From outermost termination to innermost termination

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Motivation

termination analysis of (functional) programs

- term rewriting is basic evaluation mechanism of func. programs

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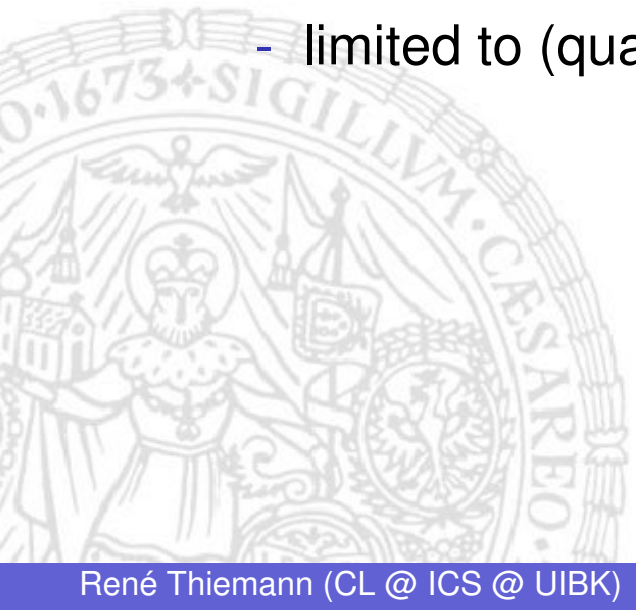
  + profits from all existing techniques for termination
  - cannot be used to disprove outermost termination
  - limited to (quasi) left-linear TRSs



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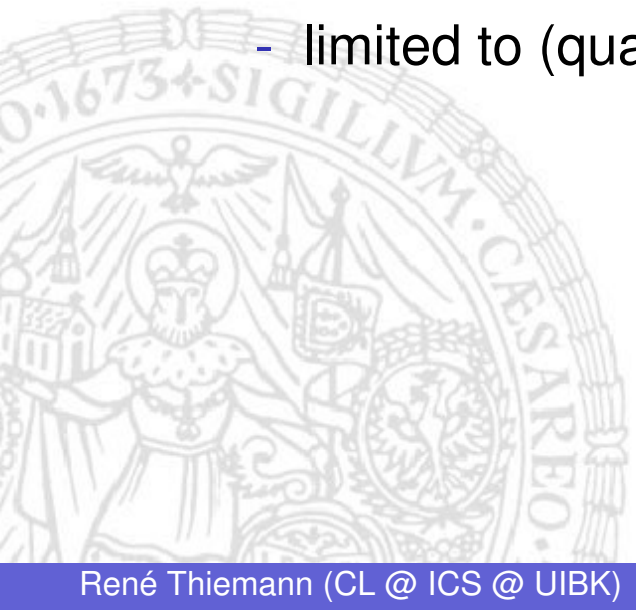

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- contribution: a new transformation $'$

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Outline

1. Motivation
2. Preliminaries
3. The Transformation
4. The Transformation - Soundness and Completeness
5. Experiments

Rewriting

Definition

- TRS: set of rules $\ell \to r$

Example

$$\mathsf{zeros} \to \mathsf{c}(0, \mathsf{zeros})$$
$$\mathsf{c}(x, xs) \to \mathsf{terminate}$$

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- rewrite step at position p: $t = C[\ell\sigma]_p \to_{\mathcal{R},p} C[r\sigma]_p = s$

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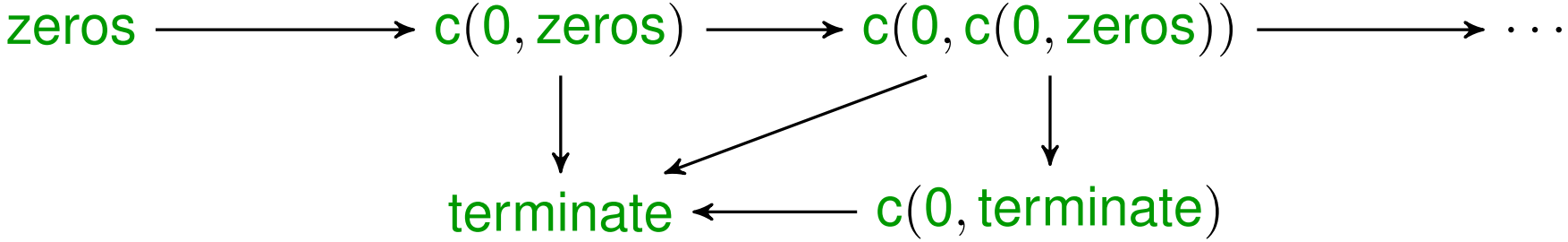

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$$\text{zeros} \xrightarrow{\text{i}} \text{c}(0, \text{zeros}) \xrightarrow{\text{i}} \text{c}(0, \text{c}(0, \text{zeros})) \xrightarrow{\text{i}} \cdots$$

$$\text{terminate} \xleftarrow{\text{i}} \text{c}(0, \text{terminate})$$

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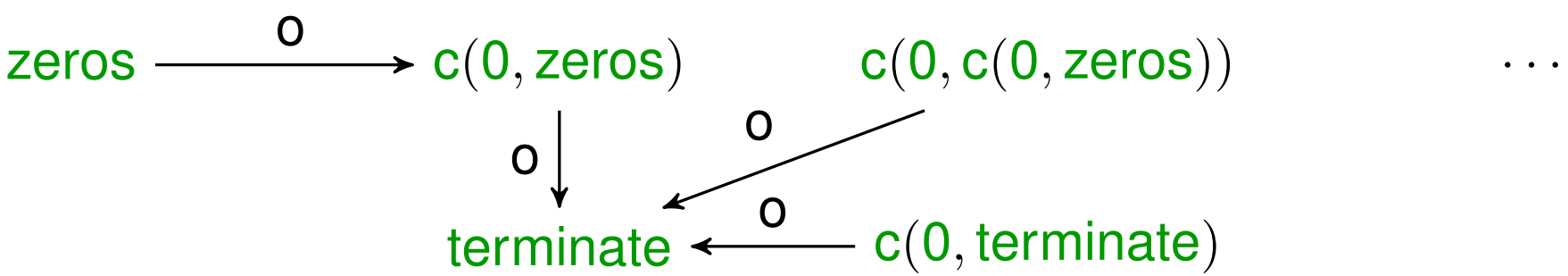


