On Stateless Restarting Automata

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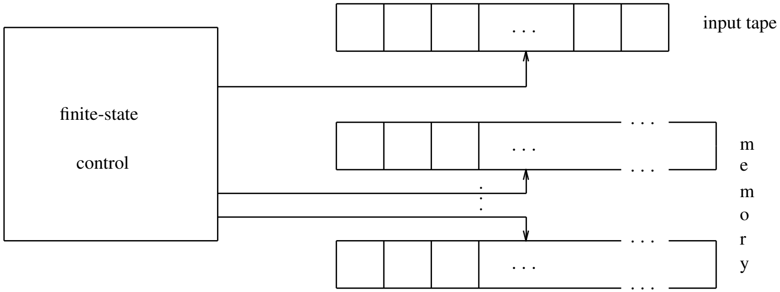
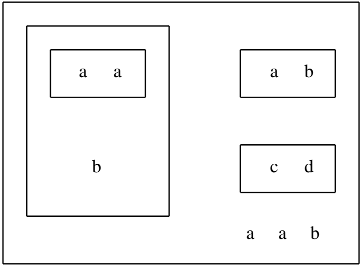
1. Introduction

Traditional automata:



Unconventional computing models:

P-systems:



membranes

DNA computing: multiset of DNA-strands

No global state.

Study traditional models of automata without states:

- How do other additional resources (heads, memory) given to finite automata relate to the absence or presence of states?
- For which computational models are states really necessary?

A stateless finite-state automaton with input alphabet Σ accepts Σ_0^* for a subset Σ_0 of Σ.

A language is accepted by a stateless pushdown automaton iff it is context-free.

A language is accepted by a stateless dpda iff it is a simple language [Har78].

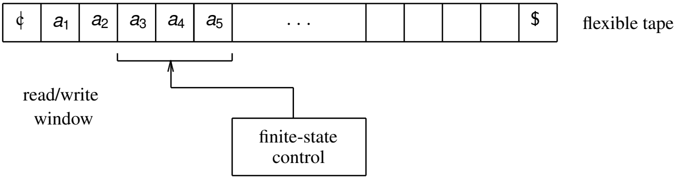
Stateless multihead finite automata and stateless multicounter systems have been investigated by O. Ibarra and his co-authors [IKO2007, YDI2007].

Theorem 1

$\forall k \geq 1 \exists$ finite language $L \subseteq \{a\}^*$: L is not accepted by any stateless two-way nondeterministic finite automaton with k heads.

Here: stateless restarting automata

2. Restarting Automata



A restarting automaton

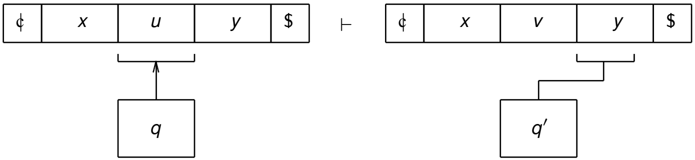
An RRWW-automaton is defined as $M = (Q, \Sigma, \Gamma, ¢, \$, q_0, k, \delta)$, where

- Q is a finite set of states,
- Σ is a finite input alphabet,
- Γ is a finite tape alphabet, $\Gamma \supseteq \Sigma$,
- $¢, \$ \notin \Gamma$ are the delimiters for the tape,
- $q_0 \in Q$ is the initial state,
- $k \in \mathbb{N}_+$ is the size of the read/write window,
- δ is the transition relation:
  $\delta : ¢ \cdot \Gamma^{k-1} \cup \Gamma^k \cup \Gamma^{\leq k-1} \cdot \$ \cup ¢ \cdot \Gamma^{\leq k-2} \cdot \$ \rightarrow P_{fin}(Ops)$.

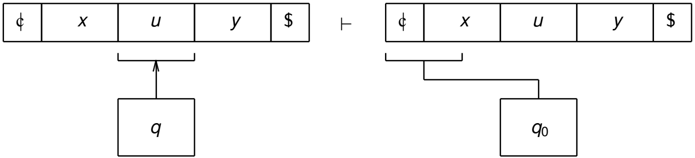
An RRWW-automaton can execute the following operations:

– A move-right step $(q', \text{MVR}) \in \delta(q, u)$

– A rewrite step $(q', v) \in \delta(q, u)$, where $|v| < |u|$:



– A restart step:



– An accept step Accept $\in \delta(q, u)$

In each computation rewrite and restart steps alternate.

