

# Natural Specifications Yield Decidability for Distributed Synthesis of Asynchronous Systems

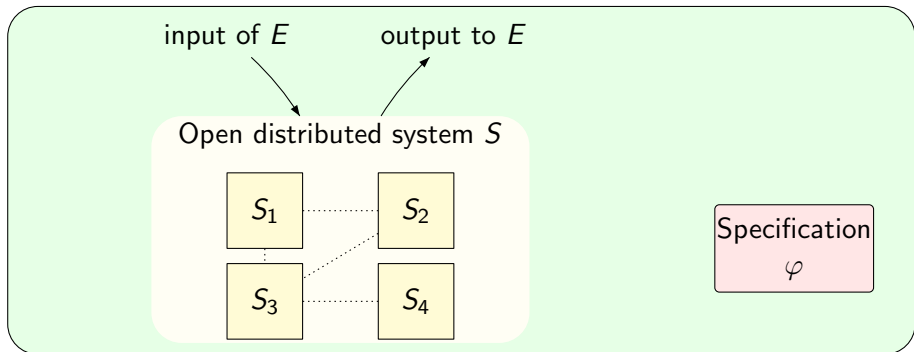
Nathalie Sznajder

LSV, ENS Cachan, France

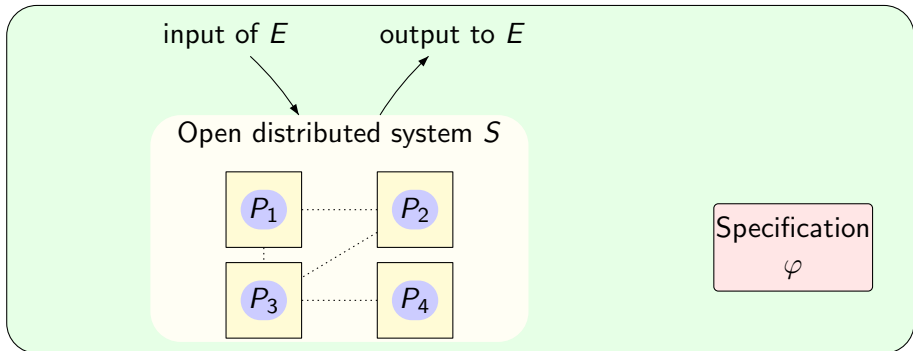
SOFSEM'09  
Czech Republic

Joint work with Thomas Chatain and Paul Gastin

# Synthesis of a distributed reactive system



# Synthesis of a distributed reactive system



## Two problems

- Decide the existence of a **distributed** program such that their **joint behavior**  $P_1 || P_2 || P_3 || P_4 || E$  satisfies  $\varphi$ , for all  $E$ .
- Synthesis : If it exists, compute such a **distributed** program.

# Distributed synthesis: Synchronous or asynchronous semantics?

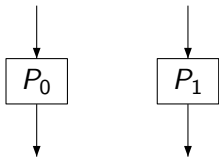
## Synchronous semantics

- At each tick of a global clock, all the processes and the environment output their new value
- Introduced in [PnueliRosner90].
- In general undecidable.

# Distributed synthesis: Synchronous or asynchronous semantics?

## Synchronous semantics

- At each tick of a global clock, all the processes and the environment output their new value
- Introduced in [PnueliRosner90].
- In general undecidable.



# Distributed synthesis: Synchronous or asynchronous semantics?

## Synchronous semantics

- At each tick of a global clock, all the processes and the environment output their new value
- Introduced in [PnueliRosner90].
- In general undecidable.



# Asynchronous semantics

## Our model

- Processes evolve asynchronously for **local** actions (i.e., communications with the environment)

# Asynchronous semantics

## Our model

- Processes evolve asynchronously for **local** actions (i.e., communications with the environment)
- They can synchronize by **signals** = common actions initiated by only one process. A process cannot refuse reception of a signal.



# Asynchronous semantics

## Our model

- Processes evolve asynchronously for **local** actions (i.e., communications with the environment)
- They can synchronize by **signals** = common actions initiated by only one process. A process cannot refuse reception of a signal.
- Specifications :
  - ▶ over **partial orders**

# Asynchronous semantics

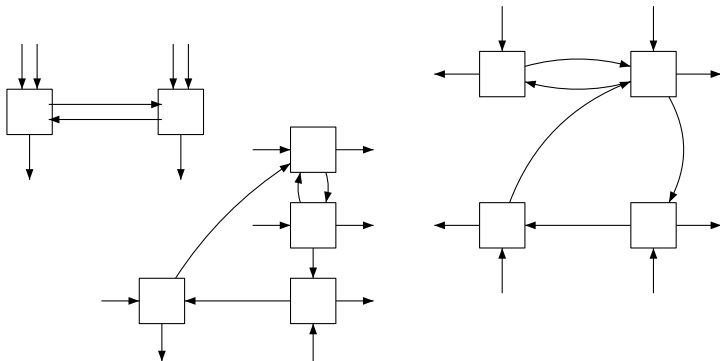
## Our model

- Processes evolve asynchronously for **local** actions (i.e., communications with the environment)
- They can synchronize by **signals** = common actions initiated by only one process. A process cannot refuse reception of a signal.
- Specifications :
  - ▶ over **partial orders**
  - ▶ will not restrain **communication abilities**

# Decidability Results

## Theorem

Synthesis problem is decidable for strongly-connected architectures



# Outline

- 1 Introduction
- 2 The model
- 3 Decidability Results

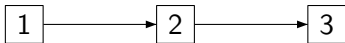
# Outline

- 1 Introduction
- 2 The model
- 3 Decidability Results

# The model

## Architectures

- Communication graph  $(Proc, E)$

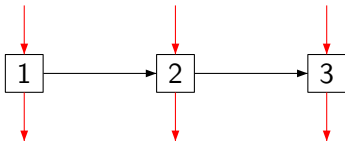


# The model

## Architectures

- Communication graph  $(Proc, E)$
- Sets of input and output signals for each process :

