

NIE-PDB: Advanced Database Systems

<http://www.ksi.mff.cuni.cz/~svoboda/courses/231-NIE-PDB/>

Lecture 12

Query Evaluation

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Lecture Outline

Algorithms

- **Access methods**
- **External sort**
- **Nested loops join**
- **Sort-merge join**
- **Hash join**

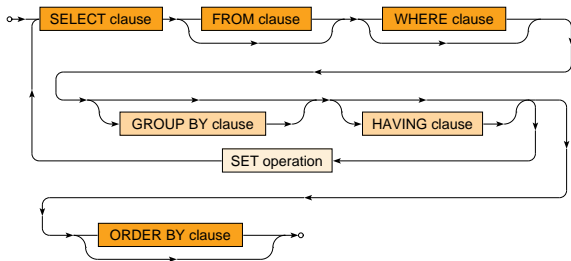
Evaluation

- **Query evaluation plans**
- **Optimization techniques**

Introduction

SQL queries

- **SELECT statements**



Introduction

Relational algebra

- Basic and inferred operations
 - Selection σ_φ , projection π_{a_1, \dots, a_n} , renaming $\rho_{b_1/a_1, \dots, b_n/a_n}$
 - Set operations: union \cup , intersection \cap , difference \setminus
 - Inner joins: cross join \times , natural join \bowtie , theta join \bowtie_φ
 - Left / right natural / theta semijoin $\ltimes, \rtimes, \ltimes_\varphi, \rtimes_\varphi$
 - Left / right natural / theta antijoin $\triangleright, \triangleleft, \triangleright_\varphi, \triangleleft_\varphi$
 - Division \div
- Extended operations
 - Left / right / full outer natural join $\ltimes, \rtimes, \ltimes, \rtimes$
 - Left / right / full outer theta join $\ltimes_\varphi, \rtimes_\varphi, \ltimes_\varphi, \rtimes_\varphi$
 - Sorting, grouping and aggregation, distinct, ...

Naïve Algorithms

Selection: $\sigma_{\varphi}(E)$

- Iteration over all tuples and removal of those filtered out

Projection: $\pi_{a_1, \dots, a_n}(E)$

- Iteration over all tuples and removal of excluded attributes
 - But also removal of duplicates within the traditional model

Distinct

- Sorting of all tuples and removal of adjacent duplicates

Inner joins: $E_R \times E_S, E_R \bowtie E_S, E_R \bowtie_{\varphi} E_S$

- Iteration over all the possible combinations via nested loops

Sorting

- Quick sort, heap sort, bubble sort, insertion sort, ...

Challenges

Blocks

- **Tuples** stored in data files are **not accessible directly**
 - Since we have **read / write operations** for **whole blocks** only
- That is true for all types of files...
 - And so not just **data files** for tables
 - But also files for **index structures** or **system catalog**

Latency

- Traditional **magnetic hard drives** are **extremely slow**
 - Efficient management of cached pages is hence essential

Memory

- Size of **available system memory** is always limited

⇒ **external algorithms** are needed

Objectives

Query evaluation plan

- Based on the **database context** and **available memory**...
... suitable **evaluation algorithms** need to be selected...
... so that the overall **evaluation cost** is **minimal**

Database context

- **Relational schema**: tables, columns, data types
- **Integrity constraints**: primary / unique / foreign keys, ...
- **Data organization**: heap / sorted / hashed file
- **Index structures**: B⁺ tree, bitmap index, hash index
- **Available statistics**: min / max values, histograms, ...

Objectives

Available system memory

- **Number of pages** allocated for the **execution of a given query**
- There are two possible scenarios...
 - Having a particular **memory size**...
 - Propose its usage and estimate the evaluation cost
 - Having a particular **cost expectation**...
 - Determine the required memory and propose its usage

Evaluation algorithms

- **Access methods**
- **Sorting:** external sort approaches
- **Joining:** nested loops, merge join, and hash join approaches
- ...

Objectives

Cost estimation

- Expressed in terms of **read / write disk operations**
 - Since hard drives are extremely slow, as already stated...
 - And so everything else can boldly be ignored
- We are interested in **estimates** only
 - Since it is unlikely we could provide accurate calculations
 - But still...
 - **The more accurate estimates, the better evaluation plans**
 - And there can really be huge differences in their efficiency...
 - Even up to **several orders of magnitude!**
- In other words...
 - **Query optimization is crucial for any database system**
 - As well as we also need to know what we are doing...

Available Statistics

Environment

- B : size of a block / page, usually $\approx 4 \text{ kB}$
- M : number of available **system memory** pages

Relation \mathcal{R}

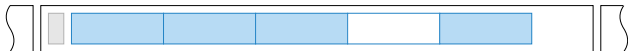
- n_R : **number of tuples**
- s_R : average / fixed tuple size
- $b_R \approx \lfloor B/s_R \rfloor$: **blocking factor**
 - Number of tuples that can be stored within one block
- $p_R \approx \lceil n_R/b_R \rceil$: **number of blocks**
- $V_{R.A}$: cardinality of the **active domain** of attribute A
 - Number of distinct values of A occurring in \mathcal{R}
- $\text{min}_{R.A}$ and $\text{max}_{R.A}$: minimal and maximal values for A

Access Methods

Data Files

Internal structure

- **Blocks** of data files for tables are **divided into slots**
 - Each slot is intended for storing exactly one tuple
 - By the way, they can easily be uniquely identified
 - Using a pair of **block and slot logical ordinal numbers**
- **Fixed-size slots**
 - Usage status of each slot just needs to be remembered



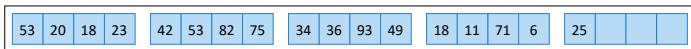
- **Variable-size slots**
 - When at least one variable-size attribute is involved
 - Slot **beginnings** and **lengths** need to be remembered



Heap File

Heap file

- Tuples are put into individual slots entirely **arbitrarily**
 - I.e., we do not have any specific knowledge of their position



Selection costs

- **Full scan** is inevitable in **almost all situations**
 - $c = p_R$
- **Equality test** with respect to a **unique attribute**
 - $c = \lceil p_R/2 \rceil$
 - Since we can stop at the moment a given tuple is found
 - However, uniform distribution of data and queries is assumed
 - And values outside of the active domain may also be queried

Sorted File

Sorted file

- Tuples are **ordered** with respect to a particular attribute

6	11	18	18	20	23	25	34	36	42	49	53	53	71	75	82	93			
---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	--	--	--

Selection costs

- **Binary search (half-interval search)** can be used in general
 - However, only when the same attribute is queried, of course
 - I.e., the same attribute as the one used for sorting
 - Otherwise, sequential read as in a heap file would be needed
- **Equality test**
 - $c = \lceil \log_2 p_R \rceil$ for a **unique** attribute
 - $c = \lceil \log_2 p_R \rceil + \lceil p_R / V_{R.A} \rceil$ for a **non-unique** attribute
- Various **range queries**

Hashed File

Hashed file

- Tuples are put into disjoint **buckets** (logical groups of blocks)
 - Based on a selected **hash function** over a particular attribute
 - E.g., $h(A) = A \bmod 3$

18	42	75	36	82	34	49	25	53	20	23	53
93	18	6						11	71		
$h(A) = 0$				$h(A) = 1$				$h(A) = 2$			

- **Hash function**
 - Its **domain** are values of a given attribute A
 - Its **range** provides H distinct values
 - There is exactly one bucket for each one of them
 - All tuples in a bucket always share the same hash value

Hashed File

File statistics

- H_R : **number of buckets**
- $C_R \approx \lceil p_R / H_R \rceil$: expected **bucket size**
 - Measured as a number of blocks in a bucket

Selection costs

- **Equality test** when the hashing attribute is queried
 - Only the corresponding bucket needs to be accessed
 - $c = C_R$ for a **non-unique** attribute
 - $c = \lceil C_R / 2 \rceil$ for a **unique** attribute
 - Similar assumptions as in the case of heap files
- Any other condition
 - $c = p_R$
 - I.e., **full scan** is needed