M works in cycles:

$q_0¢xuyz\$ \vdash^* ¢xquyz\$ \vdash ¢xvq_1yz\$ \vdash^* ¢xvyq_2z\$ \vdash q_0¢xvyz\$$

restarting configuration ... restarting configuration

Notation: $xuyz \vdash^c_M xvyz$.

Input configuration for $w \in \Sigma^*$: $q_0 ¢ w \$$

$L(M) = \{ w \in \Sigma^* \mid M$ accepts on input $w \}$ is the input language of M,

$L_C(M) = \{ x \in \Gamma^* \mid M$ accepts starting from $q_0 ¢ x \$ \}$
is the characteristic language of M,

M is deterministic, if δ is a function.

M is monotone, if in any computation of M, the distance of the place of rewriting to the right delimiter \$ does not increase from one cycle to the next.

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M is an RWW-automaton, if each rewrite step is combined with a restart step.

M is an R(R)W-automaton, if $\Sigma = \Gamma$ (i.e., no auxiliary symbols), and

M is an R(R)-automaton, if v is a (scattered) subword of u for all $(q', v) \in \delta(q, u)$.

Theorem 2 (Jančar,Mráz,Plátek,Vogel et al.)

(a) $\mathcal{L}$(det-mon-R(R)) $=$ $\mathcal{L}$(det-mon-R(R)WW) $=$ DCFL

(b) $\mathcal{L}$(mon-R(R)WW) $=$ CFL

(c) $\mathcal{L}$(det-R(R)WW) $=$ CRL $\subsetneq$ GCSL $\subsetneq$ $\mathcal{L}$(RWW) $\subseteq$ $\mathcal{L}$(RRWW)

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(a) $\mathcal{L}(\text{det-mon-R(R)}) = \mathcal{L}(\text{det-mon-R(R)WW}) = \text{DCFL}$

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(c) $\mathcal{L}(\text{det-R(R)WW}) = \text{CRL} \subsetneq \text{GCSL} \subsetneq \mathcal{L}(\text{RWW}) \subseteq \mathcal{L}(\text{RRWW})$

3. Stateless RWW-Automata

An RWW-autom. $M = (Q, \Sigma, \Gamma, ¢, \$, q_0, k, \delta)$ is stateless if $Q = \{q_0\}$.

Notation: $M = (\Sigma, \Gamma, ¢, \$, k, \delta)$.

Example

$\Sigma = \{a, b\}$, δ defined as follows:

$\delta(¢\$)$	$=$	Accept,	$\delta(¢aaa)$	$=$	MVR,	$\delta(aaaa)$	$=$	MVR,
$\delta(¢ab\$)$	$=$	Accept,	$\delta(¢aab)$	$=$	MVR,	$\delta(aaab)$	$=$	MVR,
						$\delta(aabb)$	$=$	ab.

M is a stateless R-automaton, it is deterministic and monotone, and $L(M) = \{ a^n b^n \mid n \geq 0 \}$.

Proposition 3

Given a stateless RWW*-automaton* $M = (\Sigma, \Gamma, ¢, \$, k, \delta)$*, a stateless* RWW*-automaton* $M' = (\Sigma, \Gamma, ¢, \$, k+1, \delta')$ *can be constructed such that the following properties hold:*

(1) M' *executes accept instructions only at the right end of the tape,*

(2) $L_{\mathrm{C}}(M') = L_{\mathrm{C}}(M)$,

(3) *if all rewrite instructions of* M *are deletions, then this also holds for* M'*, and*

(4) *if* M *is deterministic, then so is* M'*.*

Proposition 4

REG $\subsetneq \mathcal{L}$(stl-det-mon-R)

$L_d = \{ ca^n b^n \mid n \geq 0 \} \cup \{ da^n b^{2n} \mid n \geq 0 \}$

L_d is a deterministic linear language, that is,
it is accepted by a deterministic one-turn PDA.

