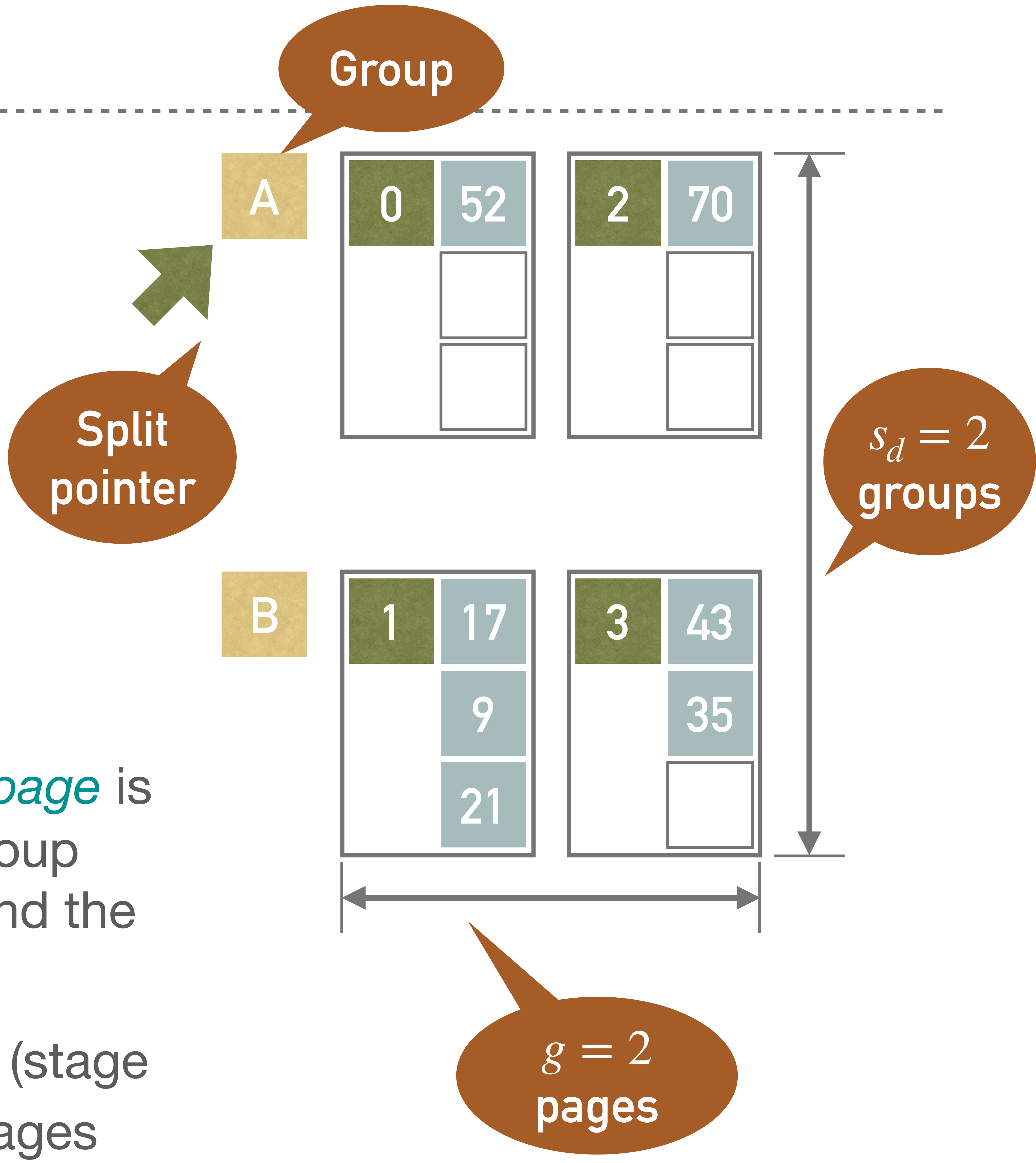
# Exercise 4.7

- Insert records with keys 27, 19, 10, and 5 into the following structure
  - I.e., start the stage $d = 2$ with $s = 4$ pages (capacity 3 records), $h(k) = k$, $p = 0$
  - Pre-defined condition: Splitting occurs after 2 inserts
- Note all the computations and illustrate the solution



# LHPE-RL

- Simplified version of LHPE
- At the *stage* $d$, the primary file consists of $p_d$ *pages*
    - Each page has *capacity* $b$
    - Pages are grouped into $s_d = p_d \div g$ groups
        - Each *group* has $g$ pages