B⁺ Tree Index

B⁺ tree index structure = self-balanced search tree

- **Logarithmic height** is guaranteed (the same across all leaves)
- Moreover, very **high fan-out** is assumed
 - I.e., our trees will tend to be significantly **wider than taller**
 - ⇒ **search times** will not only be **logarithmic**, but also really low

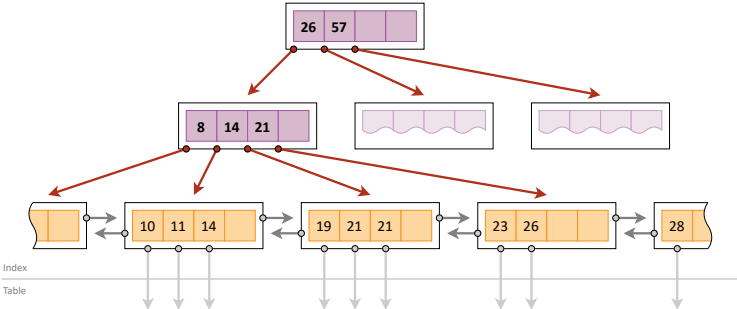
Logical structure

- **Internal node** (including a non-leaf root node)
 - Contains an ordered sequence of **dividing values** and **pointers to child nodes** representing the **sub-intervals** they determine
- **Leaf node**
 - Contains individual values and **pointers to tuples** in data file
 - Leaves are also interconnected by pointers in both directions

B⁺ Tree Index

B⁺ tree index structure (cont'd)

- Sample index for relation \mathcal{R} and its attribute A



B⁺ Tree Index

Physical structure

- **Each node** is physically represented by **one index file block**
 - And so they are treated the same way as data file blocks
 - I.e., loaded into the system memory one by one, etc.

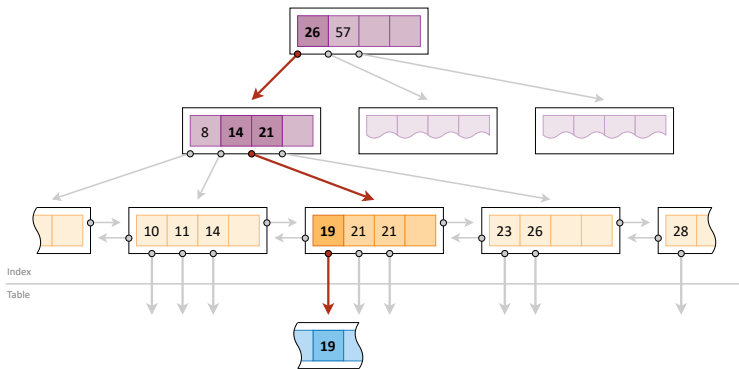
Index statistics

- $m_{R.A}$: maximal **number of children** (order of tree)
 - Usually up to small hundreds in practice
 - Actual number is guaranteed to be at least $\lceil m_{R.A}/2 \rceil$
 - Except for the root node
- $I_{R.A}$: **index height**
 - Usually just $\approx 2 - 3$ for typical real-world tables
- $p_{R.A}$: number of **leaf nodes**

B⁺ Tree Index

Search algorithm

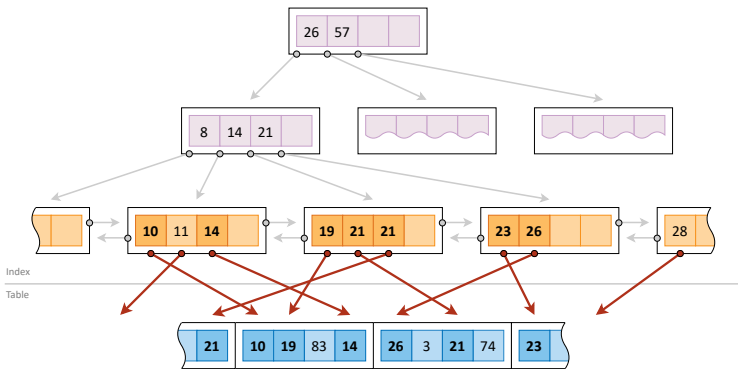
- Index is traversed from its root toward the corresponding leaf
 - Data tuple then needs to be fetched from the data file



Non-Clustered B⁺ Tree Index

Non-clustered index

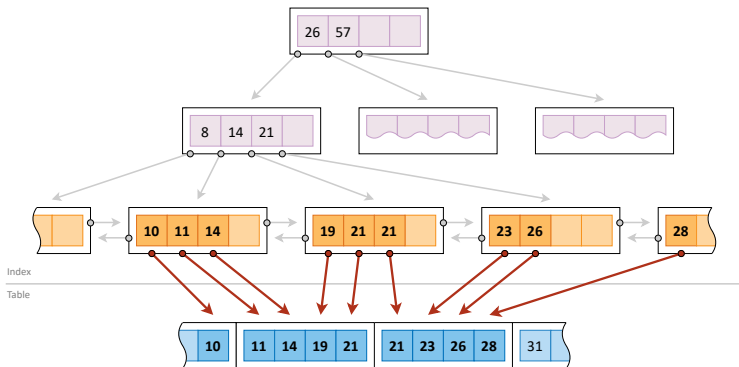
- Order of items within the leaves and data file is not the same
 - I.e., data file is organized as a **heap file of hashed file**



Clustered B⁺ Tree Index

Clustered index

- On the contrary, order of items is (at least almost) the same
 - I.e., data file is a **sorted file** (with respect to the same attribute)



Selection costs

Non-clustered B⁺ tree index

- **Equality test** for a **unique / non-unique** attribute
 - $c = I_{R.A} + 1$
 - $c = I_{R.A} + \lceil p_{R.A} / V_{R.A} \rceil + \min(p_R, \lceil n_R / V_{R.A} \rceil)$
- Various **range queries**
- ...

Clustered B⁺ tree index

- **Equality test** for a **unique / non-unique** attribute
 - $c = I_{R.A} + 1$
 - $c = I_{R.A} + \lceil p_R / V_{R.A} \rceil$
- Various **range queries**
- ...

Examples

Sample scenario #1

- **Movie** (id, title, year, ...)
 - Basic statistics
 - $n_M = 100\,000$ tuples, $b_M = 10$, $p_M = 10\,000$ blocks
 - $V_{M.id} = n_M = 100\,000$ values (since they are unique)
 - Heap file
 - Sorted file (using ids)
 - Hashed file
 - $h(M.id) = M.id \bmod 50$
 - $H_M = 50$ buckets, $C_M = 200$ blocks
 - B⁺ tree index (using ids)
 - $m_{M.id} = 100$ followers
 - $I_{M.id} = 3$, $p_{M.id} = 1\,500$ blocks

Examples

Equality test: movie with a particular identifier

- Heap file
 - $c = \lceil p_M/2 \rceil = 5\,000$
- Sorted file
 - $c = \lceil \log_2 p_M \rceil = 14$
- Hashed file
 - $c = \lceil C_M/2 \rceil = 100$
- Non-clustered index (B⁺ tree & heap file)
 - $c = I_{M.year} + 1 = 4$
- Clustered index (B⁺ tree & sorted file)
 - $c = I_{M.year} + 1 = 4$

Examples

Sample scenario #2

- **Movie** (id, title, year, ...)
 - Basic statistics
 - $n_M = 100\ 000$ tuples, $b_M = 10$, $p_M = 10\ 000$ blocks
 - $V_{M.year} = 50$ values
 - $min_{M.year} = 1943$, $max_{M.year} = 2022$ (i.e., 80 values)
 - Heap file
 - Sorted file (using years)
 - Hashed file
 - $h(M.year) = M.year \bmod 20$
 - $H_M = 20$ buckets, $C_M = 500$ blocks
 - B⁺ tree index (using years)
 - $m_{M.year} = 100$ followers
 - $I_{M.year} = 3$, $p_{M.year} = 1\ 500$ blocks