Termination

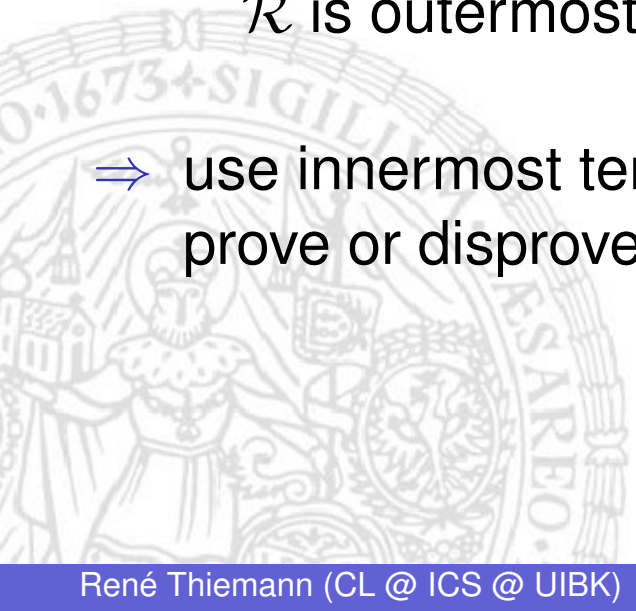
Definition

$\mathcal{R}$ is innermost / outermost terminating iff $\xrightarrow{\text{i}}_{\mathcal{R}}$ / $\xrightarrow{\text{o}}_{\mathcal{R}}$ is well-founded

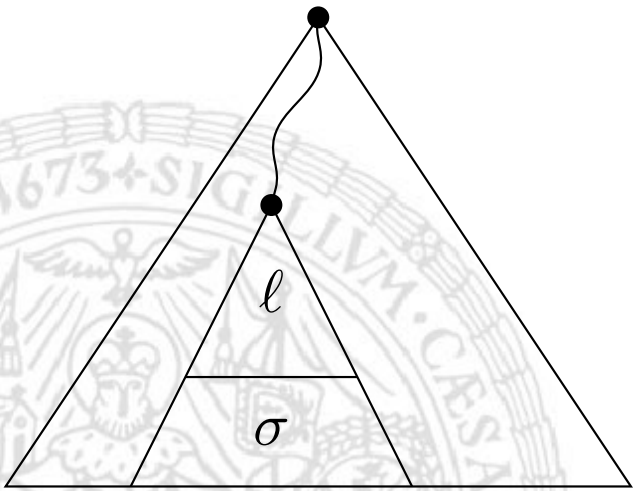
aim: transform given TRS $\mathcal{R}$ into TRS $\mathcal{R}'$ such that

$\mathcal{R}$ is outermost terminating iff $\mathcal{R}'$ is innermost terminating

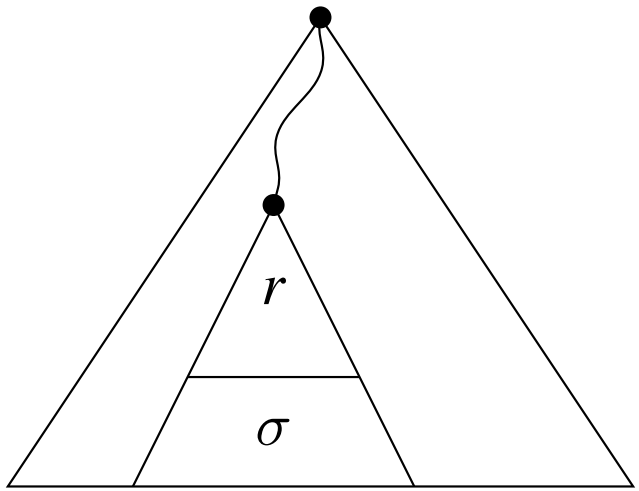
$\Rightarrow$ use innermost termination techniques to prove or disprove outermost termination



Simulation of outermost rewriting



$\xrightarrow{\mathsf{o}}_{\mathcal{R}}$



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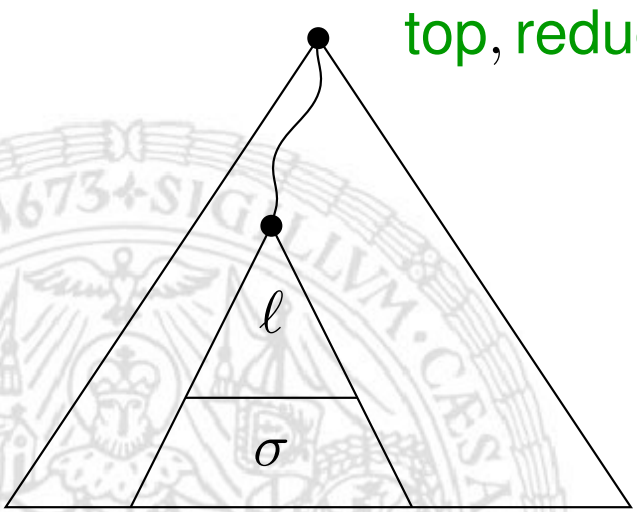