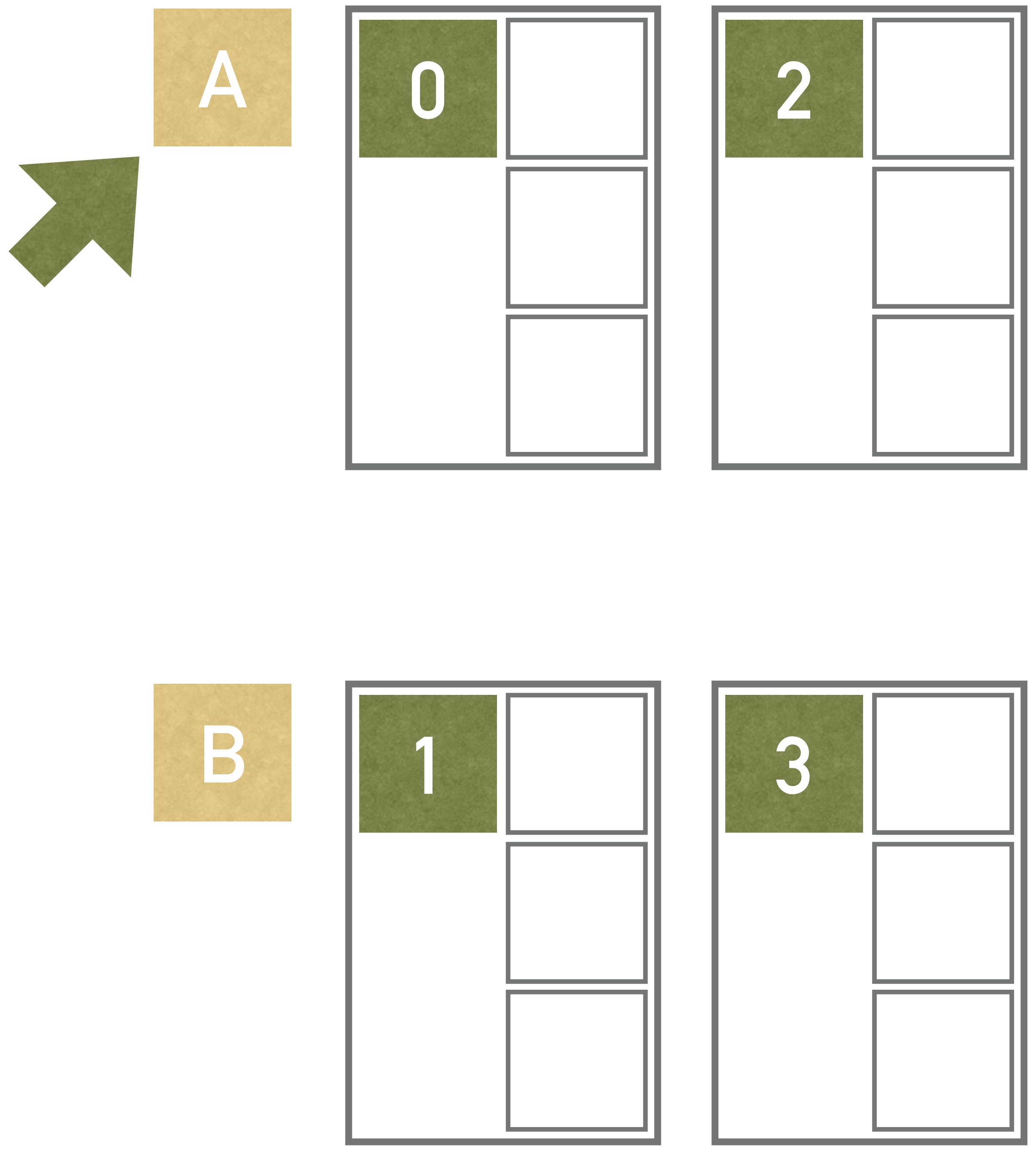



- When a *predefined condition* is met (e.g., after $L$ insertions), a *new page* is inserted at the end of the primary file and records in pages in the group pointed to the *split pointer* are redistributed between these pages and the new page (being the new member of the group)
- When the last group is redistributed, the file is (virtually) *reorganized* (stage $d + 1$) so that all the pages are again sorted into $s_{d+1} = p_{d+1} \div g$ pages
- $p_{d+1} = \lceil s_d \bullet (g + 1) \div g \rceil \bullet g$

# Example 4.8

- Insert records with keys 17, 9, 43, 21, 49, 35, 70, 52, 40, 13, 5, 80 into the following empty structure
  - Stage $d = 0$
  - The initial number of groups $s_0 = 2$
  - Page capacity $b = 3$