Examples

Equality test: movies filmed in a particular year

- Heap file
 - $c = p_M = 10\,000$
- Sorted file
 - $c = \lceil \log_2 p_M \rceil + \lceil p_M / V_{M.year} \rceil = 214$
- Hashed file
 - $c = C_M = 500$
- Non-clustered index (B⁺ tree & heap file)
 - $c = I_{M.year} + \lceil p_{M.year} / V_{M.year} \rceil + \min(p_M, \lceil n_M / V_{M.year} \rceil)$
 $= 2\,033$
- Clustered index (B⁺ tree & sorted file)
 - $c = I_{M.year} + \lceil p_M / V_{M.year} \rceil = 203$

External Sort

External Sort

N-way external merge sort

- **Sort phase (pass 1)**
 - Groups of input blocks are loaded into the system memory
 - Tuples in these blocks are then sorted
 - Any **in-memory in-place sorting algorithm** can be used
 - E.g.: *quick sort, heap sort, bubble sort, insertion sort, ...*
 - Created initial **runs** are written into a temporary file
- **Merge phase (passes 2 and higher)**
 - Groups of runs are loaded into the memory and merged
 - Newly created (longer) runs are written back on a hard drive
 - Merging is finished when exactly one run is obtained
 - And so the entire input table is sorted

Sort Phase

Pass 1

- Input data file
 - **Relational table \mathcal{R}**
 - E.g., $n_R = 18$ tuples, $b_R = 4$ tuples/block, $p_R = 5$ blocks

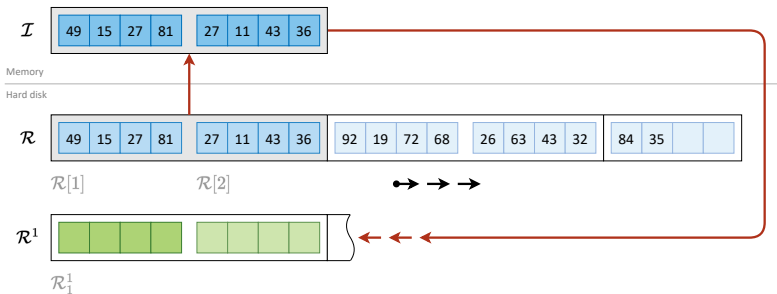
\mathcal{R}	49	15	27	81	27	11	43	36	92	19	72	68	26	63	43	32	84	35		
	$\mathcal{R}[1]$				$\mathcal{R}[2]$				$\mathcal{R}[3]$				$\mathcal{R}[4]$				$\mathcal{R}[5]$			

- System memory layout
 - **Input buffer \mathcal{I}**
 - E.g., size $M = 2$ pages

Sort Phase

Pass 1

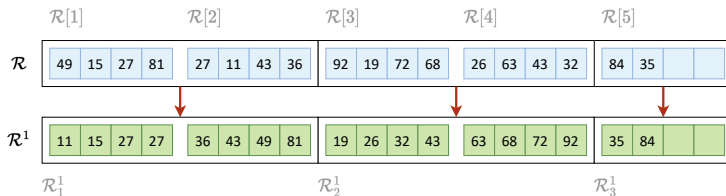
- **Groups of M blocks** are presorted and so **initial runs** created
 - Input blocks from \mathcal{R} are first loaded to \mathcal{I}
 - Individual tuples in \mathcal{I} are then sorted
 - Created runs are stored to a temporary file \mathcal{R}^1



Sort Phase

Pass 1

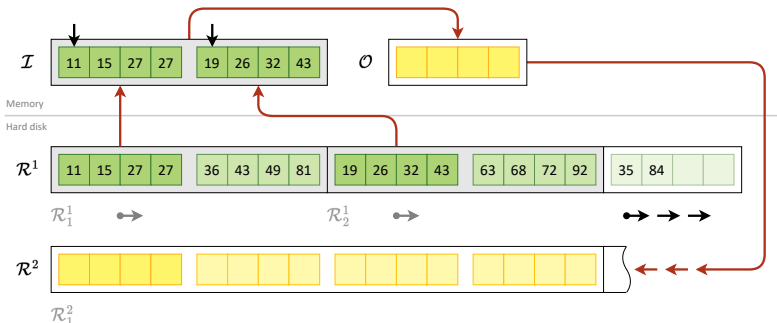
- Resulting runs in \mathcal{R}^1 within our sample scenario



Merge Phase

Pass 2

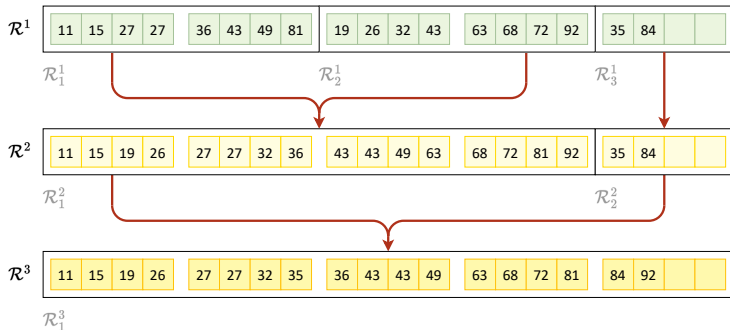
- **Groups of M runs are iteratively merged together**
 - Blocks from these input runs are gradually loaded into \mathcal{I}
 - Minimal items are then iteratively selected and moved to \mathcal{O}
 - Merged (longer) runs are written to a new temporary file \mathcal{R}^2



Merge Phase

Passes 2 and 3

- Merging continues until just a single run is acquired
 - And so the entire input table is sorted



Algorithm

Sort phase (pass 1)

- 1 $p \leftarrow 1$
 - 2 **foreach** group of blocks B_1, \dots, B_M (if any) from \mathcal{R} **do**
 - 3 read these blocks to \mathcal{I}
 - 4 sort all items in \mathcal{I}
 - 5 write all blocks from \mathcal{I} as a new run to \mathcal{R}^p
-

Algorithm

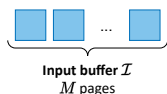
Merge phase (passes 2 and higher)

```
6 while  $\mathcal{R}^p$  has more than just one run do
7    $p \leftarrow p + 1$ 
8   foreach group of runs  $R_1, \dots, R_M$  (if any) from  $\mathcal{R}^{p-1}$  do
9     start constructing a new run in  $\mathcal{R}^p$ 
10    read the first block from each run  $R_x$  to  $\mathcal{I}[x]$ 
11    while  $\mathcal{I}$  contains at least one item do
12      select the minimal item and move it to  $\mathcal{O}$ 
13      if the corresponding  $\mathcal{I}[x]$  is empty then
14        read the next block from  $R_x$  (if any) to  $\mathcal{I}[x]$ 
15      if  $\mathcal{O}$  is full then write  $\mathcal{O}$  to  $\mathcal{R}^p$  and empty  $\mathcal{O}$ 
16    if  $\mathcal{O}$  is not empty then write  $\mathcal{O}$  to  $\mathcal{R}^p$  and empty  $\mathcal{O}$ 
```

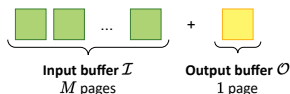
Summary

Memory layout

- Sort phase (**pass 1**): M
 - **Input buffer \mathcal{I}** : M pages



- Merge phase (**passes 2 and higher**): $M + 1$
 - **Input buffer \mathcal{I}** : $M \geq 2$ pages
 - **Output buffer \mathcal{O}** : 1 page



Summary

Time complexity

- **Single pass** (regardless of the phase)
 - $c_{\text{read}} = c_{\text{write}} = p_R$
- **Number of passes**
 - $t = \lceil \log_M(p_R) \rceil$
- **Overall cost**
 - $c_{\text{ES}} = t \cdot (c_{\text{read}} + c_{\text{write}}) = \lceil \log_M(p_R) \rceil \cdot 2p_R$

Limitation of the overall number of passes

- In general...
 - $M = \lceil \sqrt[t]{p_R} \rceil$
- Specifically for $t = 2$ (i.e., **exactly 2 passes**)
 - $M = \lceil \sqrt{p_R} \rceil$

Nested Loops Join

Nested Loops

Binary nested loops

- **Universal approach** for all types of **inner joins**
 - Natural join, cross join, theta join
 - I.e., arbitrary joining condition can be involved
 - Support possible duplicates
 - Requires no index structures
- Not the best option in all situations, though
 - Suitable for tables with significantly different sizes