Lemma 5

L_d is not accepted by any stateless RW*-automaton.*

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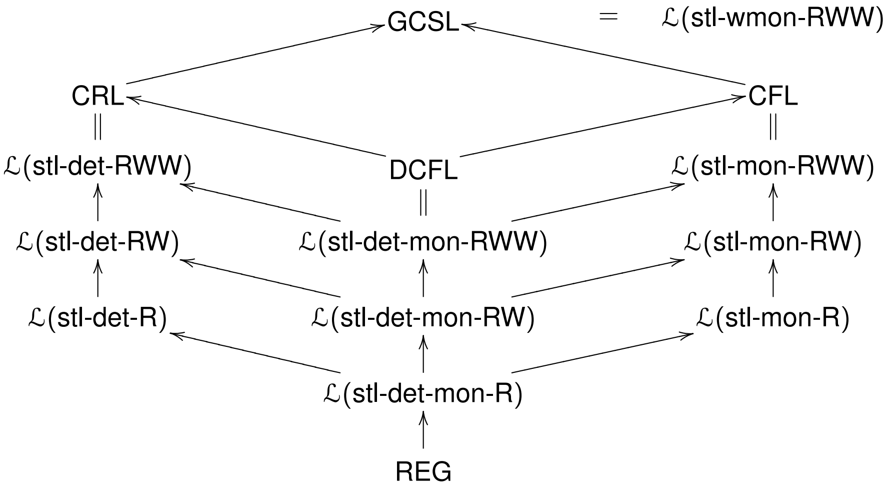
GCSL
=
$\mathcal{L}$(stl-wmon-RWW)
CRL
CFL
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DCFL
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REG

4. Stateless RRWW-Automata

Let $M = (\Sigma, \Gamma, \cent, \$, k, \delta)$ be a stateless RRWW-automaton.
Any attempt to perform a second rewrite step within a cycle is interpreted as a reject step.

Lemma 6

(a) $\{ a^n b^n \mid n \geq 0 \} \in \mathcal{L}(\text{stl-det-RR})$.

(b) $L_{aba} = \{ a^m b^{m+n} a^n \mid m, n \geq 0 \} \notin \mathcal{L}(\text{stl-det-RRW})$.

Proof idea for (b):

$$a^m b^m \in L_{aba} \quad : \quad \delta(a^\ell b^{k-\ell}) = a^{\ell-i} b^{k-\ell-i}$$
$$b^n a^n \in L_{aba} \quad : \quad \delta(b^{\ell'} a^{k-\ell'}) = b^{\ell'-i'} a^{k-\ell'-i'}$$

$$a^m b^{m+n} a^n \in L_{aba} : \underline{a^m b^{m+n}} a^n \vdash a^{m-i} b^{m-i+n} a^n \vdash^*$$
$$a^{m-i} b^{m-i} \underline{b^n a^n} \vdash a^{m-i} b^{m-i} b^{n-i'} a^{n-i'} : \text{Reject.}$$

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□

Corollary 7

$\mathrm{REG} \subsetneq \mathcal{L}(\text{stl-det-RR})$

Proof outline:

Let $L \in \mathrm{REG}(\Sigma)$, and let $A = (Q, \Sigma, q_0, F, \varphi)$ be a complete DFA for L.
If $m = |Q|$, then each word $w \in \Sigma^m$ can be written as $w = w_1 w_2 w_3$ such that $|w_2| \geq 1$ and $\varphi(q_0, w_1 w_2) = \varphi(q_0, w_1)$.
Hence, for all $z \in \Sigma^*$, $wz \in L$ iff $w_1 w_3 z \in L$.

Define a stl-det-RR-automaton M on Σ with window size $k = m + 1$:

(1) $\delta(¢ w\$) = \text{Accept}$ for all $w \in \Sigma^{<m} \cap L$,

(2) $\delta(¢ w) = ¢ w_1 w_3$ for all $w \in \Sigma^m$, where $w = w_1 w_2 w_3$ is the chosen factorization of w,

(3) $\delta(w) = \text{MVR}$ for all $w \in \Sigma^{m+1}$,

(4) $\delta(w\$) = \text{RESTART}$ for all $w \in \Sigma^{<m}$.

Then M is monotone, and $L(M) = L$. □

Theorem 8

$\mathcal{L}(\text{stl-det-mon-RRWW}) = \text{DCFL}$ *and* $\mathcal{L}(\text{stl-mon-RRWW}) = \text{CFL}$.

Proof outline:

(a) Let $L \subseteq \Sigma^+$ be context-free. Then $L = L(P)$, where $P = (Q, \Sigma, \Gamma, \delta, q_0, z_0)$ is a PDA without λ-transitions that accepts by empty pushdown.