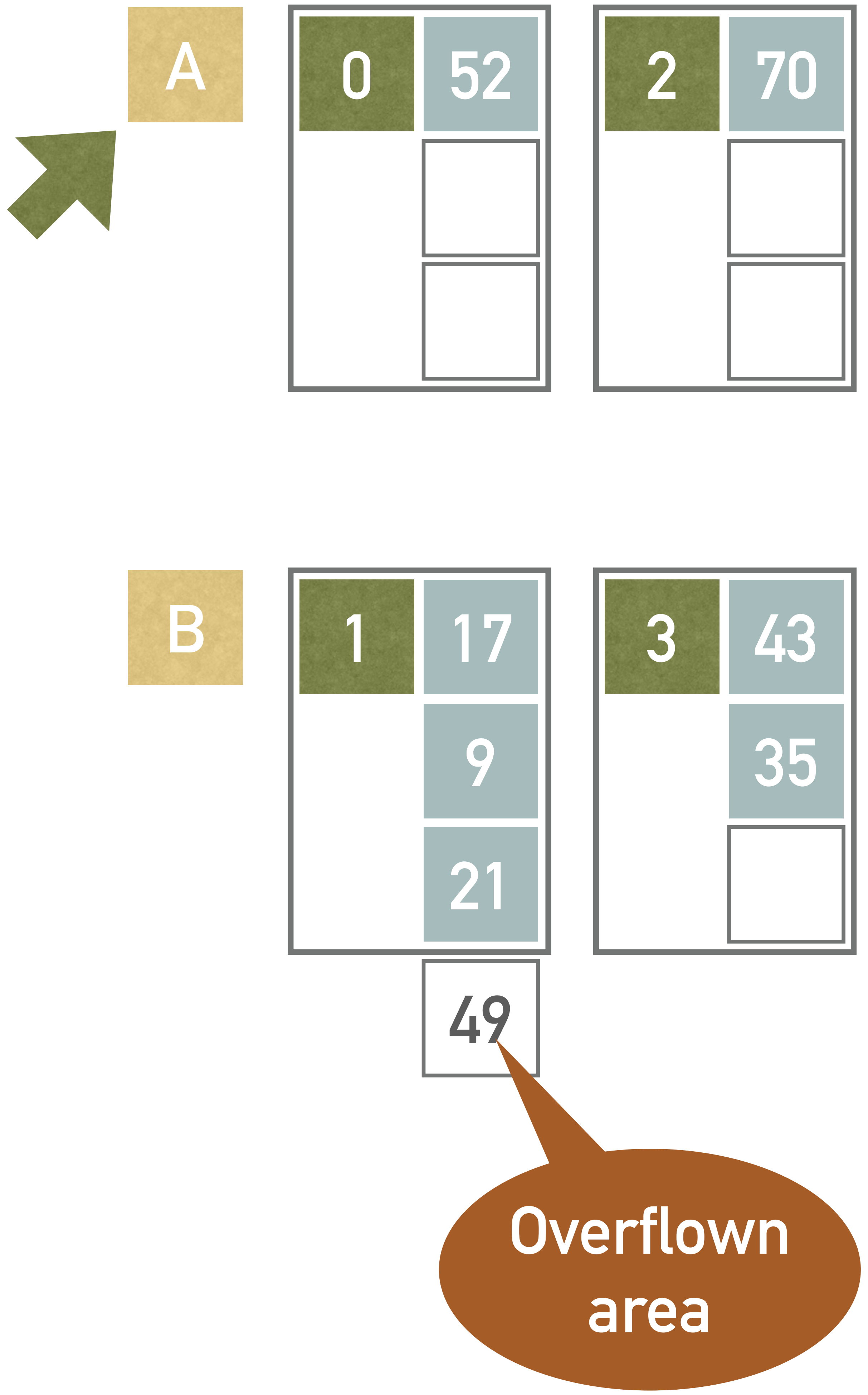
- Hash function
  - $h_0(k) = k \mod 4$
    - Determines into which of 4 initial pages a record is inserted at the beginning
  - $h_1(k) = k \mod 3$
    - Determines where the records are inserted when a group splits for the first time
  - $h_2(k) = (k \div 3) \mod 3$
    - Determines where the records are inserted when a group splits for the second time
- We are going to split regularly after two inserts, i.e., $L = 2$
  - Having $p_0 = s_0 \bullet g = 4$ pages, the first split occurs after insertion of $p_0 \bullet L = 8$ records



# Example 4.8 (Continued)

- Inserts of the first 8 keys, i.e., 17, 9, 43, 21, 49, 35, 70, and 52 are not interesting since these are inserted where the $h_0$ function says