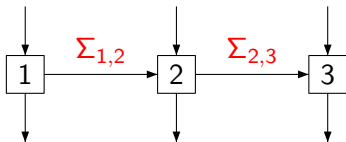
$$\bigcup_{i \in Proc} In_i \cup \bigcup_{i \in Proc} Out_i = \Gamma$$



# The model

## Architectures

- Communication graph  $(Proc, E)$
- Sets of input and output signals for each process :  
$$\bigcup_{i \in Proc} In_i \cup \bigcup_{i \in Proc} Out_i = \Gamma$$
- Processes **choose** sets  $\Sigma_{i,j}$  for  $i, j \in Proc$

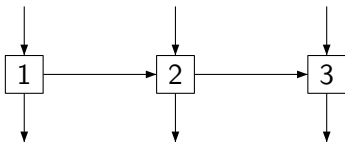




# The model

## Architectures

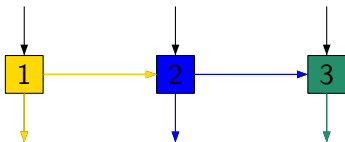
- Communication graph  $(Proc, E)$
- Sets of input and output signals for each process :  
$$\bigcup_{i \in Proc} In_i \cup \bigcup_{i \in Proc} Out_i = \Gamma$$
- Processes **choose** sets  $\Sigma_{i,j}$  for  $i, j \in Proc$
- $\Sigma = \Gamma \cup \bigcup_{(i,j) \in E} \Sigma_{i,j}$



# The model

## Architectures

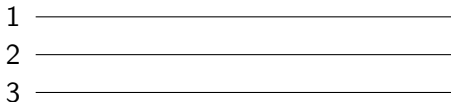
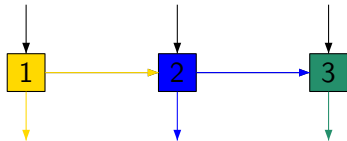
- Communication graph  $(Proc, E)$
- Sets of input and output signals for each process :  
$$\bigcup_{i \in Proc} In_i \cup \bigcup_{i \in Proc} Out_i = \Gamma$$
- Processes **choose** sets  $\Sigma_{i,j}$  for  $i, j \in Proc$
- $\Sigma = \Gamma \cup \bigcup_{(i,j) \in E} \Sigma_{i,j}$
- For each process  $i$ ,  $\Sigma_i$  is the set of signals it can send or receive, and  
$$\Sigma_i^c = Out_i \cup \bigcup_{j, (i,j) \in E} \Sigma_{i,j}$$



# The model: runs

## Runs

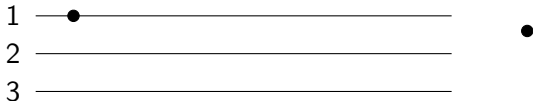
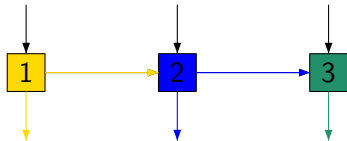
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

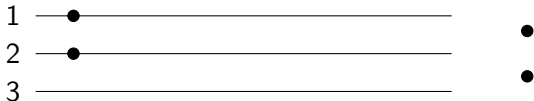
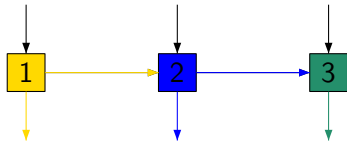
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

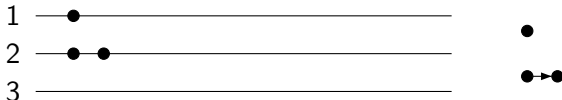
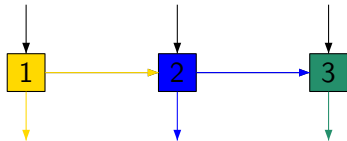
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

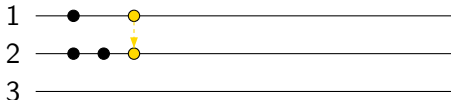
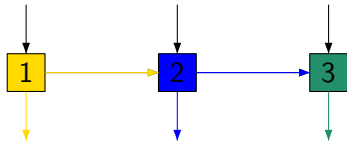
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

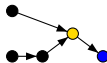
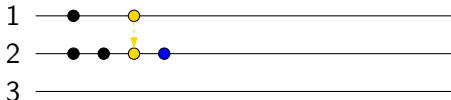
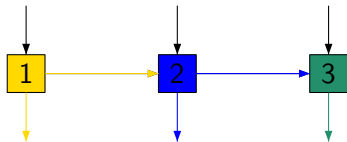
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$

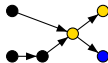
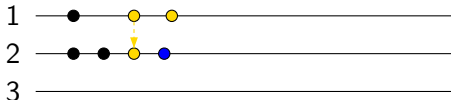
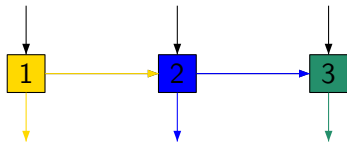




# The model: runs

## Runs

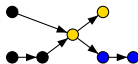
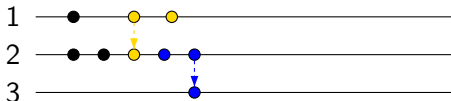
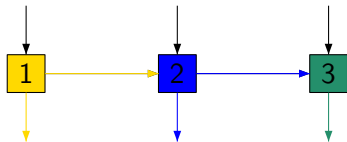
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

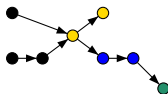
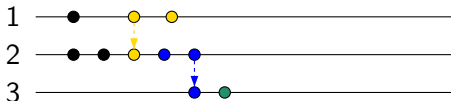
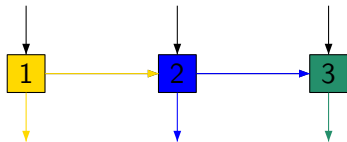
A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model: runs

## Runs

A run is a Mazurkiewicz trace  $t = (V, \lambda, \leq)$  over  $(\Sigma, D)$  where  $aDb$  if there is a process that takes part both in  $a$  and  $b$



# The model

## Strategies

- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory

# The model

## Strategies

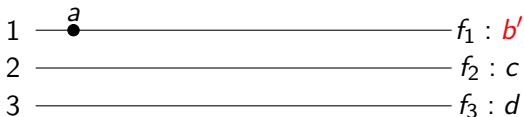
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory

1 —————  $f_1 : b$   
2 —————  $f_2 : c$   
3 —————  $f_3 : d$

# The model

## Strategies

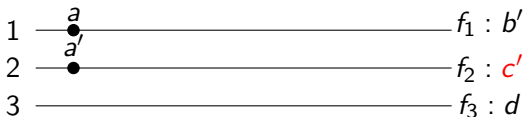
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory



# The model

## Strategies

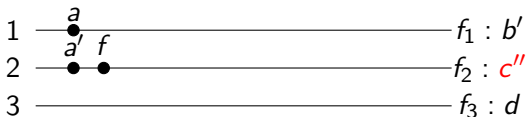
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory



# The model

## Strategies

- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory

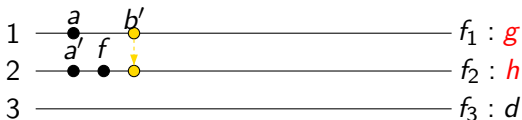




# The model

## Strategies

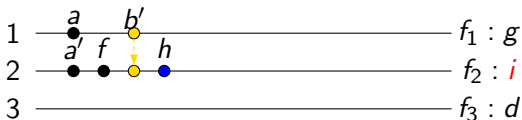
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory



# The model

## Strategies

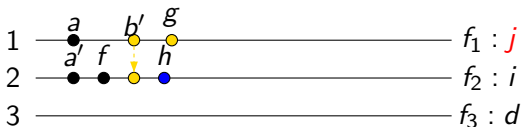
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory



# The model

## Strategies

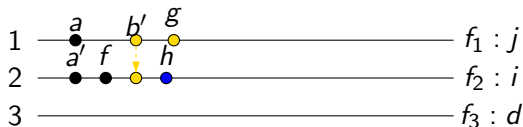
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory



# The model

## Strategies

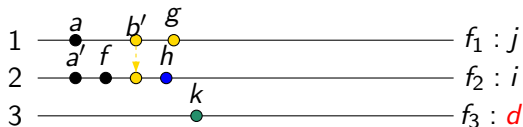
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory
- A run respects a strategy  $(f_i)_{i \in \text{Proc}}$  (is a  **$(f_i)_{i \in \text{Proc}}$ -run**) if each controllable action of process  $i$  is labelled according to the strategy  $f_i$



# The model

## Strategies

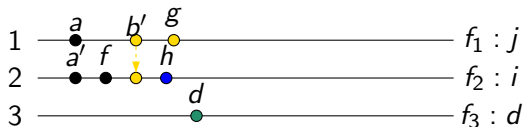
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory
- A run respects a strategy  $(f_i)_{i \in \text{Proc}}$  (is a  **$(f_i)_{i \in \text{Proc}}$ -run**) if each controllable action of process  $i$  is labelled according to the strategy  $f_i$



# The model

## Strategies

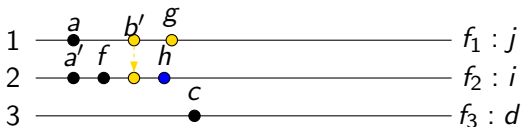
- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory
- A run respects a strategy  $(f_i)_{i \in \text{Proc}}$  (is a  **$(f_i)_{i \in \text{Proc}}$ -run**) if each controllable action of process  $i$  is labelled according to the strategy  $f_i$



# The model

## Strategies

- Strategies are functions  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  with **local** memory
- A run respects a strategy  $(f_i)_{i \in \text{Proc}}$  (is a  **$(f_i)_{i \in \text{Proc}}$ -run**) if each controllable action of process  $i$  is labelled according to the strategy  $f_i$



# The model

## Observable runs

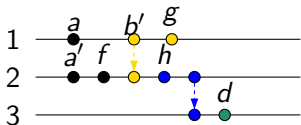
Given a run  $t = (V, \lambda, \leq)$ , we define the **observable** run by  $\pi_{\Gamma}(t) = (\Gamma, \lambda|_{\Gamma}, \leq \cap (\Gamma \times \Gamma))$



# The model

## Observable runs

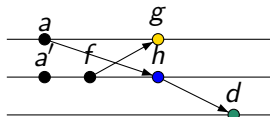
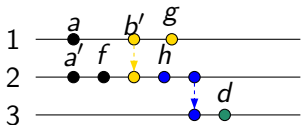
Given a run  $t = (V, \lambda, \leq)$ , we define the **observable** run by  $\pi_{\Gamma}(t) = (\Gamma, \lambda|_{\Gamma}, \leq \cap (\Gamma \times \Gamma))$



# The model

## Observable runs

Given a run  $t = (V, \lambda, \leq)$ , we define the **observable** run by  $\pi_{\Gamma}(t) = (\Gamma, \lambda|_{\Gamma}, \leq \cap (\Gamma \times \Gamma))$



# The synthesis problem

Given

- $\mathcal{A} = (\text{Proc}, E, \Gamma)$
- $\varphi$  a specification over  $\Gamma$ -labelled partial orders (observable runs)

Do there exist

- sets  $\Sigma_{i,j}$  for each  $(i,j) \in E$
- and strategies  $f_i : \Sigma_i^* \rightarrow \Sigma_i^c$  for each  $i \in \text{Proc}$

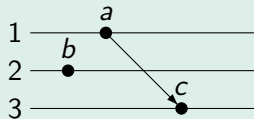
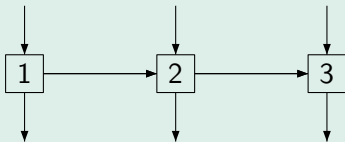
such that every  $(f_i)_{i \in \text{Proc}}$ -run  $t$  is such that  $\pi_\Gamma(t) \models \varphi$ ? If so, compute them

# Specifications

## Communication induces order relation

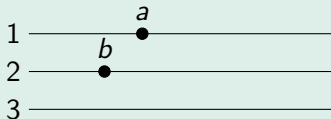
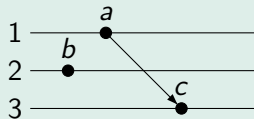
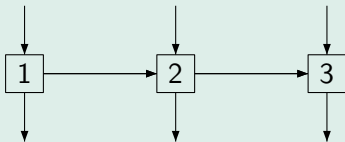
# Specifications