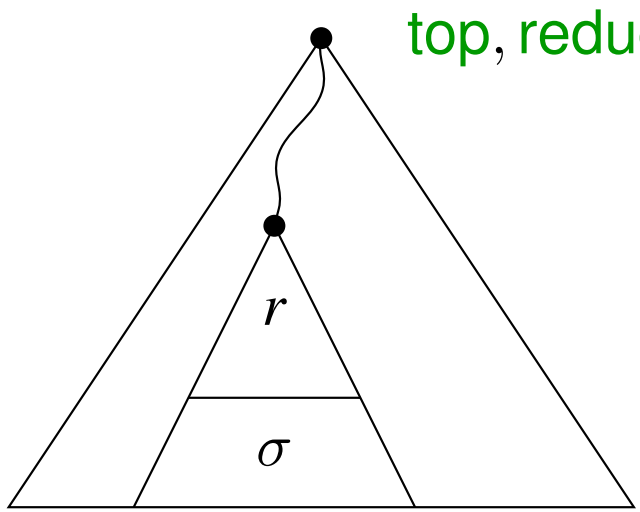

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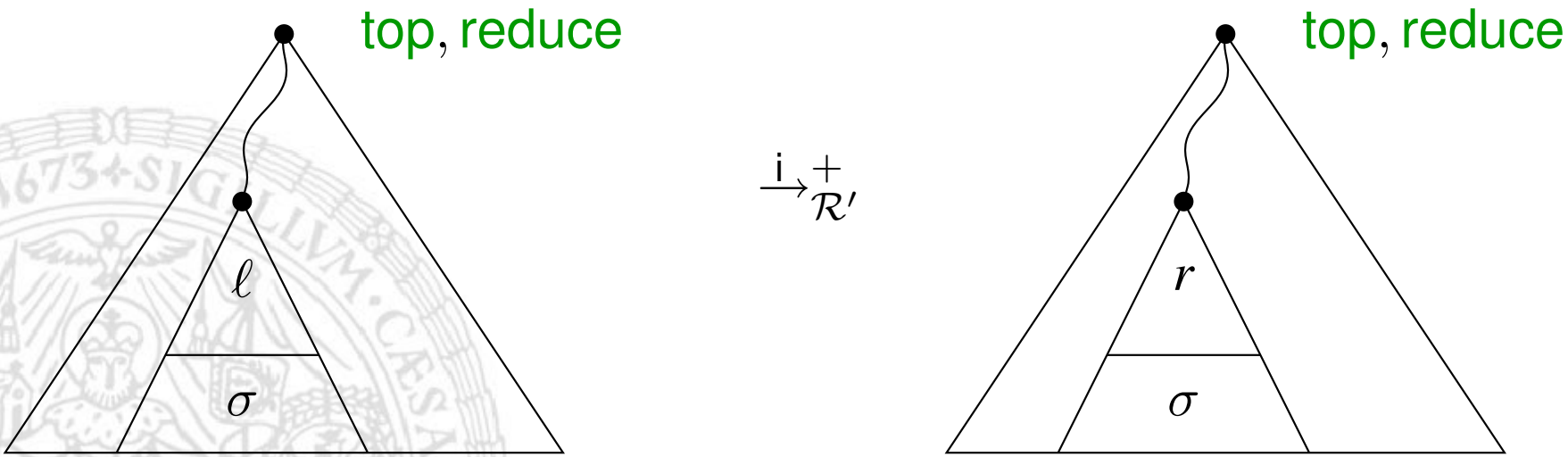

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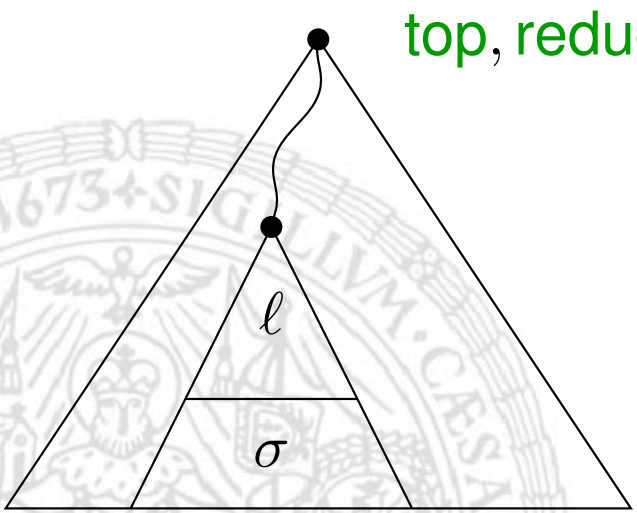

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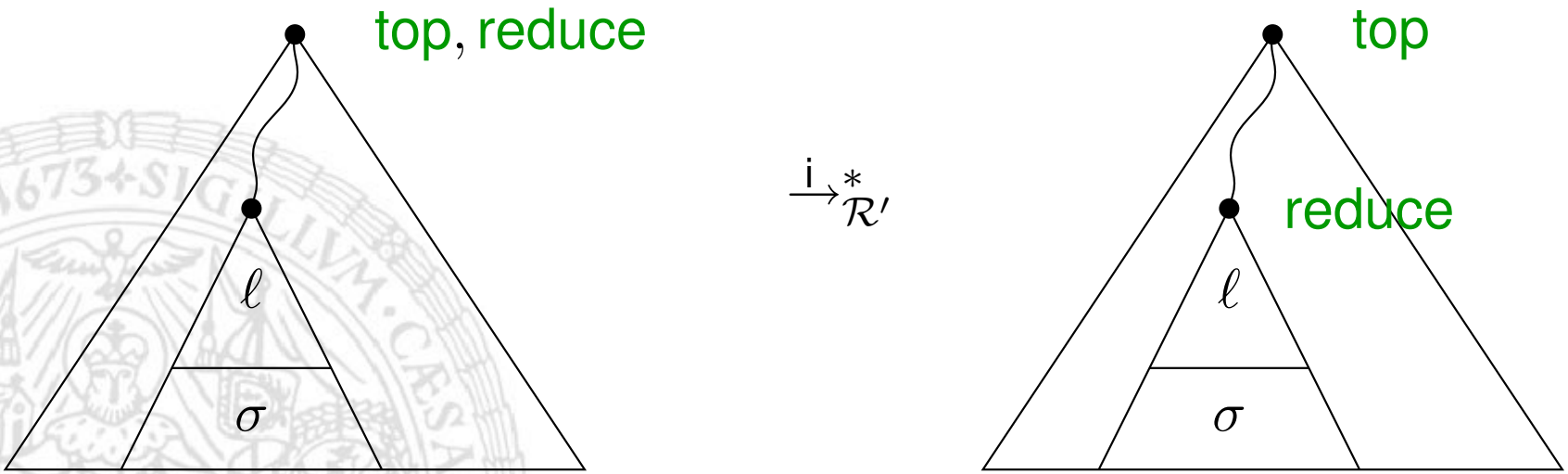