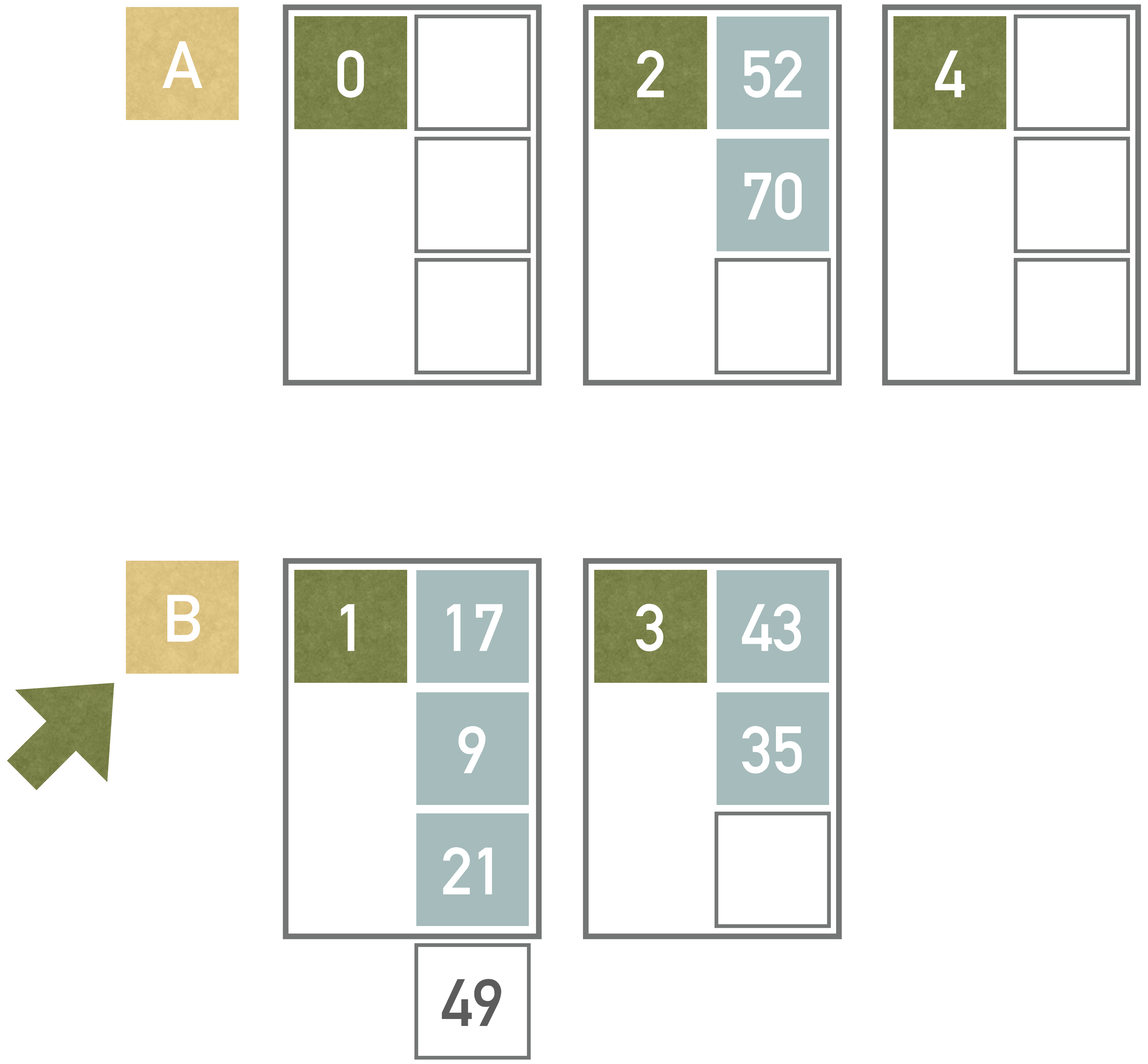
$$h_0(17) = 17 \mod 4 = 1$$
$$h_0(9) = 9 \mod 4 = 1$$
$$h_0(43) = 43 \mod 4 = 3$$
$$h_0(21) = 21 \mod 4 = 1$$
$$h_0(49) = 49 \mod 4 = 1$$
$$h_0(35) = 35 \mod 4 = 3$$
$$h_0(70) = 70 \mod 4 = 2$$
$$h_0(52) = 52 \mod 4 = 0$$




- The only problem is with key 49 which is assigned to an (already full) page 1

# Example 4.8 (Continued)

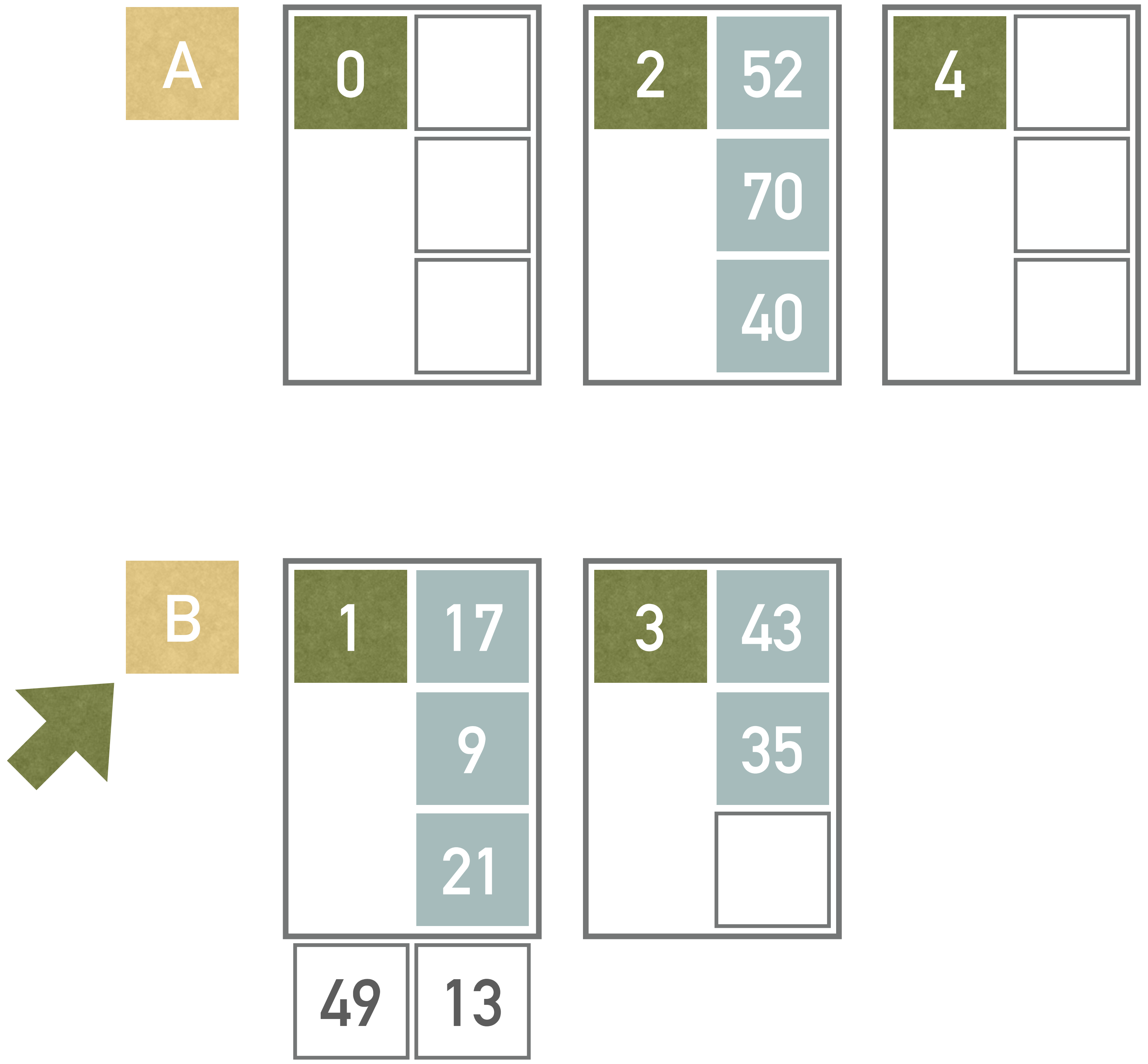
- Having inserted 8 keys, we have to split the group pointed by the split pointer, i.e., the group A having pages 0 and 2
  - Page 4 is added into group A
  - Function $h_1(k)$ is applied in order to redistribute keys in the group A
    - $h_1(k)$ returns the index of a page in a group A, i.e., $h_1(k) = 0$ for the page 0, $h_1(k) = 1$ for the page 2, and $h_1(k) = 2$ for the page 4
    - $h_1(52) = 52 \mod 3 = 1$, therefore key 52 goes into the page 2
    - $h_1(70) = 70 \mod 3 = 1$, hence the key 70 goes into the page 2
  - Split pointer is incremented




