R-TREES: SOLUTION

## EXERCISE 1

> Finish splitting of the overflown node

- Continue with Guttman's method
> The maximum number of items in a node is $M=8$
- The minimum number of items in a node is $m=3$
> If there are more options to choose, explain the reason of yours choice

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I |

## EXERCISE 1: SOLUTION

- Next, iteratively add such an object into a node which will maximise the difference in the node areas if the object was inserted into the first or second node

| Object | ABEF | GI | Difference |
| :---: | :---: | :---: | :---: |
|  | $6 \times 6-30=6$ | $5 \times 3-6=9$ | $\|6-9\|=3$ |
| C | $8 \times 5-30=10$ | $2 \times 8-6=10$ | $\|10-10\|=0$ |
| D | $6 \times 8-30=18$ | $7 \times 3-6=15$ | $\|18-15\|=3$ |

> The biggest difference shows the objects C and H , but we choose C so it

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I | will be inserted into the node which is closer, i.e., ABEF

- So now we have nodes ABCEF and GI


## EXERCISE 1: SOLUTION

- Next, iteratively add such an object into a node which will maximise the difference in the node areas if the object was inserted into the first or second node

- The biggest difference shows the object H so it will be inserted into the node which is closer, i.e., ABCEF
- So now we have nodes ABCEFH and GI

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I |

## EXERCISE 1: SOLUTION

> Finally, object D must be placed in the node GI because the minimum number of items per node is $m=3$ and

$$
|G I|=2 \text {, i.e., }|G I|<m
$$

- As a result, we have nodes ABCEFH and DGI

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I |

## EXERCISE 2

> Finish splitting of the overflown node
> Continue with Guttman's method
> The maximum number of items in a node is $M=8$
> This time, the minimum number of items in a node is $m=4$, i.e., $m=M / 2$
> If there are more options to choose, explain the reason of yours choice

- Compare and comment the results of exercises 1 and 2

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I |

## EXERCISE 2: SOLUTION

- Next, iteratively add such an object into a node which will maximise the difference in the node areas if the object was inserted into the first or second node

| Object | ABEF | GI | Difference |
| :---: | :---: | :---: | :---: |
|  | $6 \times 6-30=6$ | $5 \times 3-6=9$ | $\|6-9\|=3$ |
| C | $8 \times 5-30=10$ | $2 \times 8-6=10$ | $\|10-10\|=0$ |
| D | $6 \times 8-30=18$ | $7 \times 3-6=15$ | $\|18-15\|=3$ |

> The biggest difference shows the objects C and H , but we choose C so it

| A | A |  | F | F |  |  | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  |  | B | B |  |  |
|  |  |  | B | B | B |  |  |
| E | E | E |  |  |  |  |  |
| E | E | E |  |  |  |  |  |
|  |  |  | C | C | C | G |  |
|  | H |  |  |  |  | G |  |
|  | H |  |  |  |  | I | I | will be inserted into the node which is closer, i.e., ABEF

- So now we have nodes ABCEF and GI


## EXERCISE 2: SOLUTION

> Finally, objects H and D must be placed in the node GI because the minimum number of items per node is $m=4$ and $|G I|=2$, i.e., $|G I|<m$

- As a result, we have nodes ABCEF and DGHI
- There is a smaller death space in ABCEF node but for a price of a huge overlapping area, therefore it is already better to use smaller value of $m$ in this particular case



## EXERCISE 3

> Split the following overflown node with Greene's split method

- The maximum number of items in a node is $M=9$
- The minimum number of items in a node is $m=3$
> I.e., execute the following methods:
- PickSeeds

| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
|  | F |  |  |  |  |  |  | C | C |  |  |
|  | F |  |  |  | H | H |  | C |  |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
| D | D | D |  |  |  |  |  |  | B |  |  |

> ChooseAxis
> Distribute (ordering and placement)

## EXERCISE 3: SOLUTION

## > PickSeeds

- The largest dead space has DJ thus those will be the seeds of the splitting method

| Pair | Overall area | Area of the <br> objects | Dead space |
| :---: | :---: | :---: | :---: |
| AB | $9 \times 8=72$ | $5+4=9$ | $72-9=63$ |
| AC | $8 \times 5=40$ | $5+4=9$ | $40-9=31$ |
| $\ldots$ |  |  |  |
| BG | $11 \times 8=88$ | $4+2=6$ | $88-6=82$ |
| $\ldots$ |  |  |  |
| DJ | $12 \times 8=96$ | $3+1=4$ | $96-4=92$ |
| $\ldots$ |  |  |  |
| IJ | $6 \times 1=6$ | $2+1=3$ | $6-3=3$ |


| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
|  | F |  |  |  |  |  |  | C | C |  |  |
|  | F |  |  |  | H | H |  | C |  |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
| D | D | D |  |  |  |  |  |  | B |  |  |