Basic idea

- **Outer loop**: iteration over the blocks of the **first** table
- **Inner loop**: iteration over the blocks of the **second** table

Nested Loops

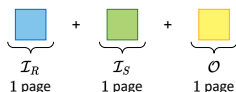
Sample input data

- Tables \mathcal{R} and \mathcal{S} to be joined using a **value equality** test

\mathcal{R}	21	84	56	19	41	72	69	35	56	84										
\mathcal{S}	31	56	75	43	88	21	43	14	92	52	25	81	72	37	64	35	14	64		

Basic setup

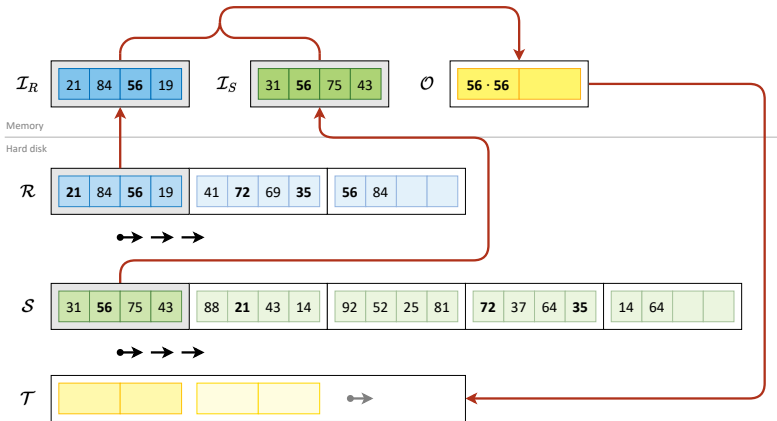
- Memory layout: $1 + 1 + 1$
 - **Input buffer** \mathcal{I}_R : 1 page
 - **Input buffer** \mathcal{I}_S : 1 page
 - **Output buffer** \mathcal{O} : 1 page



Nested Loops

Basic setup (1 + 1 + 1)

- Another pair of loops is used for joining tuples in the memory



Algorithm

Basic setup (1 + 1 + 1)

```
1 foreach block  $R$  from  $\mathcal{R}$  do
2   read  $R$  into  $\mathcal{I}_R$ 
3   foreach block  $S$  from  $\mathcal{S}$  do
4     read  $S$  into  $\mathcal{I}_S$ 
5     foreach item  $r$  in  $\mathcal{I}_R$  do
6       foreach item  $s$  in  $\mathcal{I}_S$  do
7         if  $r$  and  $s$  satisfy the join condition then
8           join  $r$  and  $s$  and put the result to  $\mathcal{O}$ 
9           if  $\mathcal{O}$  is full then write  $\mathcal{O}$  to  $\mathcal{T}$ , empty  $\mathcal{O}$ 
10 if  $\mathcal{O}$  is not empty then write  $\mathcal{O}$  to  $\mathcal{T}$  and empty  $\mathcal{O}$ 
```

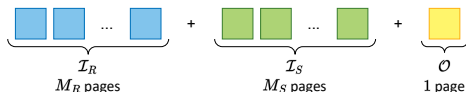
Observations

Time complexity

- Basic setup ($1 + 1 + 1$)
 - $c_{NL} = p_R + p_R \cdot p_S$
- \Rightarrow **smaller table** should always be taken as the outer one

General setup

- Multiple pages are used for both the input buffers
- Memory layout: $M_R + M_S + 1$
 - **Input buffer** \mathcal{I}_R : M_R pages
 - **Input buffer** \mathcal{I}_S : M_S pages
 - **Output buffer** \mathcal{O} : 1 page



Algorithm

General setup ($M_R + M_S + 1$)

```
1 foreach group of blocks  $R_1, \dots, R_{M_R}$  (if any) from  $\mathcal{R}$  do
2   read these blocks into  $\mathcal{I}_R$ 
3   foreach group of blocks  $S_1, \dots, S_{M_S}$  (if any) from  $\mathcal{S}$  do
4     read these blocks into  $\mathcal{I}_S$ 
5     foreach item  $r$  in  $\mathcal{I}_R$  do
6       foreach item  $s$  in  $\mathcal{I}_S$  do
7         if  $r$  and  $s$  satisfy the join condition then
8           join  $r$  and  $s$  and put the result to  $\mathcal{O}$ 
9           if  $\mathcal{O}$  is full then write  $\mathcal{O}$  to  $\mathcal{T}$ , empty  $\mathcal{O}$ 
10  if  $\mathcal{O}$  is not empty then write  $\mathcal{O}$  to  $\mathcal{T}$  and empty  $\mathcal{O}$ 
```

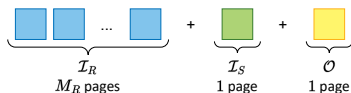
Observations

Time complexity

- General setup ($M_R + M_S + 1$)
 - $c_{NL} = p_R + \lceil p_R/M_R \rceil \cdot p_S$
- \Rightarrow there is no reason of having $M_S \geq 2$

Standard setup

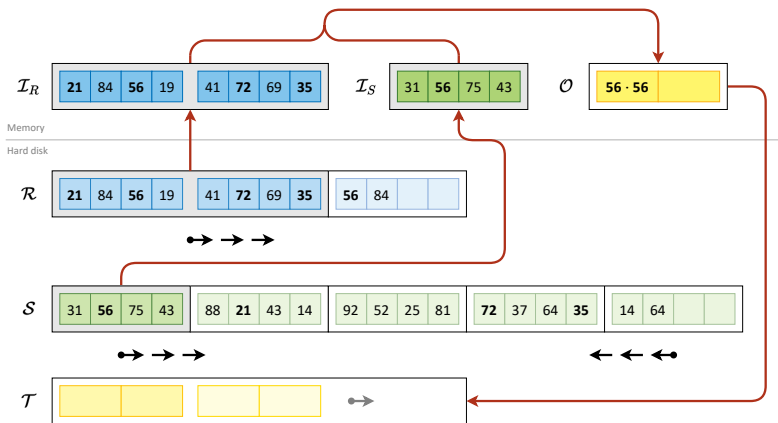
- Memory layout: $M_R + 1 + 1$
 - **Input buffer** \mathcal{I}_R : M_R pages
 - **Input buffer** \mathcal{I}_S : 1 page
 - **Output buffer** \mathcal{O} : 1 page



Standard Approach

Standard setup ($M_R + 1 + 1$) with **zig-zag** optimization

- Multiple pages are used just for the outer table



Observations

Zig-zag optimization

- Reading of the inner table \mathcal{S}
 - **Odd iterations** normally
 - **Even iterations** in reverse order

Time complexity

- Standard setup ($M_R + 1 + 1$)
 - $c_{NL} = p_R + \lceil p_R/M_R \rceil \cdot p_S$ (without zig-zag)
 - $c_{NL} = p_R + \lceil p_R/M_R \rceil \cdot (p_S - 1) + 1$ (with zig-zag)

Special cases

- Smaller table fits entirely within the memory, i.e., $p_R \leq M_R$
 - $c_{NL} = p_R + p_S$
- **Non-brute-force** replacement for **inner loops**
 - When a suitable index exists on the inner table, ...