## Communication induces order relation



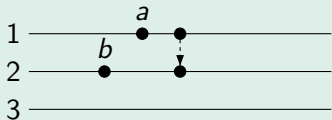
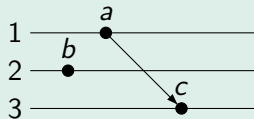
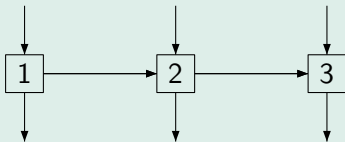
# Specifications

## Communication induces order relation



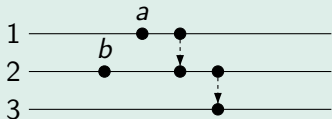
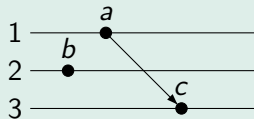
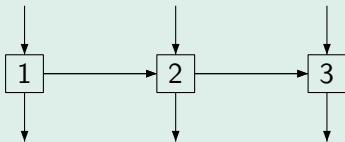
# Specifications

## Communication induces order relation



# Specifications

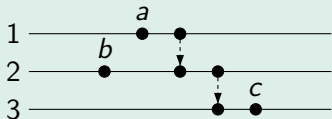
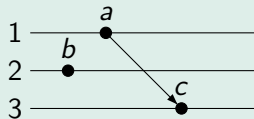
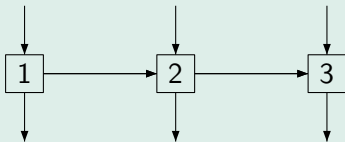
## Communication induces order relation





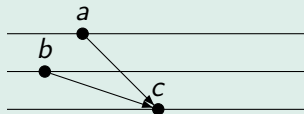
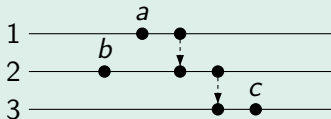
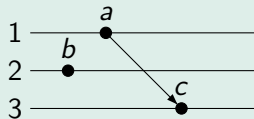
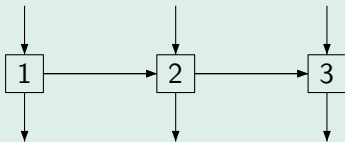
# Specifications

## Communication induces order relation



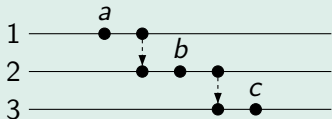
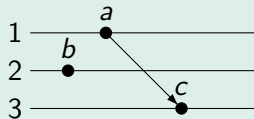
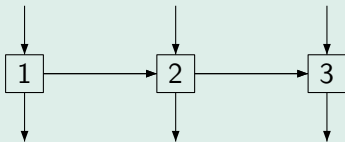
# Specifications

## Communication induces order relation



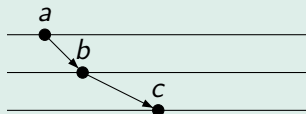
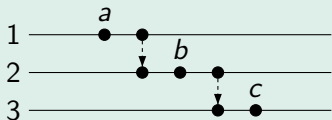
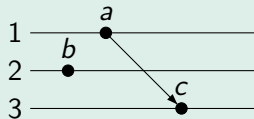
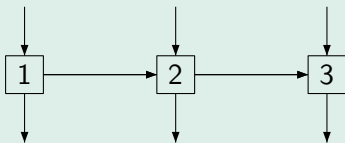
# Specifications

## Communication induces order relation



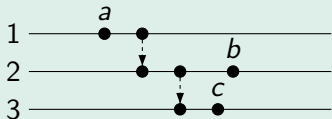
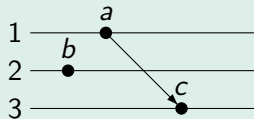
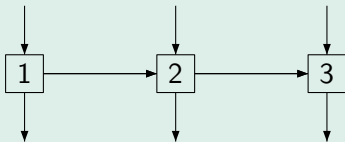
# Specifications

## Communication induces order relation



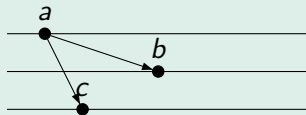
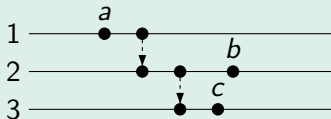
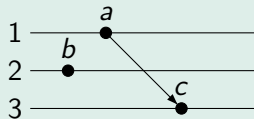
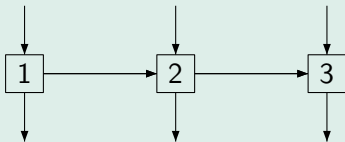
# Specifications

## Communication induces order relation



# Specifications

## Communication induces order relation



# Specifications

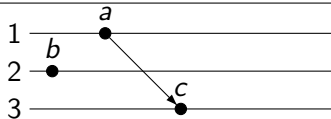
## Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions

# Specifications

## Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions

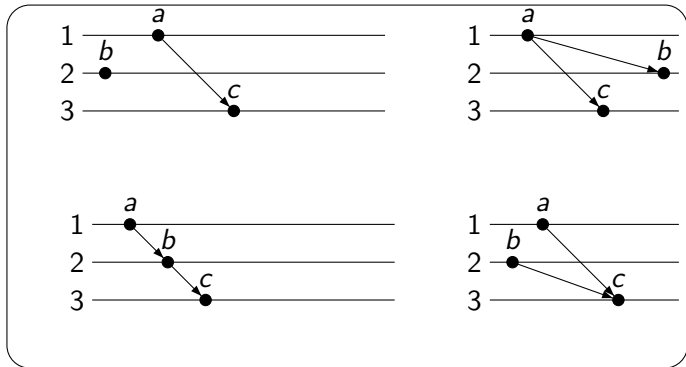




# Specifications

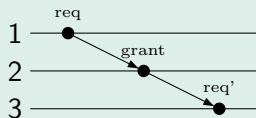
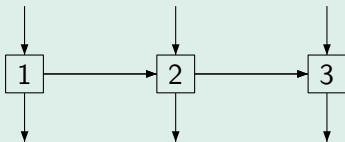
## Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions



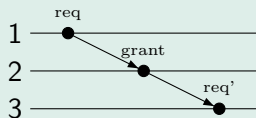
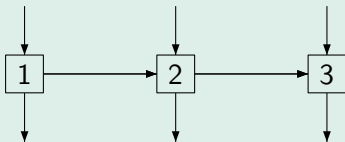
# Specifications

Input events are not controllable by processes



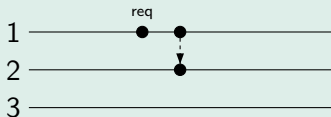
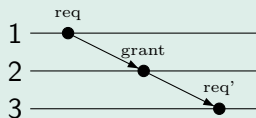
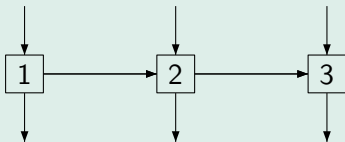
# Specifications

Input events are not controllable by processes



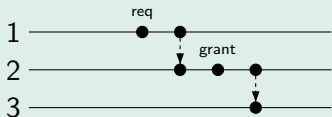
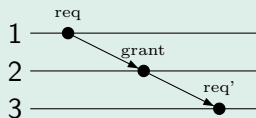
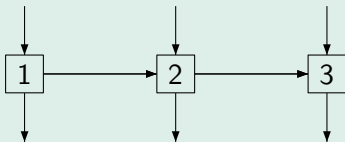
# Specifications

Input events are not controllable by processes



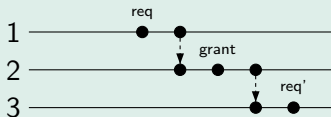
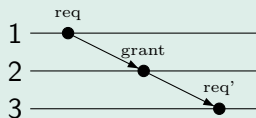
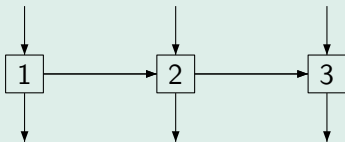
# Specifications

Input events are not controllable by processes



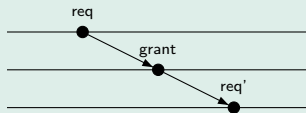
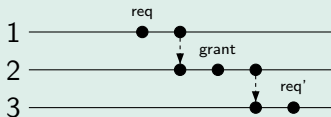
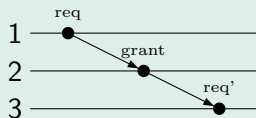
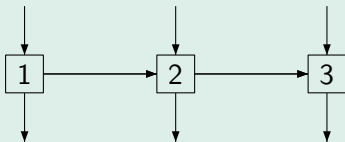
# Specifications

Input events are not controllable by processes



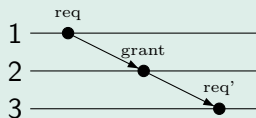
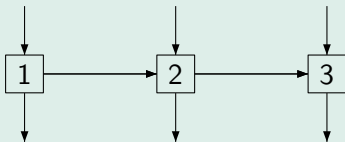
# Specifications

Input events are not controllable by processes



# Specifications

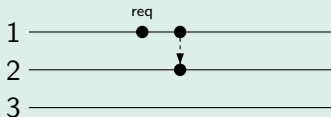
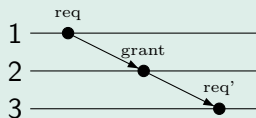
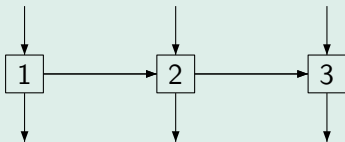
Input events are not controllable by processes





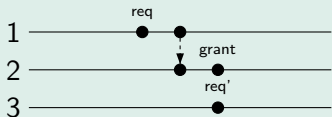
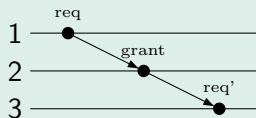
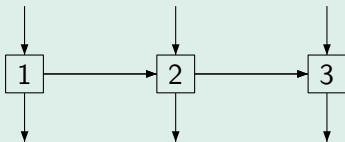
# Specifications

Input events are not controllable by processes



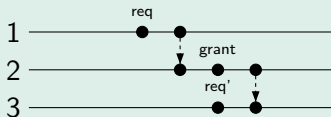
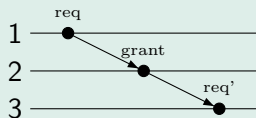
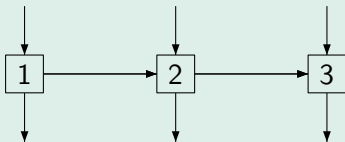
# Specifications

Input events are not controllable by processes



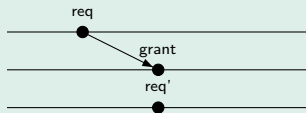
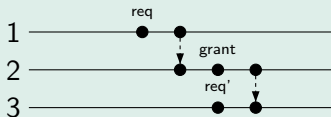
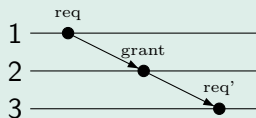
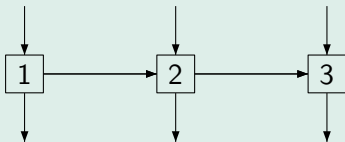
# Specifications

Input events are not controllable by processes



# Specifications

Input events are not controllable by processes



# Specifications

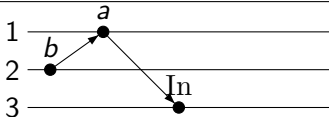
## Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions
- Specifications should not discriminate between a partial order and its "weakenings"

# Specifications

## Restrictions on specifications

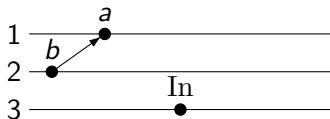
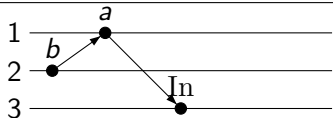
- Specifications should not discriminate between a partial order and its order extensions
- Specifications should not discriminate between a partial order and its "weakenings"



# Specifications

## Restrictions on specifications

- Specifications should not discriminate between a partial order and its order extensions
- Specifications should not discriminate between a partial order and its "weakenings"



# Outline

- 1 Introduction
- 2 The model
- 3 Decidability Results**



# Decidability Results

## Theorem

The synthesis problem over singleton architectures is decidable for regular specifications.