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- enrich signature by marker-symbols (reduce, top, go_up)
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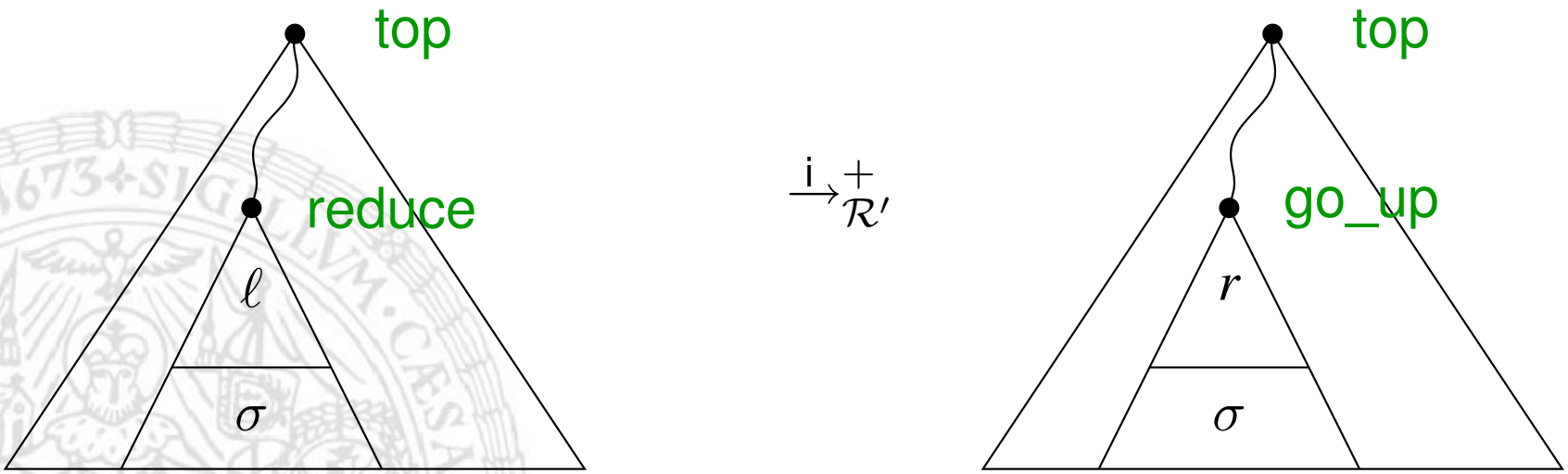

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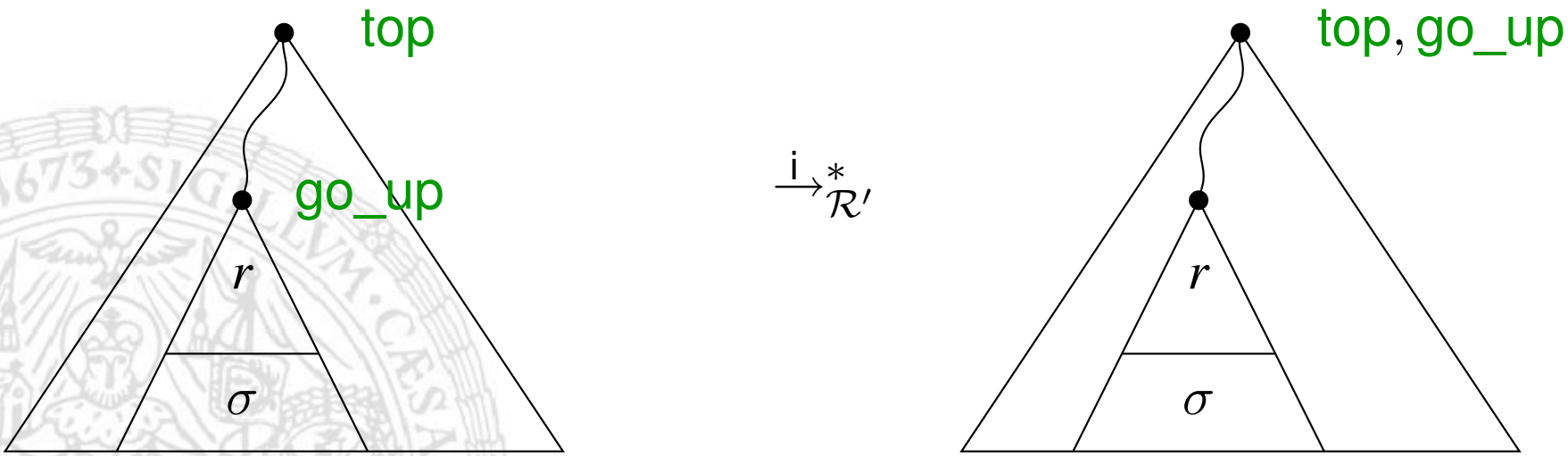

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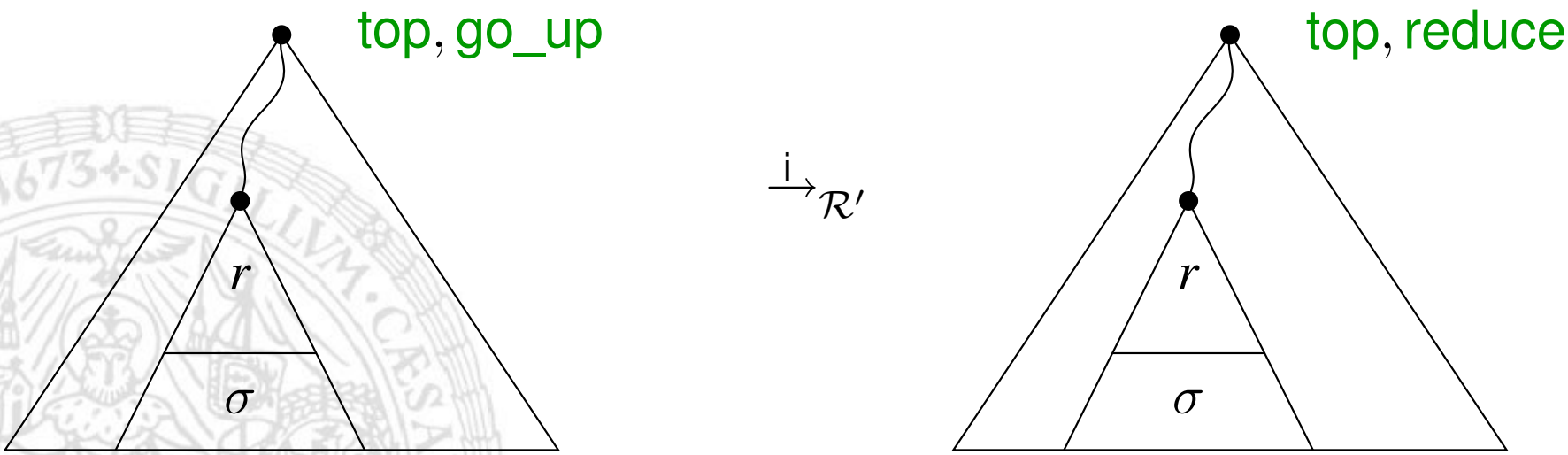