Sort-Merge Join

Sort-Merge Join

Sort-merge join algorithm (or just merge join)

- Inner joins based on **value equality tests** only
 - Basic version **without duplicates**
 - Could be extended to support them, though
- Suitable for tables with relatively similar sizes
 - Especially when they are already sorted
 - Or when the final result is expected to be sorted

Phases

- **Sort phase**
 - Both tables are externally sorted, one by one (if not yet)
- **Join phase**
 - Items are joined while simulating the merge of the two tables

Basic Approach

Sample input data

- **Input tables** \mathcal{R} and \mathcal{S}

\mathcal{R}

65	19	35	92	49	31		
----	----	----	----	----	----	--	--

\mathcal{S}

52	94	38	71	92	41	63	19	75	54	46	68	15	27	22	43	11	50	49	
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	--

Sort phase

- **Resulting sorted tables**

\mathcal{R}'

19	31	35	49	65	92		
----	----	----	----	----	----	--	--

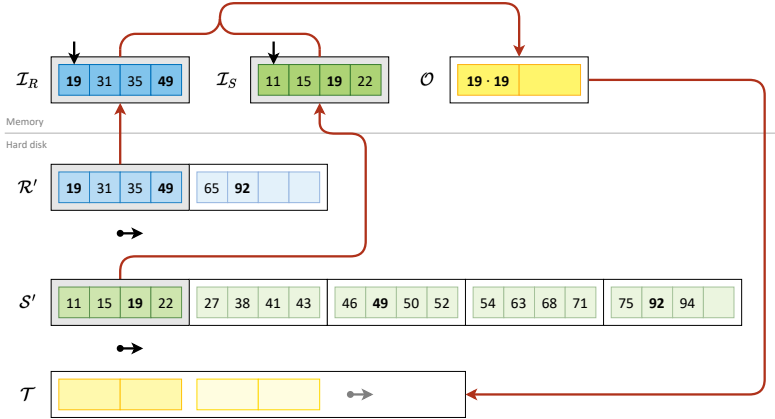
\mathcal{S}'

11	15	19	22	27	38	41	43	46	49	50	52	54	63	68	71	75	92	94	
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	--

Basic Approach

Join phase

- Blocks from the sorted tables are processed one by one



Algorithm

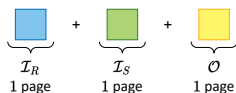
Join phase

- 1 read block $\mathcal{R}'[1]$ to \mathcal{I}_R and block $\mathcal{S}'[1]$ to \mathcal{I}_S
 - 2 **while** both \mathcal{I}_R and \mathcal{I}_S contain at least one item **do**
 - 3 let r be the minimal item in \mathcal{I}_R and s minimal item in \mathcal{I}_S
 - 4 **if** r and s can be joined **then**
 - 5 join r and s and put the result to \mathcal{O}
 - 6 **if** \mathcal{O} is full **then** write \mathcal{O} to \mathcal{T} and empty \mathcal{O}
 - 7 remove both r from \mathcal{I}_R and s from \mathcal{I}_S
 - 8 **else** remove the lower one of r from \mathcal{I}_R or s from \mathcal{I}_S
 - 9 **if** \mathcal{I}_R is empty **then** read the next block from \mathcal{R}' (if any)
 - 10 **if** \mathcal{I}_S is empty **then** read the next block from \mathcal{S}' (if any)
 - 11 **if** \mathcal{O} is not empty **then** write \mathcal{O} to \mathcal{T} and empty \mathcal{O}
-

Observations

Join phase

- Memory layout: $1 + 1 + 1$
 - **Input buffer** \mathcal{I}_R : 1 page
 - **Input buffer** \mathcal{I}_S : 1 page
 - **Output buffer** \mathcal{O} : 1 page



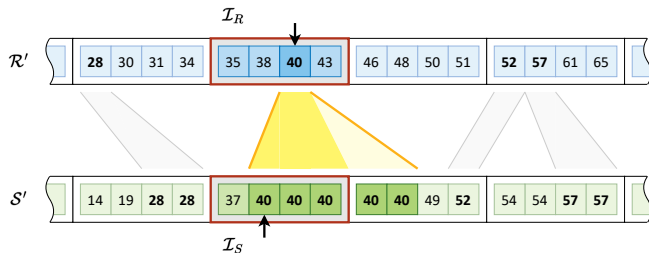
Time complexity

- Sort phase
- **Join phase**
 - $c_{MJ} = p_R + p_S$

Extended Version

Duplicate items

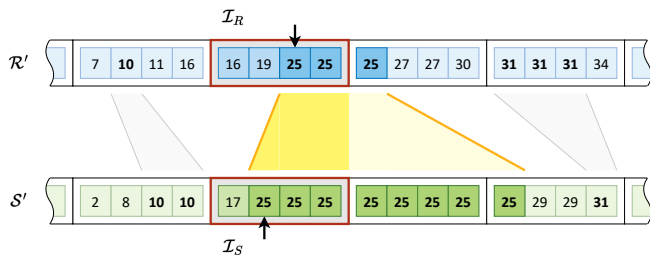
- Possible duplicates in **one table** only
 - Let it be \mathcal{S} (without loss of generality)
 - Algorithm modification is straightforward...
 - Having successfully joined r and s , we just remove s from \mathcal{I}_S and not r from \mathcal{I}_R (line 7)



Extended Version

Duplicate items

- Possible duplicates in **both tables**
 - All matching pairs of r and s just need to be joined...
 - Unfortunately, **size of input buffers** might not be sufficient
 - Since we may span block boundaries, even repeatedly



Hash Join

Hash Join

Hash join approaches

- Basic principle
 - Items of the first table are hashed into the system memory
 - Items of the second table are then attempted to be joined
- Limitations
 - Inner joins based on **value equality tests** only
 - Including possible duplicates
 - Not suitable for small **active domains**
- Particular approaches
 - **Classic** hash join, **Simple** hash join, **Partition** hash join, **Grace** hash join, and **Hybrid** hash join

Classic Hashing

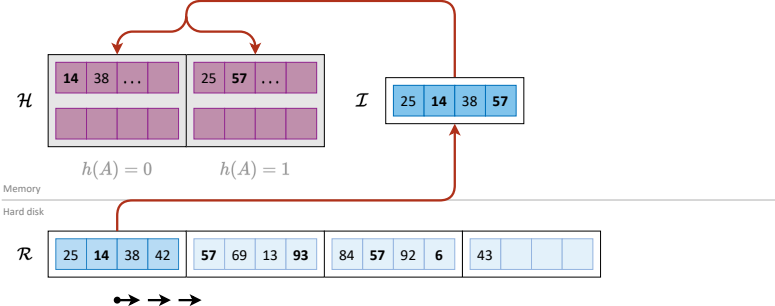
Classic hash join

- **Build phase**
 - **Smaller table** (let it be \mathcal{R}) is **hashed into the system memory**
 - I.e., it is sequentially loaded into the memory, block by block
 - All its tuples are then emplaced into the **hash container**
- **Hash function** h is assumed for this purpose
 - Its **domain** are values of the joining attribute A
 - Its **range** provides H distinct values
- **Hash container** internally contains H **buckets**
 - Its **overall size** will inevitably be somewhat larger than p_R
 - Let us say $M = \lceil F \cdot p_R \rceil$ pages for some small factor F
- **Probe phase**
 - Items from the **larger table** \mathcal{S} are **attempted to be joined**

Build Phase

Build phase

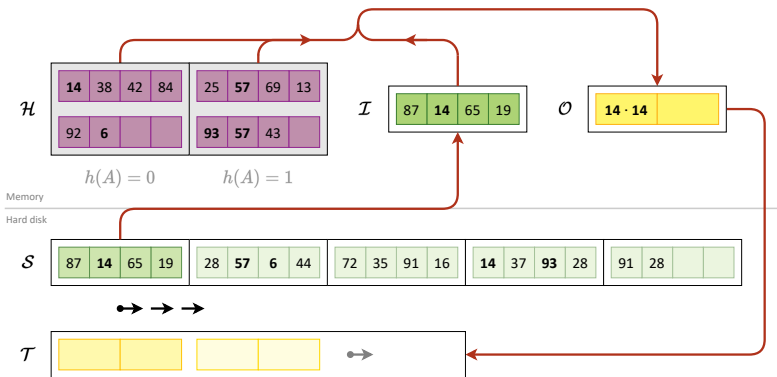
- Tuples from the smaller table are **hashed** into the memory
 - E.g., hash function $h(A) = A \bmod 2$ is assumed



Probe Phase

Probe phase

- Tuples from the larger table are attempted to be **joined**



Algorithm

Build phase

```
1 foreach block  $R$  from  $\mathcal{R}$  do  
2   read  $R$  into  $\mathcal{I}$   
3   foreach item  $r$  in  $\mathcal{I}$  do  
4     calculate hash value  $h \leftarrow h(r.A)$   
5     add  $r$  into bucket  $h$  in  $\mathcal{H}$ 
```