## EXERCISE 3: SOLUTION

> ChooseAxis

- $\mathrm{x}: 8 / 12=0.667$
> $\mathrm{y}: 6 / 8=0.750$
> In our case, the axis better separating D and J is y

|  | G |  |  | A |  |  | I | I |  |  |  | J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  |  | A |  |  |  |  | C |  |  |  |
|  |  | F |  |  |  |  |  |  | C | C |  |  |
|  |  | F |  |  |  | H | H |  | C |  |  |  |
|  |  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | E | E | E |  | B | B | B |  |
|  | D | D | D |  |  |  |  |  |  | B |  |  |

## EXERCISE 3: SOLUTION

> Distribute according to axis y

| Object | D | B | E | F | H | C | A | G | I | J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Start | 0 | 0 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 7 |
| end | 0 | 1 | 1 | 4 | 3 | 5 | 7 | 7 | 7 | 7 |

- The solution:

| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
| F |  |  |  |  |  |  |  |  |  |  |  |
|  | F |  |  |  |  | H | H |  | C |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
|  | D | D | D |  |  |  |  |  |  | B |  |

> BDEFH || ACGIJ

## EXERCISE 4

> Split the following overflown node with R *Tree split
method
> The maximum number of items in a node is $M=9$

- The minimum number of items in a node is $m=3$
> I.e., execute the following methods:
> ChooseSplitAxis
> Distribute

| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
|  | F |  |  |  |  |  |  | C | C |  |  |
|  | F |  |  |  | H | H |  | C |  |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
| D | D | D |  |  |  |  |  |  | B |  |  |

- Illustrate the result


## EXERCISE 4: SOLUTION

> Ordering* based on the x-axis: GDFAEHICBJ
> margin-value $($ GDF $\|$ AEHICBJ $)=(3+8) * 2+(10+8) * 2=22+36=58$
$>$ margin-value $($ GDFA $|\mid$ EHICBJ $)=(5+8) * 2+(8+8) * 2=26+32=58$
$>$ margin-value $($ GDFAE $|\mid$ HICBJ $)=(7+8) * 2+(7+8) * 2=30+30=60$
$>$ margin-value $($ GDFAEH $|\mid$ ICBJ $)=(7+8) * 2+(6+8) * 2=30+28=58$
$>$ margin-value $($ GDFAEHI $|\mid$ CBJ $)=(8+8) * 2+(4+8) * 2=32+24=56$

- Sum $=58+58+60+58+56=290$
> Ordering* based on the y-axis: DBEFHCAGIJ
$>$ margin-value $(\mathrm{DBE}|\mid$ FHCAGIJ $)=(11+2) * 2+(12+6) * 2=26+36=62$
$>$ margin-value $($ DBEF $|\mid \mathrm{HCAGIJ})=(11+5) * 2+(12+5) * 2=32+34=66$
> margin-value $(\mathrm{DBEFH}|\mid \mathrm{CAGIJ})=(11+5) * 2+(12+5) * 2=32+34=66$
$>$ margin-value $\left(\mathrm{DBEFHC}|\mid \mathrm{AGIJ})=(11+6)^{*} 2+(12+3) * 2=34+30=64\right.$

| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
|  | F |  |  |  |  |  |  | C | C |  |  |
|  | F |  |  |  | H | H |  | C |  |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
| D | D | D |  |  |  |  |  |  | B |  |  |

DBEFHCAGIJ
AEHFBCGID
$>$ margin-value $(\mathrm{DBEFHCA}|\mid \mathrm{GIJ})=(11+8) * 2+(12+2) * 2=38+24=62$

- Sum $=62+66+66+64+62=320$


## EXERCISE 4: SOLUTION

- We chose splitting along the x -axis (smaller sum)
> overlap-value $($ GDF $|\mid$ AEHICBJ $)=8($ column AD $)$
$>$ overlap-value $($ GDFA $|\mid E H I C B J)=8($ column AE $)$
> overlap-value (GDFAE ||HICBJ) $=16$ (columns HE; IHE)
> overlap-value $($ GDFAEH $|\mid$ ICBJ $)=8$ (column IHE)
> overlap-value (GDFAEHI||CBJ) $=0$
> There is only one distribution having the smallest overlap, therefore the area-value does not have to be computed

| G |  |  | A |  |  | I | I |  |  |  | J |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G |  | A | A | A |  |  |  |  |  |  |  |
|  |  |  | A |  |  |  |  | C |  |  |  |
|  | F |  |  |  |  |  |  | C | C |  |  |
|  | F |  |  |  | H | H |  | C |  |  |  |
|  | F |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | E | E | E |  | B | B | B |  |
| D | D | D |  |  |  |  |  |  | B |  |  |

- The result is: GDFAEHI||CBJ