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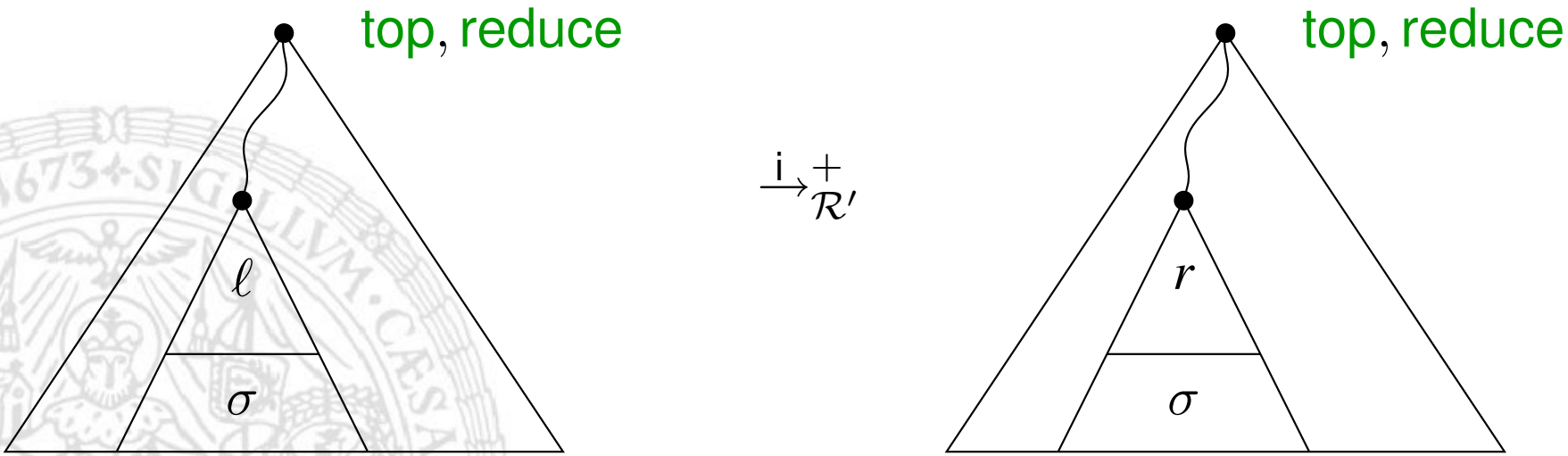
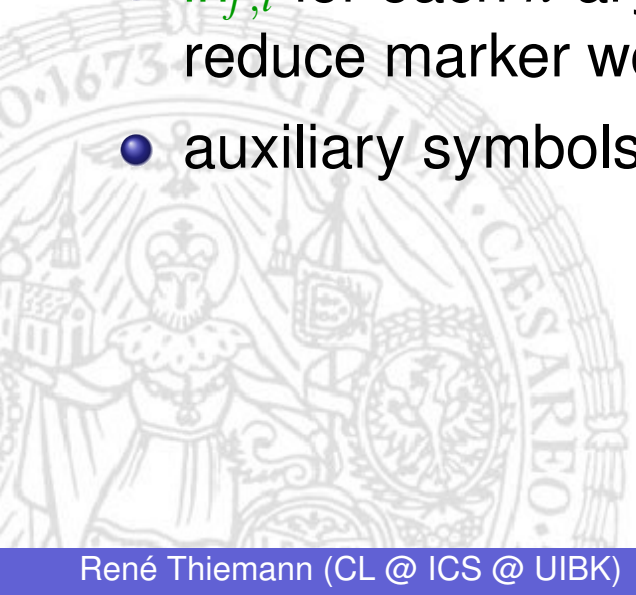


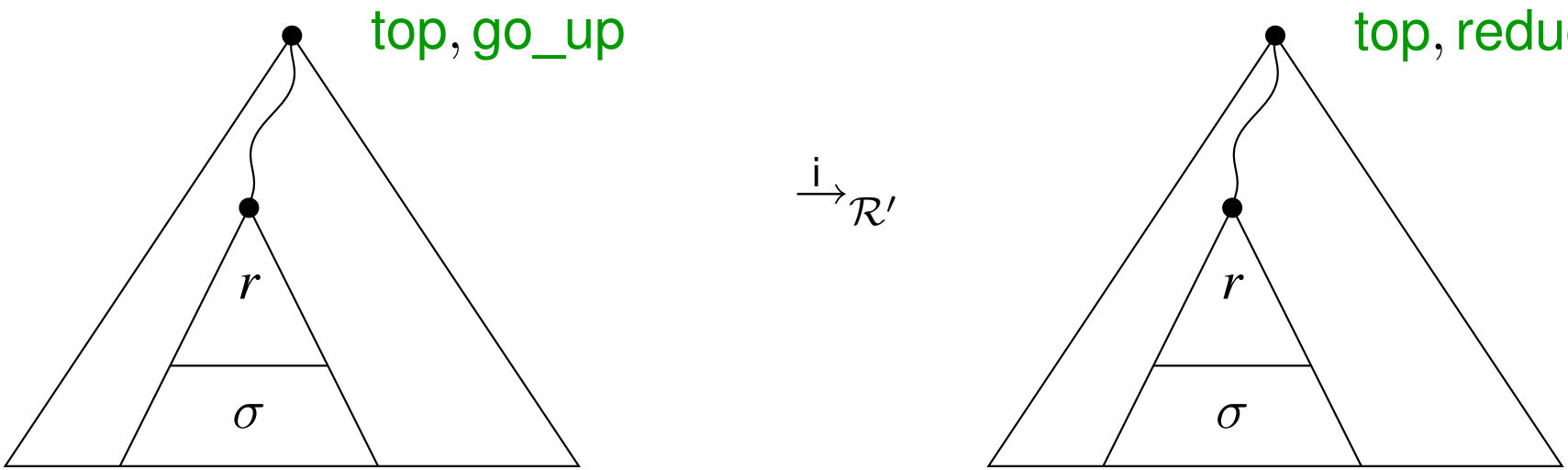
Enriching the signature

the enriched signature Σ' consists of

- all symbols of the old signature Σ
- top to mark the top of a term
- reduce to mark where we are allowed to reduce
- go_up to mark that we have to go up again
- $\text{in}_{f,i}$ for each n-ary symbol $f \in \Sigma$ and $1 \leq i \leq n$ to mark that the reduce marker went below an f into the i-th argument
- auxiliary symbols result, check_f, and redex_f