- The key in the overflow area, i.e., 49, does not belong neither to page 0 nor to page 2, thus stays where it is

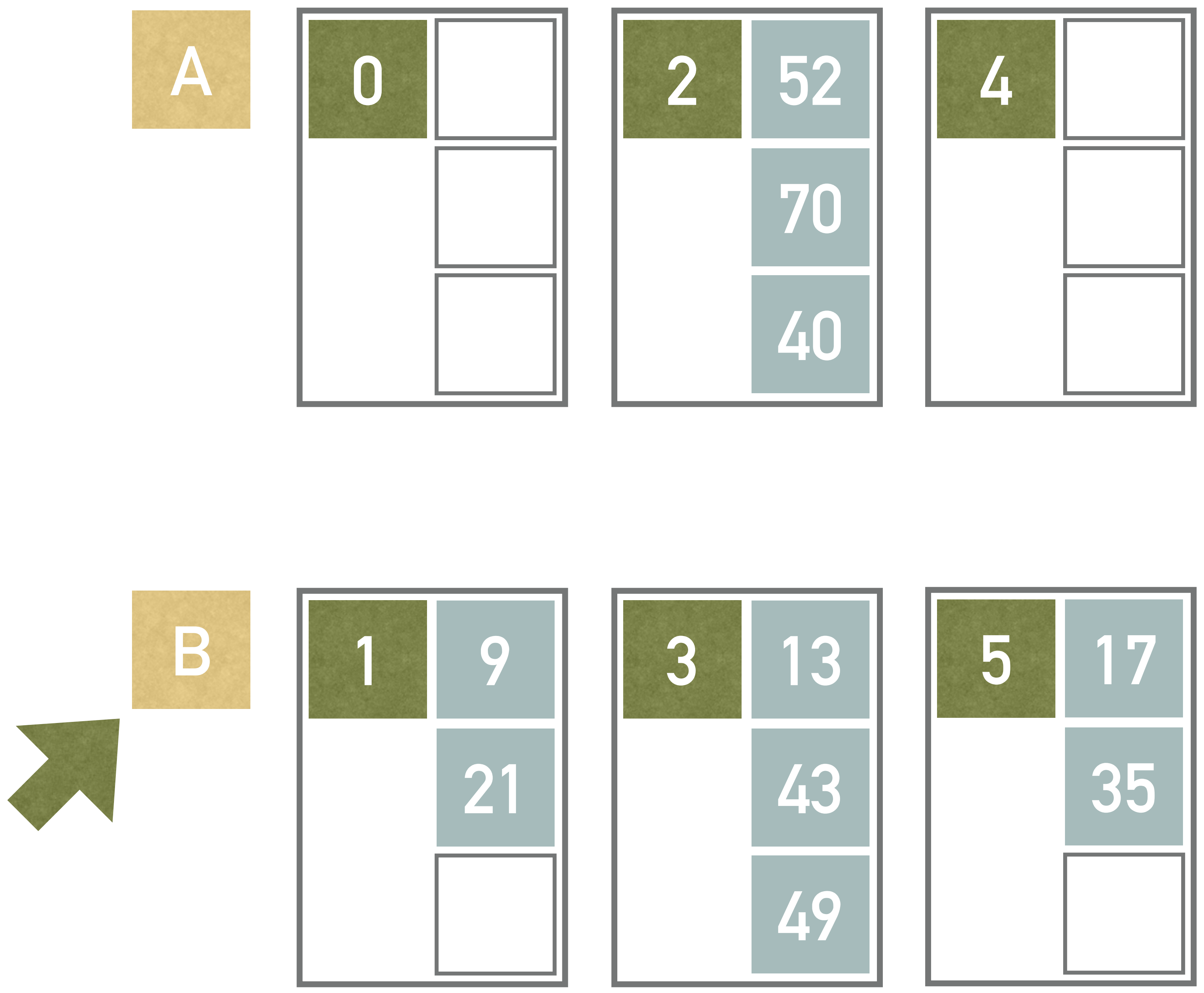
# Example 4.8 (Continued)