Algorithm

Probe phase

```
1 foreach block  $S$  from  $\mathcal{S}$  do
2   read  $S$  into  $\mathcal{I}$ 
3   foreach item  $s$  in  $\mathcal{I}$  do
4     calculate hash value  $h \leftarrow h(s.A)$ 
5     foreach item  $r$  in bucket  $h$  in  $\mathcal{H}$  do
6       if  $r$  and  $s$  can be joined then
7         join  $r$  and  $s$  and put the result to  $\mathcal{O}$ 
8         if  $\mathcal{O}$  is full then write  $\mathcal{O}$  to  $\mathcal{T}$  and empty  $\mathcal{O}$ 
9 if  $\mathcal{O}$  is not empty then write  $\mathcal{O}$  to  $\mathcal{T}$  and empty  $\mathcal{O}$ 
```

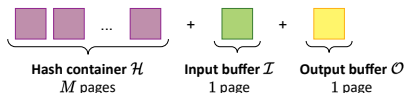
Observations

Memory layout

- Build phase: $M + 1$
 - **Hash container \mathcal{H}** : $M = \lceil F \cdot p_R \rceil$ pages
 - **Input buffer \mathcal{I}** : 1 page



- Probe phase: $M + 1 + 1$
 - **Hash container \mathcal{H}** : M pages (preserved from the build phase)
 - **Input buffer \mathcal{I}** : 1 page
 - **Output buffer \mathcal{O}** : 1 page



Observations

Time complexity

- Build and probe phases
 - $c_{\text{build}} = pR$
 - $c_{\text{probe}} = pS$
- **Overall cost**
 - $c_{\text{CH}} = c_{\text{build}} + c_{\text{probe}} = pR + pS$

Summary

- Interesting approach as for its efficiency
 - However, usable only when the smaller table can entirely be hashed into the system memory...

Partition Hashing

Partition hash join

- Basic principle
 - **Both tables are first partitioned**
 - Using partition function p
 - **Pairs of the corresponding partitions** are then **joined** together
 - Using the classic hash join approach
 - Or actually even nested loops if desired

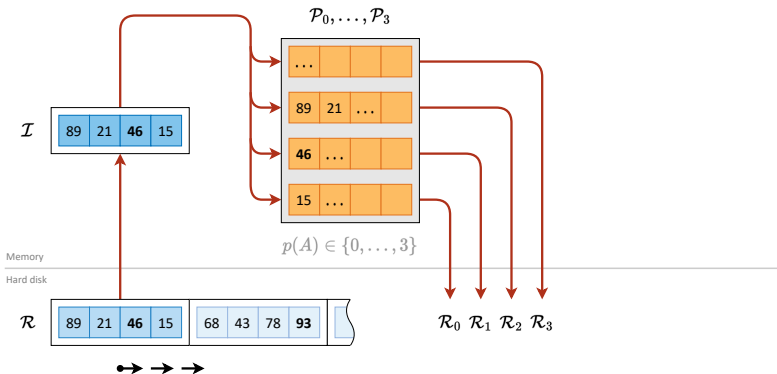
Overall procedure

-
- 1 split \mathcal{R} and create partitions $\mathcal{R}_0, \dots, \mathcal{R}_P$
 - 2 split \mathcal{S} and create partitions $\mathcal{S}_0, \dots, \mathcal{S}_P$
 - 3 **foreach** partition $p \in \{0, \dots, P - 1\}$ **do**
 - 4 └ join partitions \mathcal{R}_p and \mathcal{S}_p
-

Partition Phase

Partition phase (for table \mathcal{R})

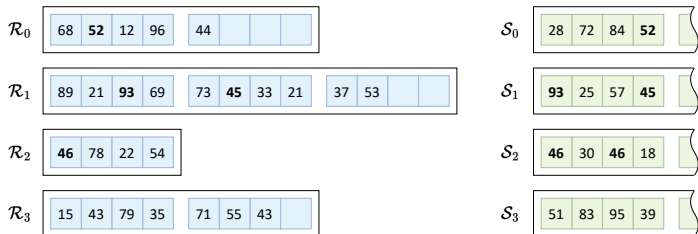
- Tuples of a given table are split to **disjoint partitions**



Join Phase

Partition phase

- Resulting partitions for our sample scenario



Join phase

- Pairs of the corresponding partitions are then joined together
 - \mathcal{R}_0 and \mathcal{S}_0 , \mathcal{R}_1 and \mathcal{S}_1 , ...

Algorithm

Partition phase

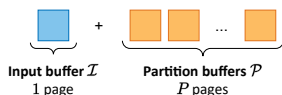
- Table \mathcal{R} is assumed, partitioning of \mathcal{S} is analogous

```
1 foreach block  $R$  from  $\mathcal{R}$  do
2   read  $R$  into  $\mathcal{I}$ 
3   foreach item  $r$  in  $\mathcal{I}$  do
4     calculate partition value  $p \leftarrow p(r.A)$ 
5     add  $r$  into partition buffer  $\mathcal{P}_p$ 
6     if  $\mathcal{P}_p$  is full then write  $\mathcal{P}_p$  to  $\mathcal{R}_p$  and empty  $\mathcal{P}_p$ 
7 foreach partition  $p \in \{0, \dots, P-1\}$  do
8   if  $\mathcal{P}_p$  is not empty then write  $\mathcal{P}_p$  to  $\mathcal{R}_p$  and empty  $\mathcal{P}_p$ 
```

Observations

Memory layout

- Partition phase: $1 + P$
 - **Input buffer \mathcal{I}** : 1 page
 - **Partition buffers \mathcal{P}** : P pages



Time complexity

- **Partitioning phase**
 - $c_{\text{split}} \approx 2 \cdot p_R + 2 \cdot p_S$
- **Overall cost** (with classic hash join involved)
 - $c_{\text{PH}} = c_{\text{split}} + P \cdot c_{\text{CH}} \approx c_{\text{split}} + P \left[\frac{p_R}{P} + \frac{p_S}{P} \right] \approx 3 \cdot (p_R + p_S)$

Query Evaluation

Sample Query

Database schema

- **Movie** (id, title, year, ...)
- **Actor** (movie, actor, character, ...)
 - FK: Actor[movie] \subseteq Movie[id]

Sample query

- **Actors and characters they played in movies filmed in 2000**

- SQL expression

```
SELECT title, actor, character
FROM Movie JOIN Actor
WHERE (year = 2000) AND (id = movie)
```

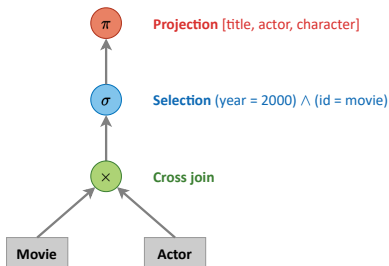
- RA expression

$$\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000) \wedge (\text{id}=\text{movie})} (\text{Movie} \times \text{Actor}) \right)$$

Sample Query

Sample query (cont'd)

- **Actors and characters they played in movies filmed in 2000**
 - $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000) \wedge (\text{id}=\text{movie})} (\text{Movie} \times \text{Actor}) \right)$



Query Evaluation

Basic idea

- SQL query → RA query → evaluation plan → query result

Evaluation process

- (1) **Scanning** [scanner]
 - **Lexical analysis** is performed over the input SQL expression
 - Lexemes are recognized and then tokens generated
- (2) **Parsing** [parser]
 - **Syntactic analysis** is performed
 - Derivation tree is constructed according to the SQL grammar
- (3) **Translation**
 - **Query tree with relational algebra operations** is constructed

Query Evaluation

Evaluation process (cont'd)

- (4) **Validation** [validator]
 - **Semantic validity** is checked
 - Compliance of relation schemas with intended operations
- (5) **Optimization** [optimizer]
 - **Alternative evaluation plans** are devised and compared
 - In order to find the most efficient plan
 - Based on their evaluation **cost estimates**
- (6) **Code generation** [generator]
 - Execution code is generated for the chosen plan
- (7) **Execution** [processor]
 - Intended query is finally evaluated
 - And the yielded result provided to the user