# Decidability Results

## Theorem

The synthesis problem over singleton architectures is decidable for regular specifications.

## Theorem

The distributed synthesis problem over strongly connected architectures is decidable for acceptable specifications.

# Decidability Results

## Theorem

The synthesis problem over singleton architectures is decidable for regular specifications.

## Theorem

The distributed synthesis problem over strongly connected architectures is decidable for acceptable specifications.

## Proof

By reduction to the singleton case.

# Strongly connected architectures

## Proposition

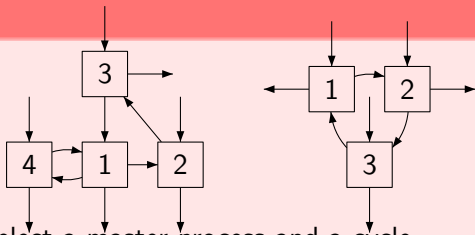
If there is a winning strategy over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected one.

# Strongly connected architectures

## Proposition

If there is a winning strategy over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected one.

## Proof



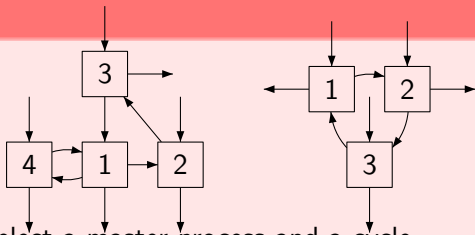
- We select a master process and a cycle.

# Strongly connected architectures

## Proposition

If there is a winning strategy over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected one.

## Proof



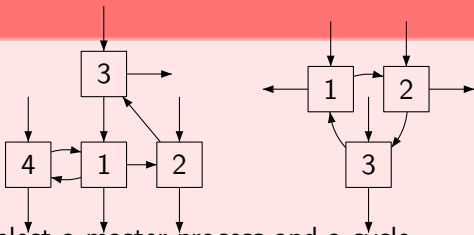
- We select a master process and a cycle.
- Master process will centralize information and tell other processes which value to output

# Strongly connected architectures

## Proposition

If there is a winning strategy over the singleton architecture then one can define internal signals sets and a distributed winning strategy for the strongly connected one.

## Proof

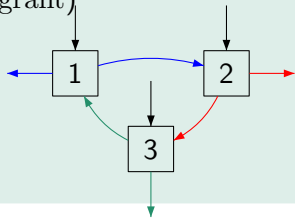


- We select a master process and a cycle.
- Master process will centralize information and tell other processes which value to output
- Objective: create a run that will be a **weakening** of a  $f$ -run over the singleton

# Centralize information

## Example

$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

Master collect information by sending a signal `Msg` through the cycle

1 \_\_\_\_\_

$t$ : 2 \_\_\_\_\_

3 \_\_\_\_\_

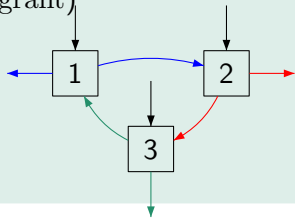
$t'$ : \_\_\_\_\_



# Centralize information

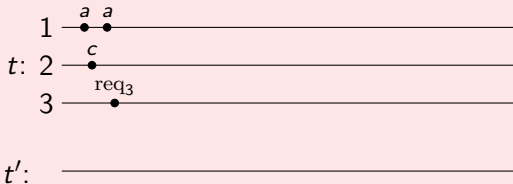
## Example

$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

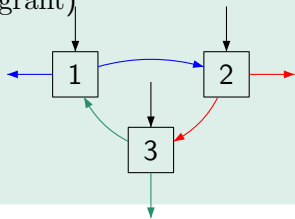
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Centralize information

## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

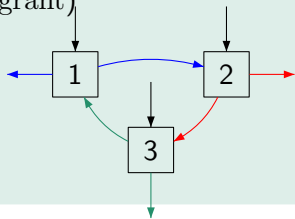
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Centralize information

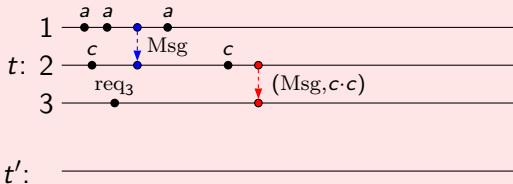
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

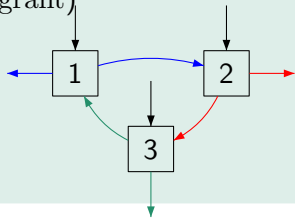
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Centralize information

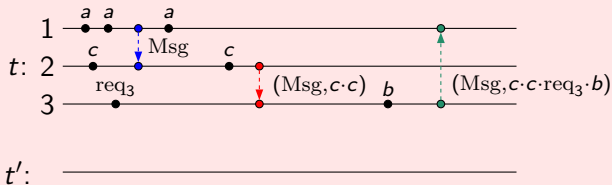
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

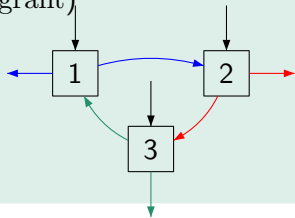
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Centralize information

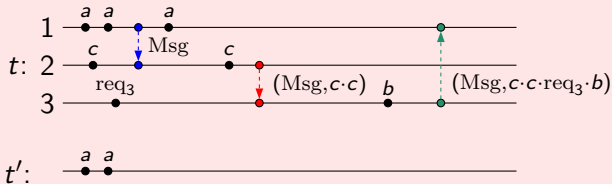
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