(b) Here we can assume that L is accepted by a DPDA P for which each λ-transition simply pops a symbol from the pushdown [Autebert,Berstel,Boasson 97]. □

Theorem 9

$\mathcal{L}(\text{stl-det-RRWW}) = \text{CRL}$.

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5. Stateless Two-Phase RRWW-Automata

A stateless two-phase RRWW-automaton (stl-2-RRWW) is described by a 7-tuple $M = (\Sigma, \Gamma, ¢, \$, k, \delta_1, \delta_2)$:

δ_1 : transition relation for the first phase of a cycle,

δ_2 : transition relation for the second phase of a cycle.

Lemma 10

(a) $\mathcal{L}$(stl-(det)-R(W)(W)) $\subseteq$ $\mathcal{L}$(stl-(det)-2-RR(W)(W))

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Theorem 11

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$$L_d = \{ ca^n b^n \mid n \geq 0 \} \cup \{ da^n b^{2n} \mid n \geq 0 \} \notin \mathcal{L}(\text{stl-2-RRW})$$

Corollary 12

$\mathcal{L}(\text{stl-(det)-2-RR(W)}) \subsetneq \mathcal{L}(\text{(det)-RR(W)})$

$$L^{(1)}_{\text{expo}} = \{ a^{2^n} \mid n \geq 0 \} \cup \{ a^i b a^j \mid i, j \geq 0, \exists m \geq 1 : i + 2 \cdot j = 2^m \}$$

Lemma 13

$L^{(1)}_{\text{expo}} \in \mathcal{L}(\text{stl-det-2-RRW})$

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Lemma 14

$L^{(1)}_{\text{expo}} \notin \mathcal{L}(\text{stl-RW}) \cup \mathcal{L}(\text{stl-det-RRW})$

Proof idea:

Let M be a stl-RW-automaton for $L^{(1)}_{\text{expo}}$:

If M accepts ba^{2^n} in a tail, then it also accepts $ba^{2^n+1} \notin L^{(1)}_{\text{expo}}$.

If $ba^{2^n} \vdash^c_M w$, then $w = a^{2^n}$, and $ba^{2^n}ba^{2^{n-1}} \vdash^c_M a^{2^n}ba^{2^{n-1}} \in L^{(1)}_{\text{expo}}$.

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This contradicts the determinism of M. □

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Lemma 13 and Lemma 14 imply the following.

Corollary 15

(a) $\mathcal{L}(\text{stl-det-RW}) \subsetneq \mathcal{L}(\text{stl-det-2-RRW})$

(b) $\mathcal{L}(\text{stl-det-RRW}) \subsetneq \mathcal{L}(\text{stl-det-2-RRW})$

Corollary 16

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Lemma 17

$\overline{L}^{(1)}_{\text{expo}} := \{a, b\}^* \smallsetminus L^{(1)}_{\text{expo}}$ *is not accepted by any stateless deterministic* 2-RRW*-automaton that executes accept instructions only at the right end of the tape.*

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6. Concluding Remarks

The following (non-) closure properties have been obtained for the language families that are specified by stateless deterministic restarting automata:

	Operation								
	$\cup$	$\cap$	$\sim$	$\cap_{reg}$	R	$\cdot$	$*$	h^{-1}	h_λ
$\mathcal{L}$(stl-det-R)	–	–	+	–	–	–	–	–	–
$\mathcal{L}$(stl-det-RW)	–	–	+	–	–	–	–	–	–
$\mathcal{L}$(stl-det-RR)	–	–	–	–	–	–	–	–	–
$\mathcal{L}$(stl-det-RRW)	–	–	–	–	–	–	–	–	–
$\mathcal{L}$(stl-det-2-RR)	–	–	+	–	–	–	–	–	–
$\mathcal{L}$(stl-det-2-RRW)	–	–	+	–	–	–	–	–	–

Open Problems:

- What is the exact relationship between $\mathcal{L}$(stl-det-R(W)) and $\mathcal{L}$(stl-det-RR(W)) ?
- Is the expressive power of stl-RRWW-automata changed, if we require them to perform accept steps only at the right end of the tape ?
- Closure properties for language families accepted by non-deterministic types of stateless restarting automata ?
- What is the expressive power of stateless RRWW-automata that are allowed to perform several rewrite steps per cycle ?