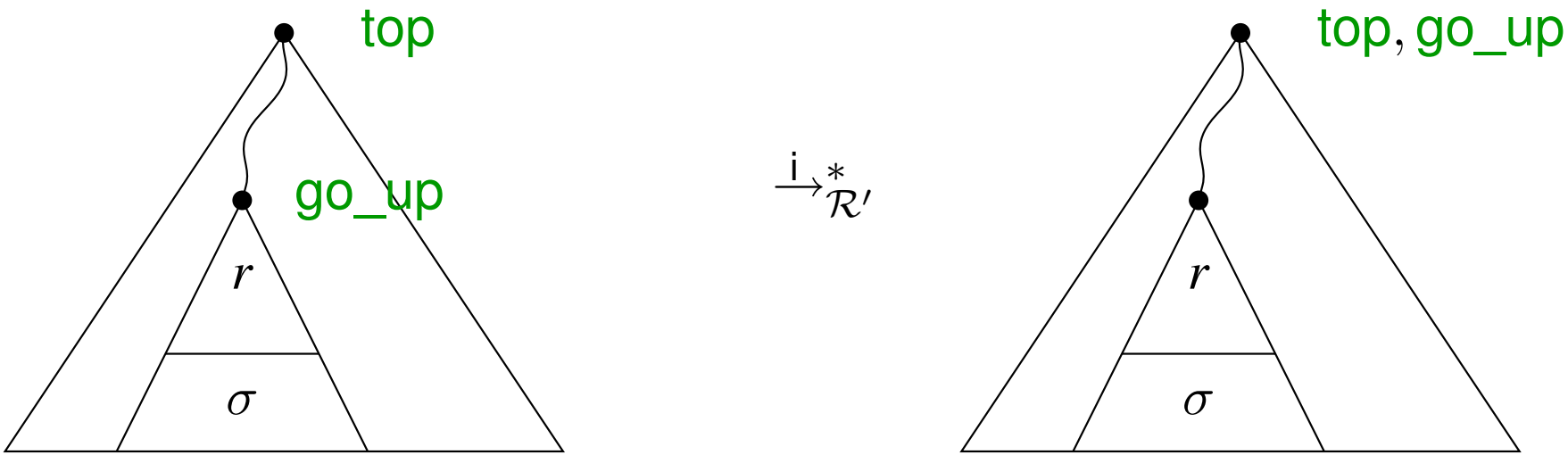
4. phase: switch from go_up to reduce at top position



add following rule to $\mathcal{R}'$:

$$\mathsf{top}(\mathsf{go\_up}(x)) \to \mathsf{top}(\mathsf{reduce}(x))$$

3. phase: move go_up back to top

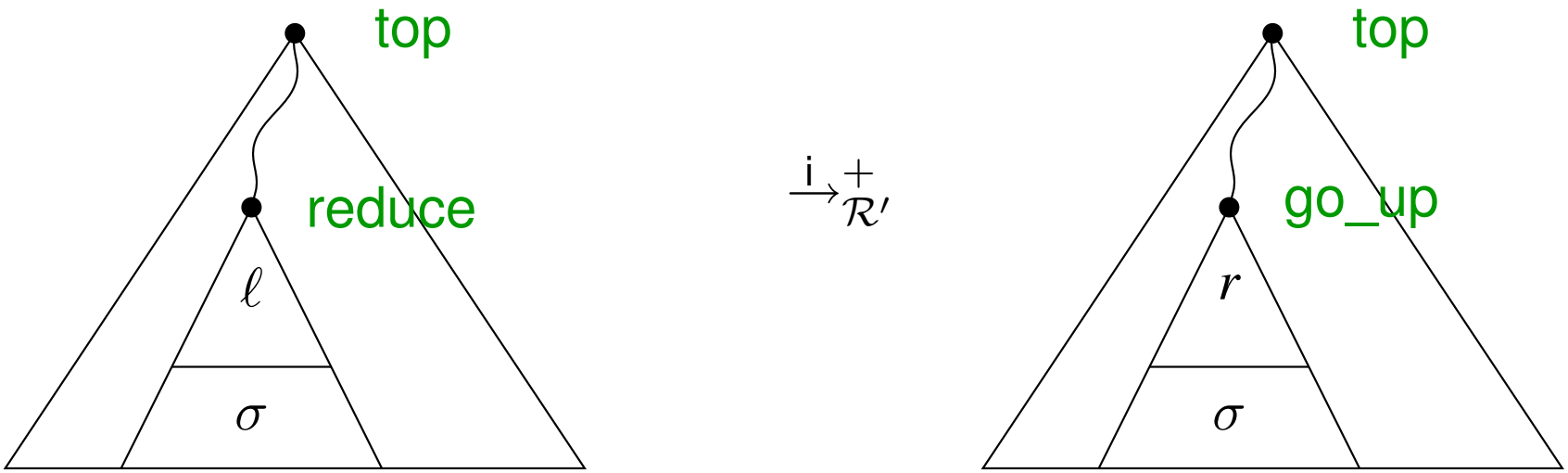


add following rules to $\mathcal{R}'$:

$$\mathsf{in}_{f,i}(x_1, \ldots, \mathsf{go\_up}(x_i), \ldots, x_n) \to \mathsf{go\_up}(f(x_1, \ldots, x_n))$$

for all $f \in \Sigma, 1 \leq i \leq n$

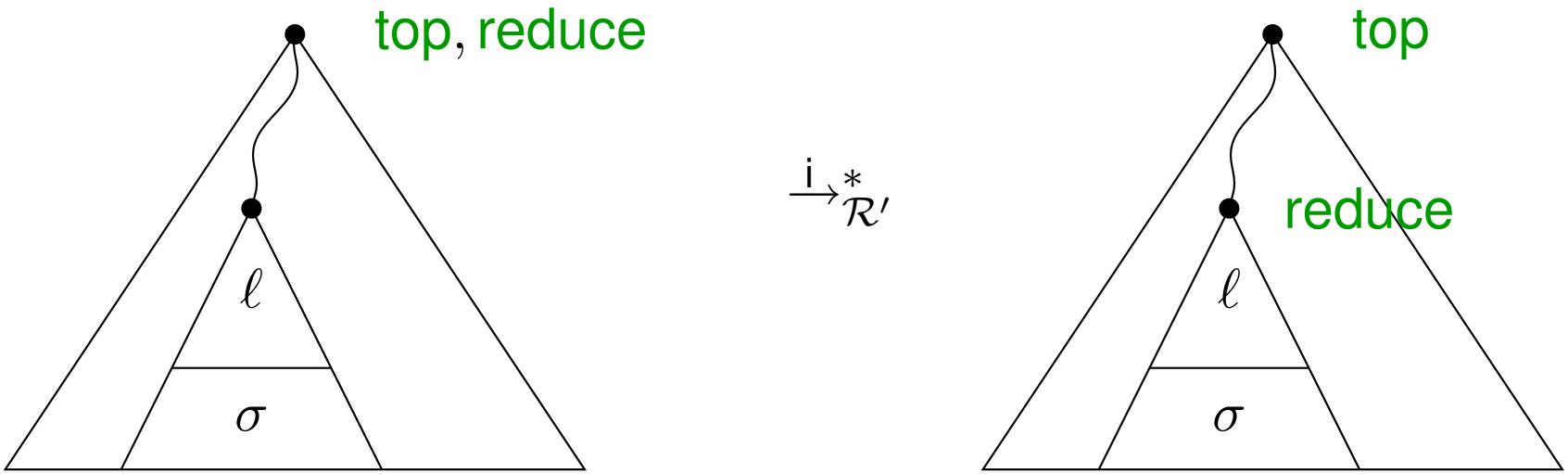
2. phase: perform reduction and switch to go_up