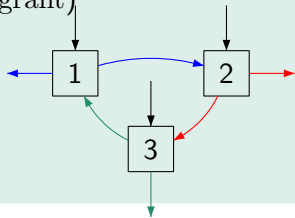
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Centralize information

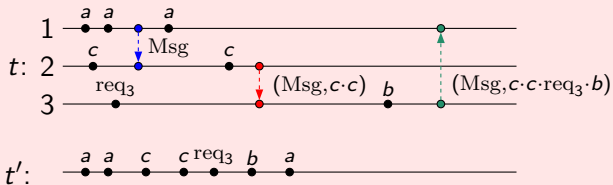
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_3$  and no alert

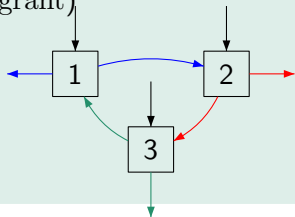
Master collect information by sending a signal  $\text{Msg}$  through the cycle



# Tell processes what to output

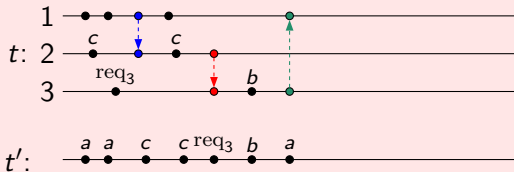
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

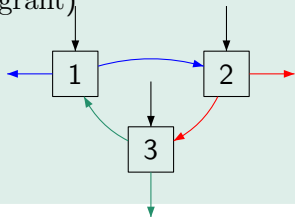
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output

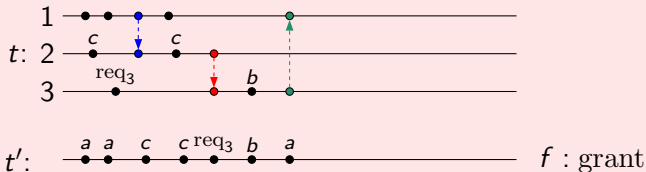
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

Master sends orders to other processes to simulate the strategy  $f$

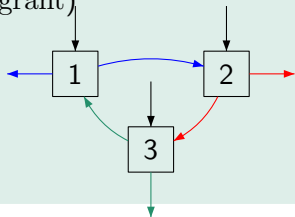




# Tell processes what to output

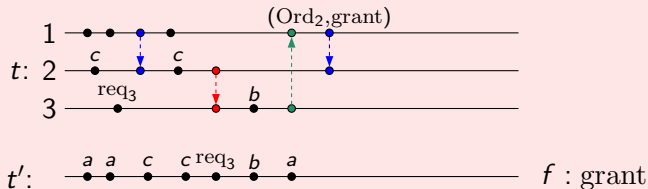
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

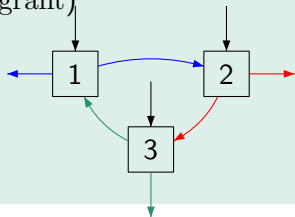
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output

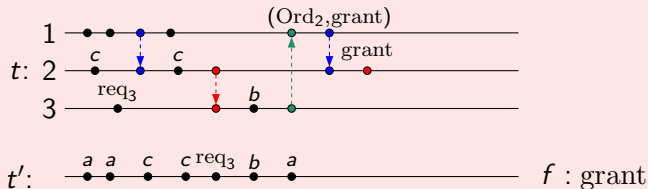
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

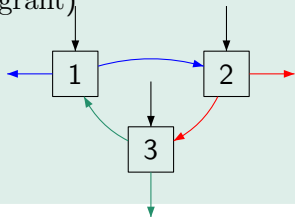
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output

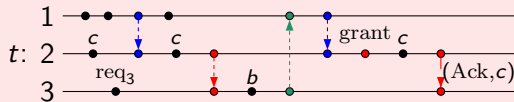
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

Master sends orders to other processes to simulate the strategy  $f$

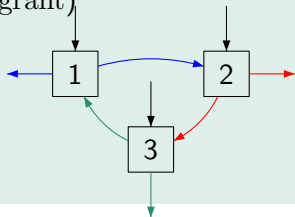


$t'$ :  $\bullet \bullet \bullet \bullet \bullet \bullet \bullet$   $f : \text{grant}$

# Tell processes what to output

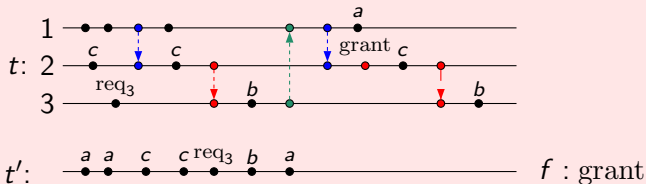
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

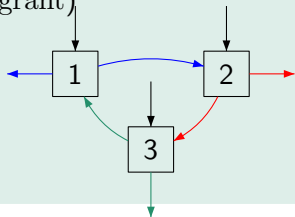
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output

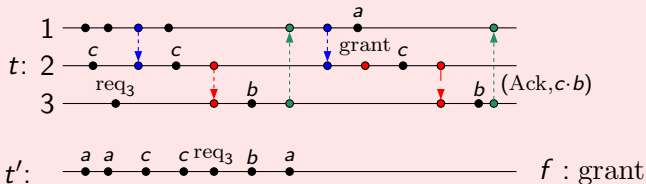
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

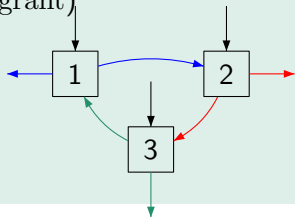
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output

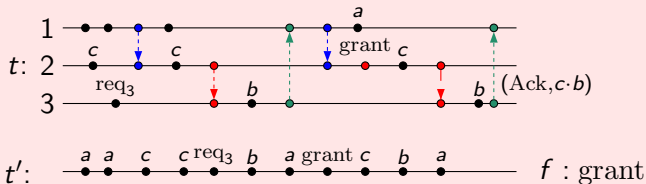
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

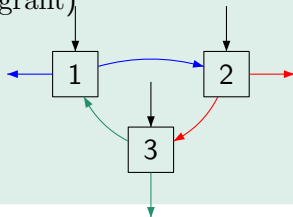
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output (2)