- Next, we insert record with key 40
  - $h_0(40) = 40 \mod 4 = 0$
  - Based on the function $h_0$, the record with key 40 should be assigned to the page 0 but this page has already been split
  - Thus we need to use $h_1$ which sends it into the second page in the group A
    - $h_1(40) = 40 \mod 3 = 1$(i.e., page 2)
- Next, we insert record with key 13
  - $h_0(13) = 13 \mod 4 = 1$
  - Based on the function $h_0$, the record with key belongs to the page 1, which has not been split yet
    - No need to use $h_1$
  - The page 1 is already full, therefore the overflow area is used



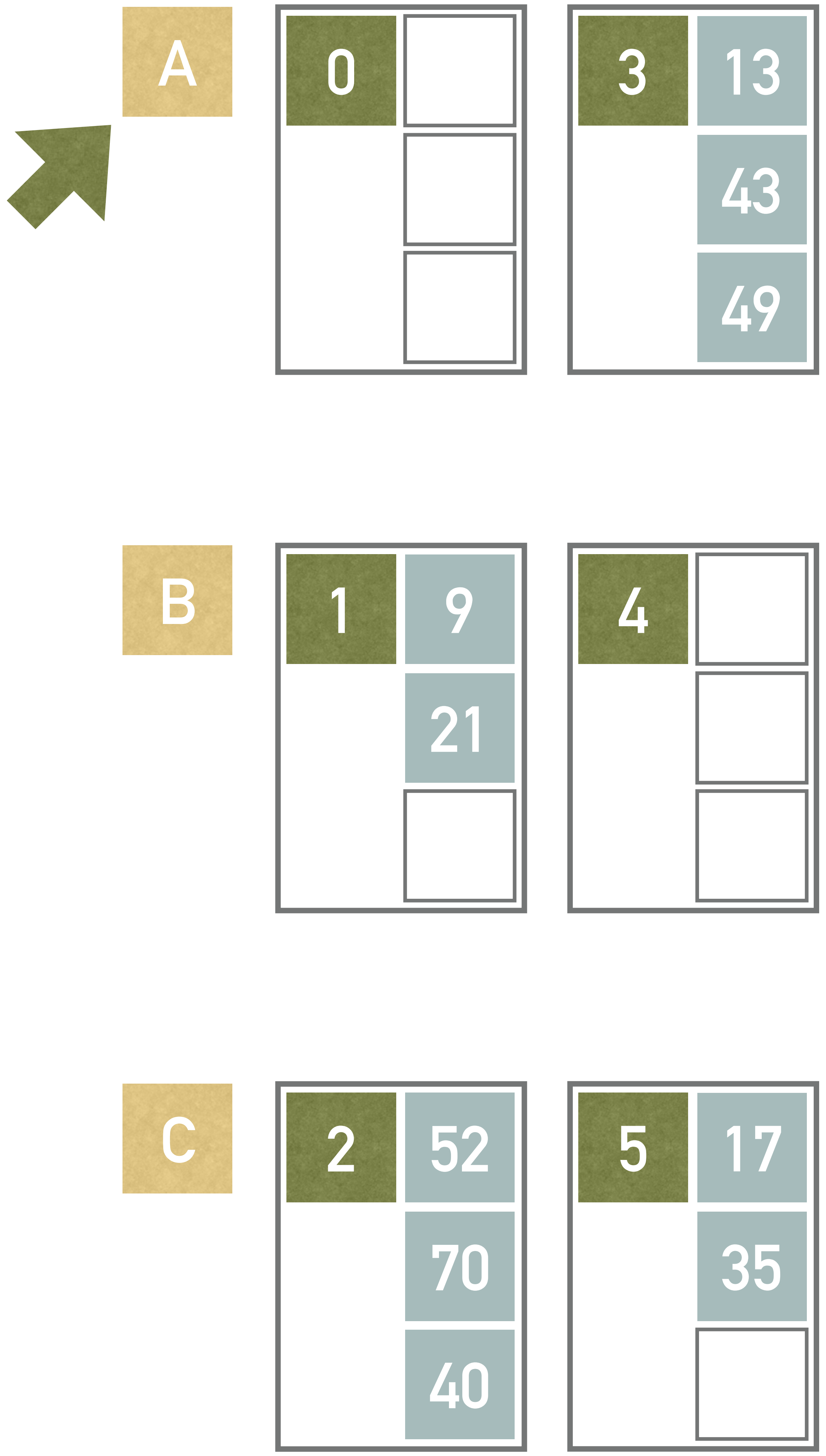
# Example 4.8 (Continued)