add following rules to $\mathcal{R}'$:

$$\mathsf{reduce}(f(x_1,\ldots,x_n)) \to \mathsf{check}_f(\mathsf{redex}_f(x_1,\ldots,x_n)) \qquad \text{for all } f \in \Sigma$$
$$\mathsf{redex}_f(\ell_1,\ldots,\ell_n) \to \mathsf{result}(r) \qquad \text{for all } f(\ell_1,\ldots,\ell_n) \to r \in \mathcal{R}$$
$$\mathsf{check}_f(\mathsf{result}(x)) \to \mathsf{go\_up}(x) \qquad \text{for all } f \in \Sigma$$

1. phase: move reduce to outermost redexes



recall existing rules of $\mathcal{R}'$:

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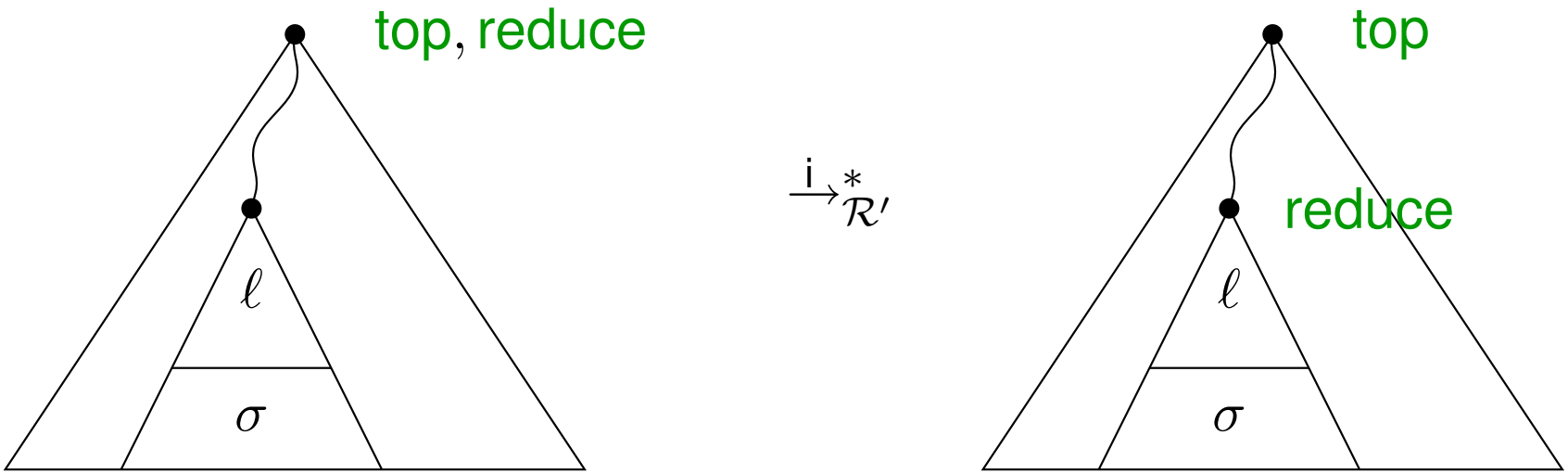

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The transformed TRS $\mathcal{R}'$

$$\begin{aligned}
\mathsf{reduce}(f(x_1,\ldots,x_n)) &\to \mathsf{check}_f(\mathsf{redex}_f(x_1,\ldots,x_n))\\
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\mathsf{top}(\mathsf{go_up}(x)) &\to \mathsf{top}(\mathsf{reduce}(x))
\end{aligned}$$

Theorem

$\mathcal{R}$ is outermost terminating iff $\mathcal{R}'$ is innermost terminating

soundness proof.

from

$$t_1 \xrightarrow{\mathrm{o}}_{\mathcal{R}} t_2 \xrightarrow{\mathrm{o}}_{\mathcal{R}} t_3 \xrightarrow{\mathrm{o}}_{\mathcal{R}} \cdots$$

obtain

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Completeness: proof sketch

1. if $\mathcal{R}'$ is not innermost terminating then there is infinite derivation

$$\mathsf{top}(\mathsf{go\_up}(t_1)) \xrightarrow{\mathsf{i}}_{\mathcal{R}',\epsilon} \mathsf{top}(\mathsf{reduce}(t_1)) \xrightarrow{\mathsf{i}}^*_{\mathcal{R}',>\epsilon} \mathsf{top}(\mathsf{go\_up}(t_2)) \xrightarrow{\mathsf{i}}_{\mathcal{R}',\epsilon} \cdots$$

2. from

$$\mathsf{reduce}(t_i) \xrightarrow{\mathsf{i}}^*_{\mathcal{R}'} \mathsf{go\_up}(t_{i+1})$$

conclude

$$\mathcal{O}(t_i) \xrightarrow{\mathsf{o}}_{\mathcal{R}} \mathcal{O}(t_{i+1})$$

where $\mathcal{O}$ maps terms of extended signature to terms of original signature

$\Rightarrow$ obtain infinite outermost derivation

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Completeness: proof sketch

1. if $\mathcal{R}'$ is not innermost terminating then there is infinite derivation

$$\mathsf{top}(\mathsf{go\_up}(t_1)) \xrightarrow{\mathsf{i}}_{\mathcal{R}',\epsilon} \mathsf{top}(\mathsf{reduce}(t_1)) \xrightarrow{\mathsf{i}}^*_{\mathcal{R}',>\epsilon} \mathsf{top}(\mathsf{go\_up}(t_2)) \xrightarrow{\mathsf{i}}_{\mathcal{R}',\epsilon} \cdots$$