Query Evaluation

Query tree

- Internal tree structure
 - **Leaf nodes** = input **tables**
 - **Inner nodes** = individual RA **operations** (σ , π , \times , \bowtie , ...)
- Root node represents the entire query
 - Nodes are evaluated from leaves toward the root

Query evaluation plan

- **Query tree**
- For each inner node...
 - Calculated **statistics** (number of tuples, blocking factor, ...)
 - Selected **algorithm** (limited by **context** and available **memory**)
 - Estimated **cost**
- **Overall cost**

Sample Plan #1

Cross join

$$n_1 = n_M \cdot n_A = 100\,000\,000\,000$$

$$b_1 = (b_M \cdot b_A) / (b_M + b_A) = 8$$

$$p_1 = n_1 / b_1 = 12\,500\,000\,000$$

Nested loops

$$M_1 = 25 + 1 + 1 = 27$$

$$c_1^r = p_M + (p_M/25) \cdot p_A = 10\,010\,000$$

$$c_1^w = p_1 = 12\,500\,000\,000$$

Sorted file (year)

$$n_M = 100\,000$$

$$b_M = 10$$

$$p_M = 10\,000$$

$$V_{M,\text{year}} = 50$$

B⁺ tree index (year)

$$m_{M,\text{year}} = 100$$

$$I_{M,\text{year}} = 3$$

Projection [title, actor, character]

$$n_3 = n_2 = 20\,000$$

$$b_3 \leftarrow 50$$

$$p_3 = n_3 / b_3 = 400$$

$$c_3^r = p_2 = 2\,500$$

$$c_3^w = p_3 = 400$$

Selection (year = 2000) \wedge (id = movie)

$$n_2 = n_1 \cdot (1/V_{M,\text{year}}) \cdot (1/n_M) = 20\,000$$

$$b_2 = b_1 = 8$$

$$p_2 = n_2 / b_2 = 2\,500$$

$$c_2^r = p_1 = 12\,500\,000\,000$$

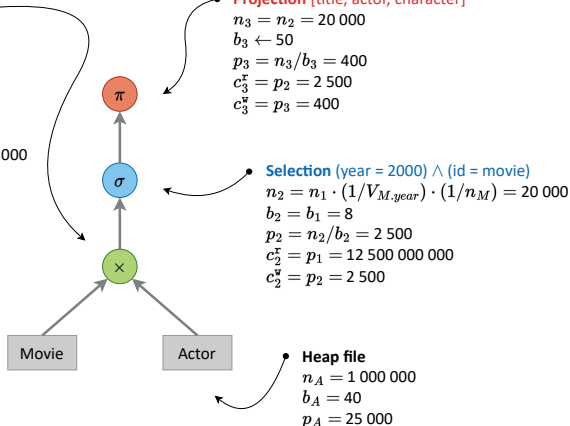
$$c_2^w = p_2 = 2\,500$$

Heap file

$$n_A = 1\,000\,000$$

$$b_A = 40$$

$$p_A = 25\,000$$



Evaluation Plan Cost

Overall evaluation cost

- Let us first assume that all **intermediate results are always written to temporary files** and so each involved operation...
 - Reads its inputs from / writes its output to a hard drive
- **Overall cost** then equals to the **sum of all the partial costs**

Cost of Plan #1

- $M = 25 + 1 + 1$ memory pages
- $c = [c_1^r + c_1^w] + [c_2^r + c_2^w] + [c_3^r]$
- $c = [p_M + (p_M/25) \cdot p_A + p_1] + [p_1 + p_2] + [p_2]$
- $c = [10\ 010\ 000 + 12\ 500\ 000\ 000] + [12\ 500\ 000\ 000 + 2\ 500] + [2\ 500]$
- $c = 25\ 010\ 015\ 000$

Sample Query

Intuitive optimization

- **Actors and characters they played in movies filmed in 2000**

- SQL expression

```
SELECT title, actor, character
FROM Movie JOIN Actor ON (id = movie)
WHERE (year = 2000)
```

- RA expression

$$\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000)} \left(\text{Movie} \bowtie_{(\text{id}=\text{movie})} \text{Actor} \right) \right)$$

Sample Plan #2

Theta join [id = movie]

$$n_1 = n_A = 1\,000\,000$$

$$b_1 = (b_M \cdot b_A) / (b_M + b_A) = 8$$

$$p_1 = n_1 / b_1 = 125\,000$$

Nested loops

$$M_1 = 25 + 1 + 1 = 27$$

$$c_1^r = p_M + (p_M / 25) \cdot p_A = 10\,010\,000$$

$$c_1^w = p_1 = 125\,000$$

Sorted file (year)

$$n_M = 100\,000$$

$$b_M = 10$$

$$p_M = 10\,000$$

$$V_{M.year} = 50$$

B⁺ tree index (year)

$$m_{M.year} = 100$$

$$I_{M.year} = 3$$

Projection [title, actor, character]

$$n_3 = n_2 = 20\,000$$

$$b_3 \leftarrow 50$$

$$p_3 = n_3 / b_3 = 400$$

$$c_3^r = p_2 = 2\,500$$

$$c_3^w = p_3 = 400$$

Selection (year = 2000)

$$n_2 = n_1 \cdot (1 / V_{M.year}) = 20\,000$$

$$b_2 = b_1 = 8$$

$$p_2 = n_2 / b_2 = 2\,500$$

$$c_2^r = p_1 = 125\,000$$

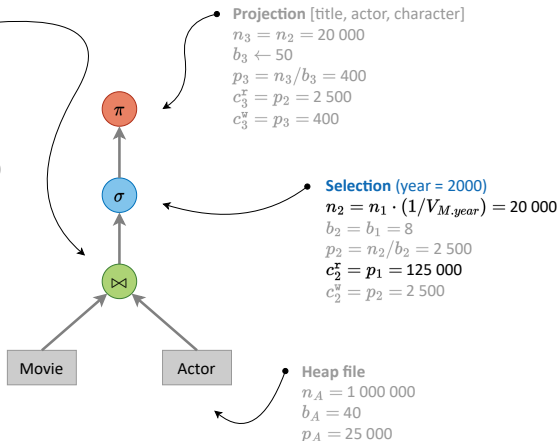
$$c_2^w = p_2 = 2\,500$$

Heap file

$$n_A = 1\,000\,000$$

$$b_A = 40$$

$$p_A = 25\,000$$



Sample Plan #2

Cost of Plan #2

- Again $M = 25 + 1 + 1$ memory pages
- $c = [c_1^r + c_1^w] + [c_2^r + c_2^w] + [c_3^r]$
- $c = [p_M + (p_M/25) \cdot p_A + p_1] + [p_1 + p_2] + [p_2]$
- $c = [10\,010\,000 + 125\,000] + [125\,000 + 2\,500] + [2\,500]$
- $c = 10\,265\,000$
 - That is approximately 2 400 times better than the first plan

Pipelining

Pipelining mechanism

- **Intermediate results are passed** between the operations **directly without** the usage of **temporary files** on a disk
 - And so just within the system memory
 - It may even be possible to do it **in-place** without extra pages
- Unfortunately, such an approach is not always possible...