- Once again, we have to split the group (we have already inserted $L = 2$ records)
  - Split pointer points to the group B, i.e., pages 1 and 3 will be split
  - Page 5 is added
  - Function $h_1(k)$ will be applied in order to redistribute keys in the group B
    - $h_1(17) = 17 \mod 3 = 2$, therefore goes to the page 5
    - $h_1(9) = 9 \mod 3 = 0$, therefore goes to the page 1
    - $h_1(21) = 21 \mod 3 = 0$, therefore goes to the page 1
    - $h_1(43) = 43 \mod 3 = 1$, therefore goes to the page 3
    - $h_1(35) = 35 \mod 3 = 2$, therefore goes to the page 5
    - $h_1(49) = 49 \mod 3 = 1$, therefore goes to the page 3
    - $h_1(13) = 13 \mod 3 = 1$, therefore goes to the page 3



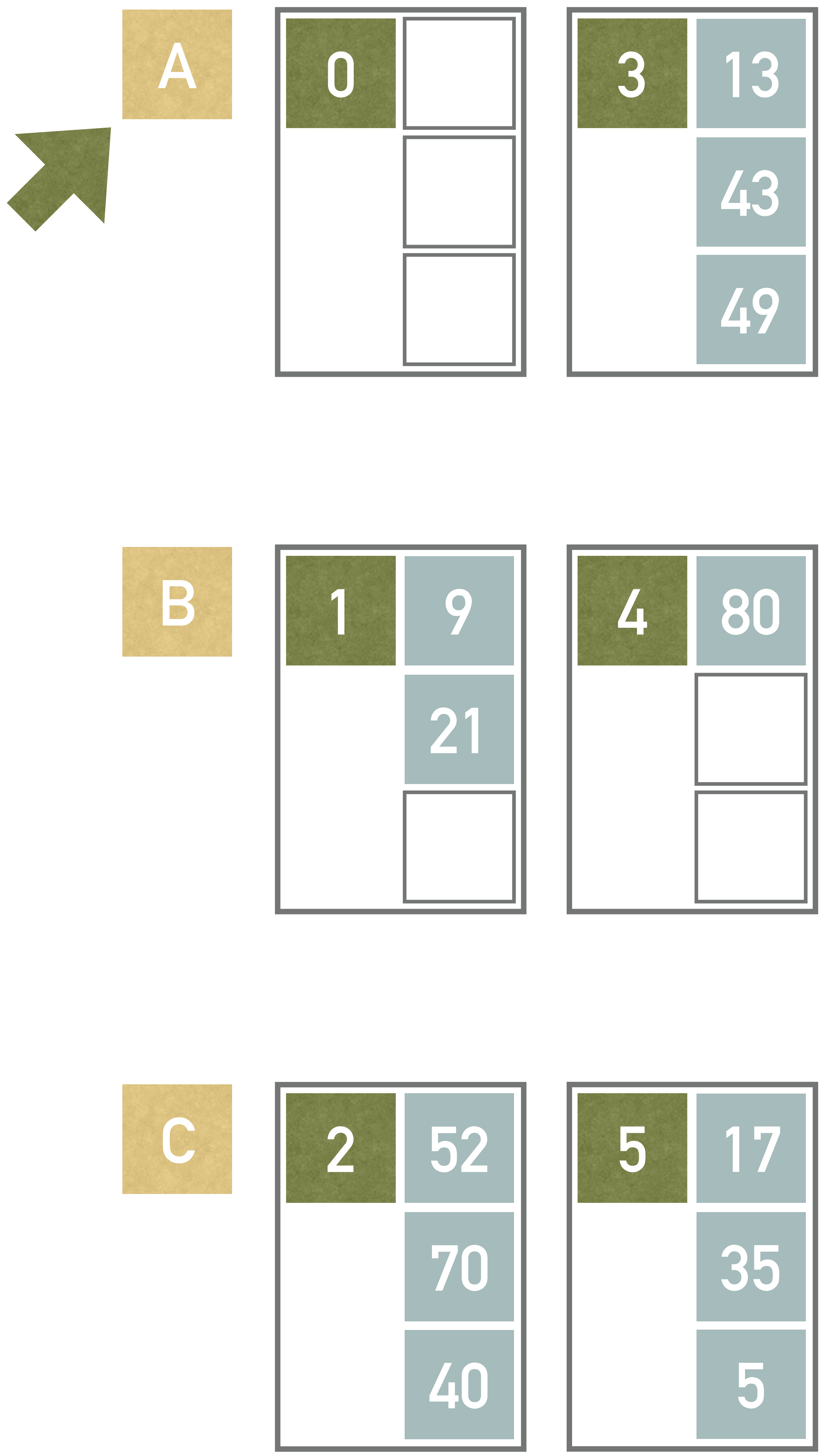
# Example 4.8 (Continued)

- Having all the group processed (i.e., split), the end of the stage $d = 0$ occurs
  - Hence, the file is reorganized into 3 groups, each having 2 pages
- The reorganization is only virtual
  - The page numbers are kept, we just think of the pages differently