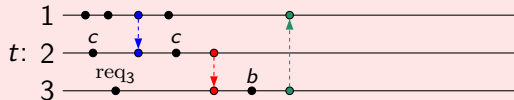
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

Master sends orders to other processes to simulate the strategy  $f$

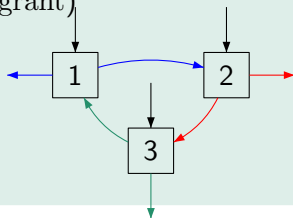


$t'$ :  $a \ a \ c \ c \ \text{req}_3 \ b \ a$        $f : \text{grant}$

# Tell processes what to output (2)

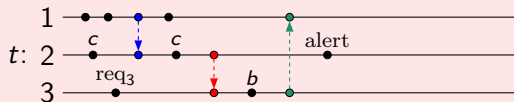
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

Master sends orders to other processes to simulate the strategy  $f$



$t'$ :  $\bullet a \bullet a \bullet c \bullet c \bullet \text{req}_3 \bullet b \bullet a$        $f : \text{grant}$

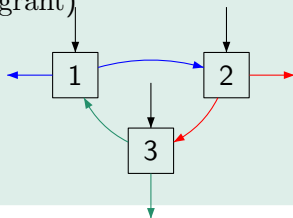




# Tell processes what to output (2)

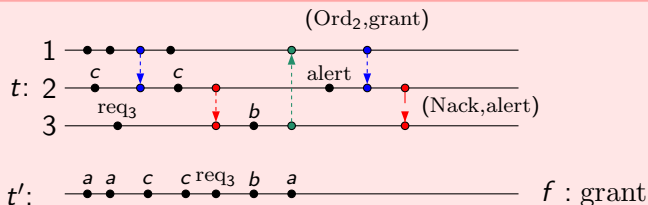
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

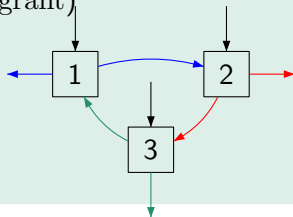
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output (2)

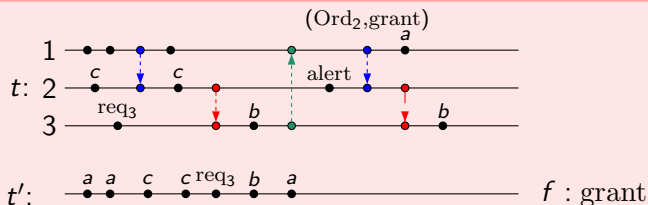
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

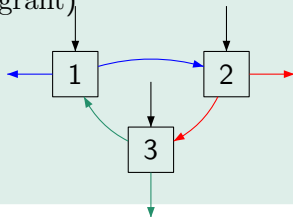
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output (2)

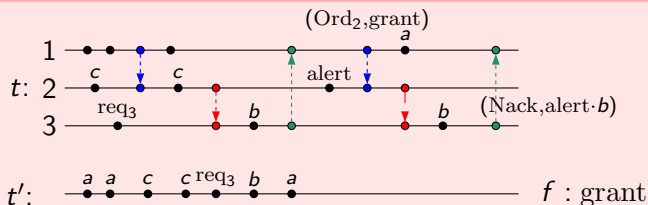
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

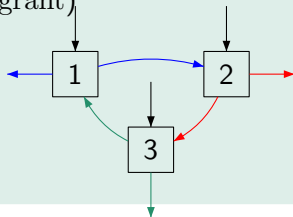
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output (2)

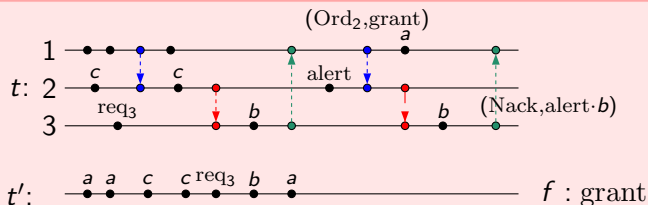
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

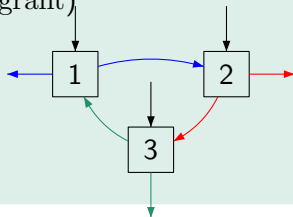
Master sends orders to other processes to simulate the strategy  $f$



# Tell processes what to output (2)

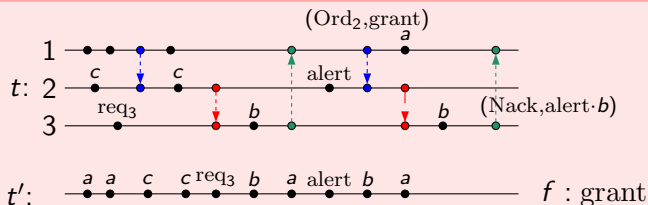
## Example

$$\text{req}_3 \rightarrow F_{32}(\neg Y_2 \text{ alert} \leftrightarrow \text{grant})$$



$f(\sigma) = \text{grant}$  iff  $\sigma$  contains  $\text{req}_2$  and no alert

Master sends orders to other processes to simulate the strategy  $f$



# Proof - end

## Lemma

$t'$  is an  $f$ -run.

## Lemma

$\pi_{\Gamma}(t)$  is a weakening of  $t'$ .

## Conclusion

Then  $t' \models \varphi$  and, by closure property  $\pi_{\Gamma}(t) \models \varphi$ .

# Proof - end

## Lemma

$t'$  is an  $f$ -run.

## Lemma

$\pi_{\Gamma}(t)$  is a weakening of  $t'$ .

## Conclusion

Then  $t' \models \varphi$  and, by closure property  $\pi_{\Gamma}(t) \models \varphi$ .

## Proposition

If there are communication sets  $\Sigma_{i,j}$  for  $(i,j) \in E$  and a winning distributed strategy on the strongly connected architecture, then there is a winning strategy on the singleton.

## Proof

Easier.



# Conclusion

- Asynchrony removes undecidability causes
- We have defined a new model of communication
- We have identified a class of decidable architectures
- Hopefully, many more to come!