Cost of **Plan #2 with pipelining**

- Still $M = 25 + 1 + 1$ memory pages
- $c = [c_1^r + \cancel{c_1^w}] + [\cancel{c_2^r} + \cancel{c_2^w}] + [\cancel{c_3^r}]$
 - Joined tuples are filtered and projected immediately in-place
- $c = 10\ 010\ 000$

Query Optimization

Objective = finding the most **optimal query evaluation plan**

- It is not possible to consider all plans, though
 - Simply because there are far too many of them
 - And so **pruning and heuristics** need to be incorporated

Optimization strategies

- **Algebraic**
 - Proposal of **alternative plans** using query tree transformations
- **Statistical**
 - Estimation of **costs and result sizes** based on available statistics
- **Syntactic**
 - Manual modification of query expressions by users themselves
 - In order to involve plans that would otherwise be unreachable
 - Breaches the principle of **declarative querying**, though

Statistical Optimization

Objective

- Capability of **calculating necessary result characteristics...**
 - Of both the final result as well as all intermediate ones
 - I.e., all individual nodes within a given evaluation plan tree
- ... so that the **overall cost** can be estimated
 - And thus alternative plans mutually compared

Basic statistics

- **Data file** for table \mathcal{R}
 - n_R number of tuples, s_R tuple size, b_R blocking factor
 - p_R number of pages
 - Hashed file: H_R number of buckets, C_R bucket size
- **Index file** for attribute A from table \mathcal{R}
 - B^+ tree: $I_{R.A}$ tree height, $p_{R.A}$ number of leaf nodes

Statistical Optimization

Additional statistics

- Provide deeper insight into the **active domain**
 - May even be implicitly **derivable** from index structures
 - Unfortunately, they may also be **missing or unavailable**
 - Especially as for intermediate results
- $V_{R.A}$ number of distinct values
- $min_{R.A}$ and $max_{R.A}$ minimal and maximal values
- **Histograms**
 - Provide even more accurate understanding of the domain
 - And so better estimates
 - Especially useful for **non-uniform distributions**

Size Estimates: Selection

Selection: $T = \sigma_{\varphi}(E)$

Tuple size

- $s_T = s_E$
 - Tuples are just filtered out and so their size remains untouched

Blocking factor

- $b_T = b_E$

Number of tuples

- Basic idea: $n_T = \lceil n_E \cdot r_{\varphi} \rceil$
- $r_{\varphi} \in [0, 1]$ is an estimated **reduction factor**
 - Describes how much the original tuples will be reduced
 - Depends on a particular condition φ
 - As well as particular available statistics...

Size Estimates: Projection

Projection: $T = \pi_{a_1, \dots, a_n}(E)$

Tuple size

- s_T is simply calculated using sizes of all preserved attributes

Blocking factor

- $b_T = \lfloor B/s_T \rfloor$

Number of tuples

- Default SQL projection without the DISTINCT modifier
 - I.e., removal of potential duplicates is not performed
 - $n_T = n_E$
- With duplicates removal enabled
 - $n_T = n_E$ if at least one key of E is preserved
 - ...

Size Estimates: Joins

Inner joins: $T = E_R \times E_S$ or $E_R \bowtie E_S$ or $E_R \bowtie_{\varphi} E_S$

Tuple size

- $s_T \approx s_R + s_S$
 - Less for natural join since shared attributes are not repeated

Blocking factor

- $b_T \approx \left\lfloor \frac{B}{s_T} \right\rfloor \approx \left\lfloor \frac{B}{s_R + s_S} \right\rfloor \approx \left\lfloor \frac{B}{B/b_R + B/b_S} \right\rfloor \approx \left\lfloor \frac{b_R \cdot b_S}{b_R + b_S} \right\rfloor$
 - Can be calculated exactly from the actual resulting tuple size
 - As well as **estimated just using the original blocking factors**

Number of tuples

- $n_T = \lceil n_R \cdot n_S \cdot r_{\varphi} \rceil$ with $r_{\varphi} \in [0, 1]$ for **joining condition** φ
 - Similar approach with **reduction factors** as in selections

Algebraic Optimization

Objective

- Capability of **finding alternative query evaluation plans**
 - Based on various **equivalence rules**
 - E.g.: commutativity of selection, associativity of inner joins, ...
- Ultimate challenge
 - **Space of all possible plans** may be enormous
 - And so **significant pruning** must be involved

Basic strategy for **SPJ queries** = **select-project-join** queries

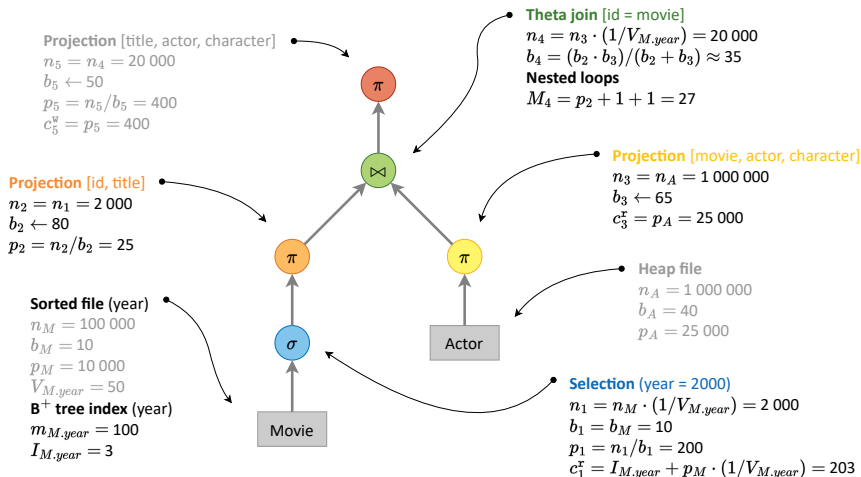
- They allow to be approached at two separate levels...
 - **Single-relation** plans = best **access method** for each table
 - **Multi-relation** plans = best **join plan** for all the tables
- But still an **NP-complete problem**

Examples

Sample transformations

- $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000) \wedge (\text{id}=\text{movie})} (\text{Movie} \times \text{Actor}) \right) // \#1$
- $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{id}=\text{movie})} \left(\varphi_{(\text{year}=2000)} (\text{Movie} \times \text{Actor}) \right) \right)$
- $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000)} \left(\varphi_{(\text{id}=\text{movie})} (\text{Movie} \times \text{Actor}) \right) \right)$
- $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000)} (\text{Movie} \bowtie_{(\text{id}=\text{movie})} \text{Actor}) \right) // \#2$
- $\pi_{\text{title,actor,character}} \left(\varphi_{(\text{year}=2000)} (\text{Movie}) \bowtie_{(\text{id}=\text{movie})} \text{Actor} \right)$
- $\pi_{\text{title,actor,character}} \left(\pi_{\text{id,title}} \left(\varphi_{(\text{year}=2000)} (\text{Movie}) \right) \bowtie_{(\text{id}=\text{movie})} \right)$
 $\pi_{\text{movie,actor,character}} (\text{Actor}) // \#3$

Sample Plan #3



Sample Plan #3

Cost of Plan #3 with pipelining

- $M = 25 + 1 + 1$ memory pages for buffers \mathcal{I}_1 , \mathcal{I}_2 and \mathcal{O}
 - I.e., still the same amount of system memory pages used
- $c = [c_1^f + \cancel{c_1^f}] + [\cancel{c_2^f} + \cancel{c_2^f}] + [c_3^f + \cancel{c_3^f}] + [\cancel{c_4^f} + \cancel{c_4^f}] + [\cancel{c_5^f}]$
 - \mathcal{I}_2 is used for index traversal and then reading of movies
 - All filtered and projected movies are put into \mathcal{I}_1
 - Actors are read into \mathcal{I}_2 , their projection is postponed
 - Joined tuples are put into \mathcal{O} and projected
- $c = [I_{M.year} + p_M \cdot (1/V_{M.year})] + [p_A]$
- $c = [203] + [25\ 000]$
- $c = 25\ 203$
 - That is approximately 400 times better than the second plan
 - And so almost 1 million times better than the first plan

Explain Statements

EXPLAIN statement

- Allows to **retrieve the evaluation plan** for a given query
 - When ANALYZE modifier is provided...
 - Query is also executed and the actual run times are returned



Example

- **EXPLAIN**
SELECT title, actor, character
FROM Movie JOIN Actor
WHERE (year = 2000) AND (id = movie)

Observations

False assumptions and simplifications

- **Variable size** of tuples
- **Unused slots** and **inner fragmentation** within blocks
- **Overflow areas** in sorted / hashed files
- **Outer fragmentation** of files on a hard drive
- Impact of the **caching manager**
- Extent of **available statistics** and their **lazy maintenance**
- **Non-uniform distribution** of data / queries
- **Independence of conditions** in reduction factors
- ...

Conclusion

Evaluation **algorithms**

- Access methods
- Sorting
 - **External merge sort** algorithm
- Joining
 - Binary **nested loops join** with / without zig-zag
 - **Sort-merge join**
 - Classic / partition **hash join**

Query **evaluation** and **optimization**

- Evaluation **plans**
 - Cost estimates, **pipelining**
- Statistical / algebraic optimization