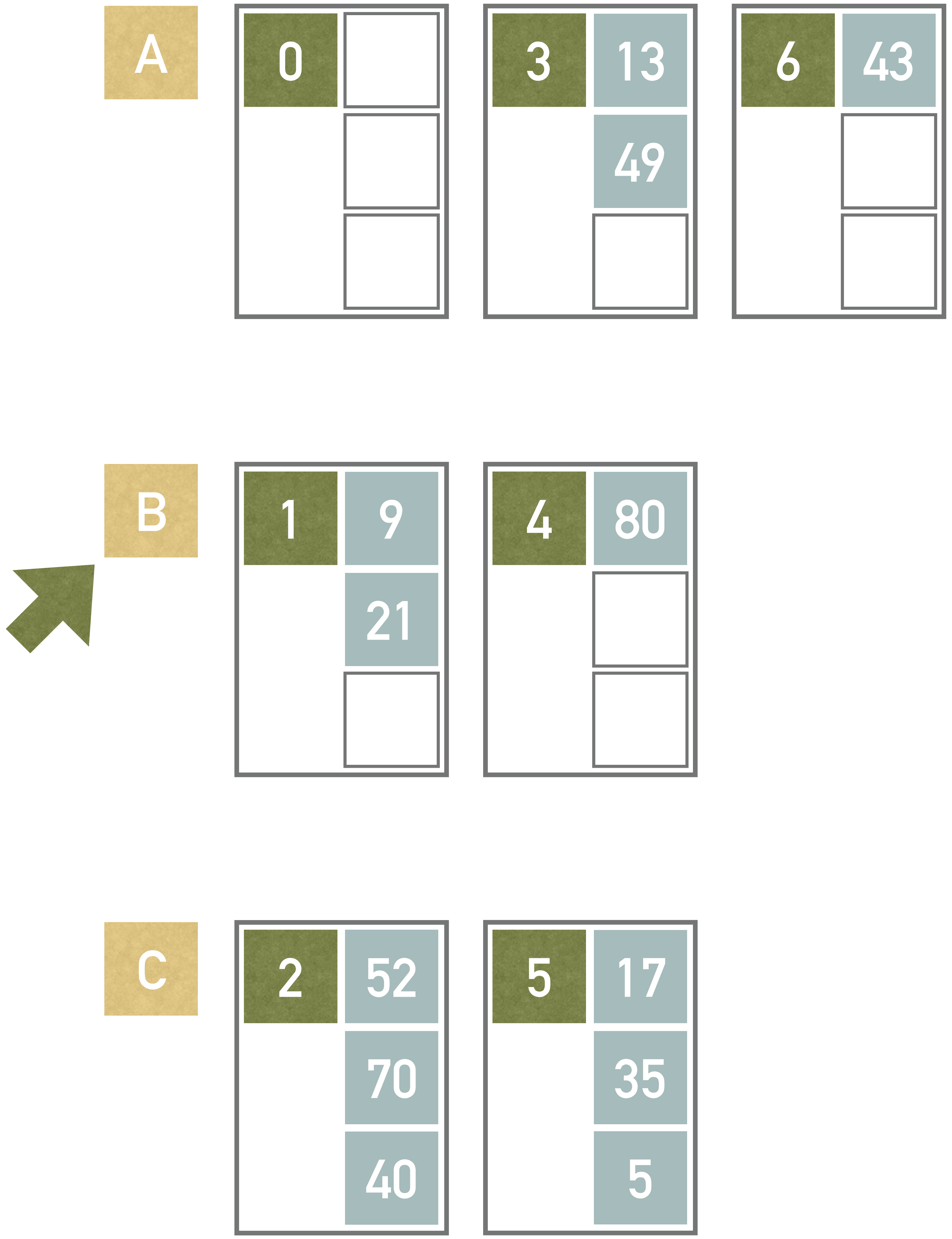
# Example 4.8 (Continued)

- Now, we insert record with key 5
  - $h_0(5) = 5 \mod 4 = 1$
  - Based on the function $h_0$, this record belongs to the page 1, which has been split once
    - Therefore we have to use $h_1$
    - $h_1(5) = 5 \mod 3 = 2$ (note that redistribution is only virtual)
  - The record comes into page 5
- Next, we insert record with key 80
  - $h_0(80) = 80 \mod 4 = 0$
  - Based on the function $h_0$, this record belongs to the page 0, which has been split once
    - Therefore we have to use $h_1$
    - $h_1(80) = 80 \mod 3 = 2$ (once again, redistribution is only virtual)



# Example 4.8 (Continued)

- Having inserted additional $L = 2$ records, we must split once again
  - The split pointer points to the group A, i.e., pages 0 and 3
  - Page 6 is added into the group A
  - Function $h_2(k)$ is applied in order to redistribute keys in the group A
    - $h_2(k)$ returns the index of a page in a group A, i.e., $h_2(k) = 0$ for the page 0, $h_2(k) = 1$ for the page 3, $h_2(k) = 2$ for the page 6
    - $h_2(43) = (43 \div 3) \mod 3 = 2$ (i.e., page 6)
    - $h_2(49) = (49 \div 3) \mod 3 = 1$ (i.e., page 3)
    - $h_2(13) = (13 \div 3) \mod 3 = 1$ (i.e., page 3)
- Finally, the split pointer is incremented



# Exercise 4.9

- Insert record with keys 37 into the structure from example 6 (see the picture)
  - Stage $d = 1$
  - Page capacity $b = 3$
  - Predefined condition $L = 2$
  - Hash functions:
    - $h_0(k) = k \mod 4$
    - $h_1(k) = k \mod 3$
    - $h_2(k) = (k \div 3) \mod 3$
- Note all the computations and illustrate the solution