2. from

$$\mathsf{reduce}(t_i) \xrightarrow{\mathsf{i}}^*_{\mathcal{R}'} \mathsf{go\_up}(t_{i+1})$$

conclude

$$\mathcal{O}(t_i) \xrightarrow{\mathsf{o}}_{\mathcal{R}} \mathcal{O}(t_{i+1})$$

where $\mathcal{O}$ maps terms of extended signature to terms of original signature

$\Rightarrow$ obtain infinite outermost derivation

$$\mathcal{O}(t_1) \xrightarrow{\mathsf{o}}_{\mathcal{R}} \mathcal{O}(t_2) \xrightarrow{\mathsf{o}}_{\mathcal{R}} \mathcal{O}(t_3) \xrightarrow{\mathsf{o}}_{\mathcal{R}} \cdots$$

Experimental Results

- consider those 434 TRSs of TPDB 4.0 where termination prover AProVE could not show termination
- one minute timeout, Intel Core 2 Duo processor, 2GB RAM
- four approaches
  - Cariboo
  - Raffelsieper Zantema transformation + AProVE
  - $\mathcal{R}'$ + AProVE
  - improved version $\mathcal{R}''$ of our transformation + AProVE

Transformation	Cariboo	RZ	$\mathcal{R}'$	$\mathcal{R}''$
outermost t/ proven	(≥ 6)	6 (160s)	7 (158s)	7 (139s)

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first method to automatically disprove outermost termination

Improved Transformation $\mathcal{R}''$

special treatment for constructors C and constants

$$\mathsf{reduce}(f(x_1,\ldots,x_n)) \to \mathsf{check}_f(\mathsf{redex}_f(x_1,\ldots,x_n)) \quad \text{for all } f \in \Sigma$$

$$\mathsf{check}_f(\mathsf{redex}_f(x_1,\ldots,x_n)) \to \mathsf{in}_{f,i}(x_1,\ldots,\mathsf{reduce}(x_i),\ldots,x_n)$$
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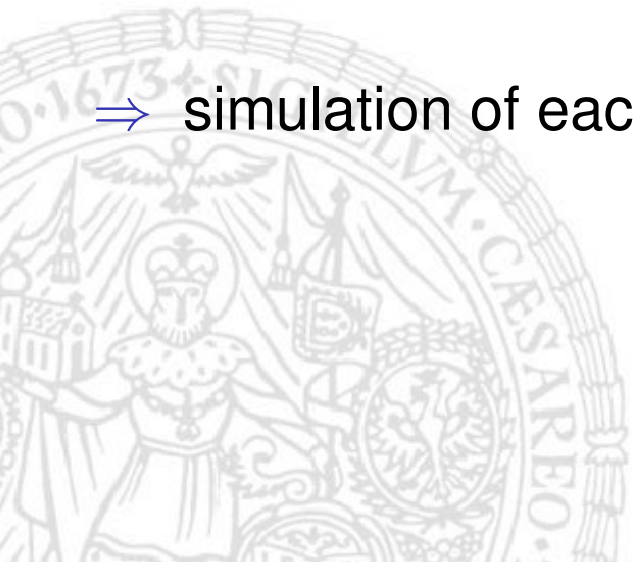
$\Rightarrow$ simulation of each outermost step in less innermost steps



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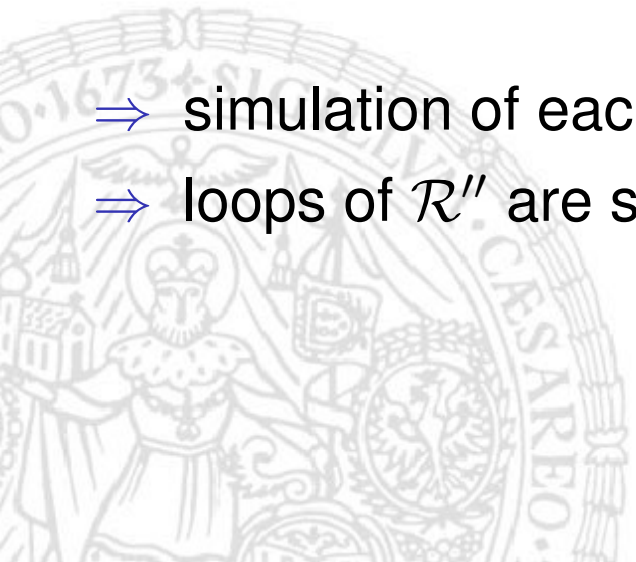

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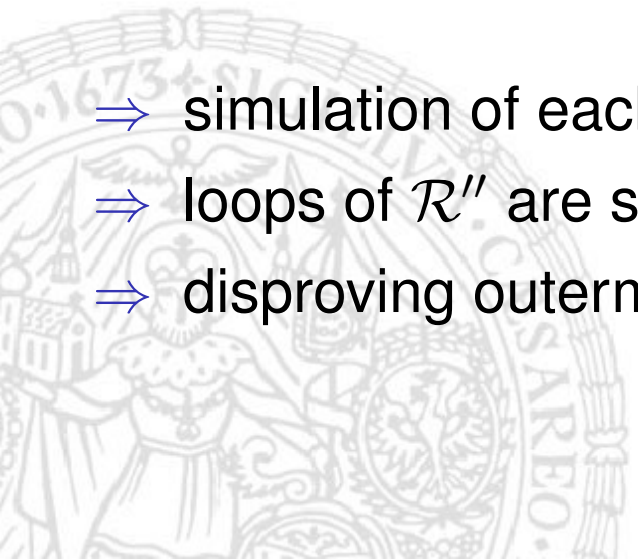

- $\Rightarrow$ simulation of each outermost step in less innermost steps
- $\Rightarrow$ loops of $\mathcal{R}''$ are shorter than loops of $\mathcal{R}'$
- $\Rightarrow$ disproving outermost termination becomes easier

Summary

- complete transformations such that

  $\mathcal{R}$ is outermost terminating iff $\mathcal{R}'$ or $\mathcal{R}''$ is innermost terminating
- first automatic approach to disprove outermost termination

Future own work

- not all outermost loops of $\mathcal{R}$ are transformed to innermost loops of $\mathcal{R}'$ or $\mathcal{R}''$

  $\rightarrow$ develop direct method which can detect all outermost loops

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